2D Z. Trafo

$$X(z_1, z_2) = \sum_{n_1=-\infty}^{+\infty} \sum_{n_2} x(n_1, n_2)\, z_1^{-n_1}\, z_2^{-n_2}$$

2D. DTFT $\{x(n_1, n_2)\} = X(\omega_1, \omega_2) = \left[X(z_1, z_2) \right]_{z_1 = e^{j\omega_1},\ z_2 = e^{j\omega_2}}$

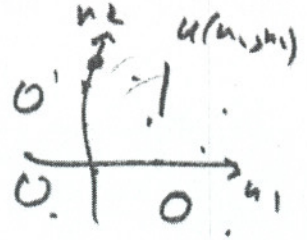
unit bi-circle.

$z = re^{j\omega}$ → In 1-D ROC only depends on r, not ω.

in 2D ROC depends r_1, r_2.

Ex $x(n_1, n_2) = a^{n_1} b^{n_2} u(n_1, n_2)$

$$X(z_1, z_2) = \sum_{n_1} \sum_{n_2} a^{n_1} b^{n_2} z_1^{-n_1} z_2^{-n_2}$$

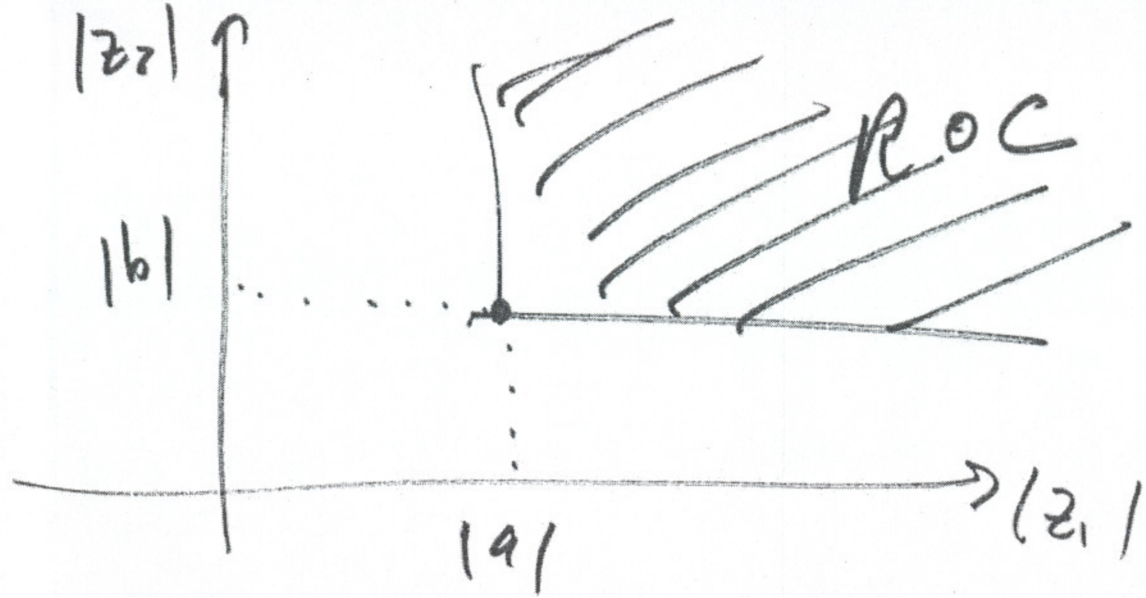


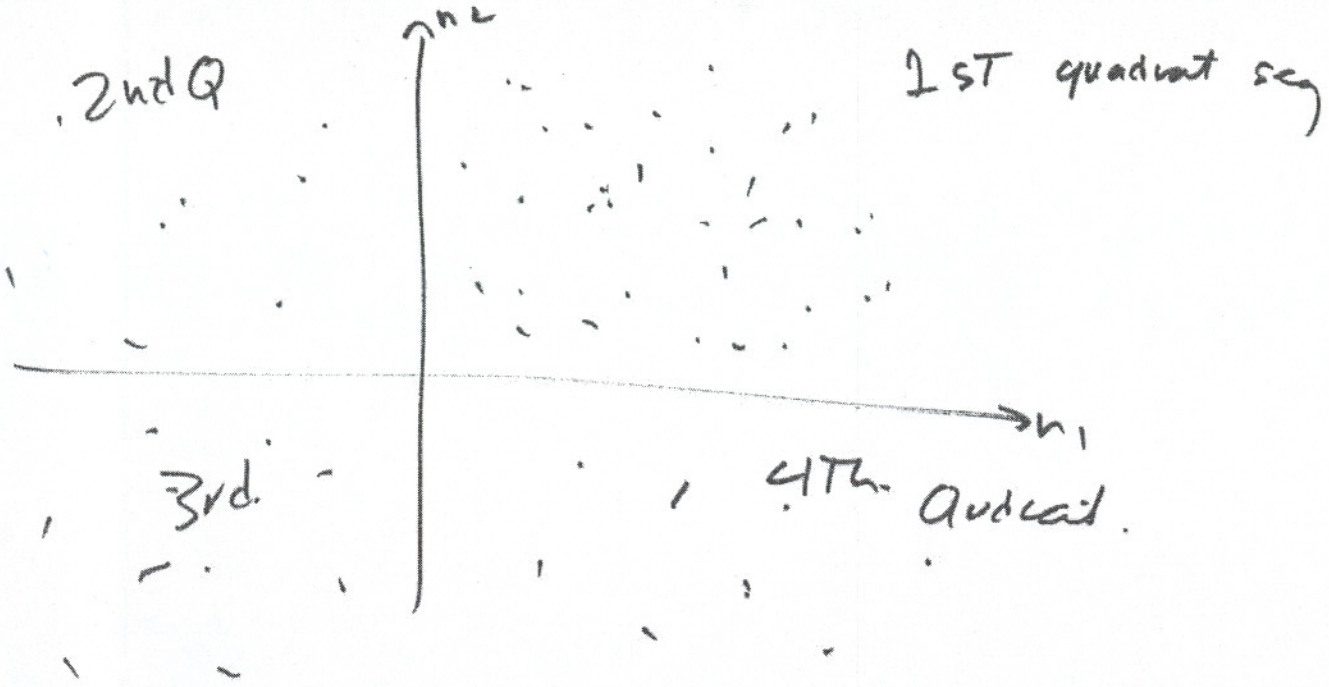
$$= \frac{1}{1 - b z_2^{-1}} \cdot \frac{1}{1 - a z_1^{-1}}$$

ROC: $|z_2| > |b|$ and $|z_1| > |a|$

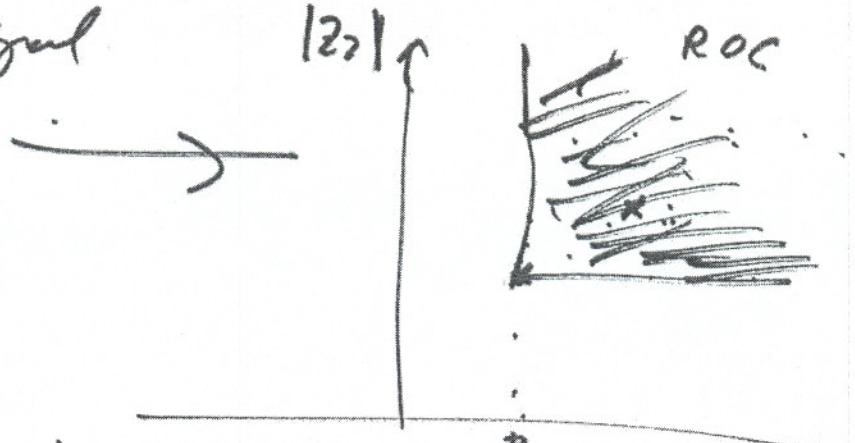
$$\left(\sum_{n_2=0}^{\infty} b^{n_2} z_2^{-n_2} \right) \left(\sum_{n_1=0}^{\infty} a^{n_1} z_1^{-n_1} \right)$$

→ Region convergence.


|z2|
|b|
ROC
|a|
|z1|


2nd Q
1st quadrant sq
n_2
n_1
3rd
4th Quad

If 1st quad signal

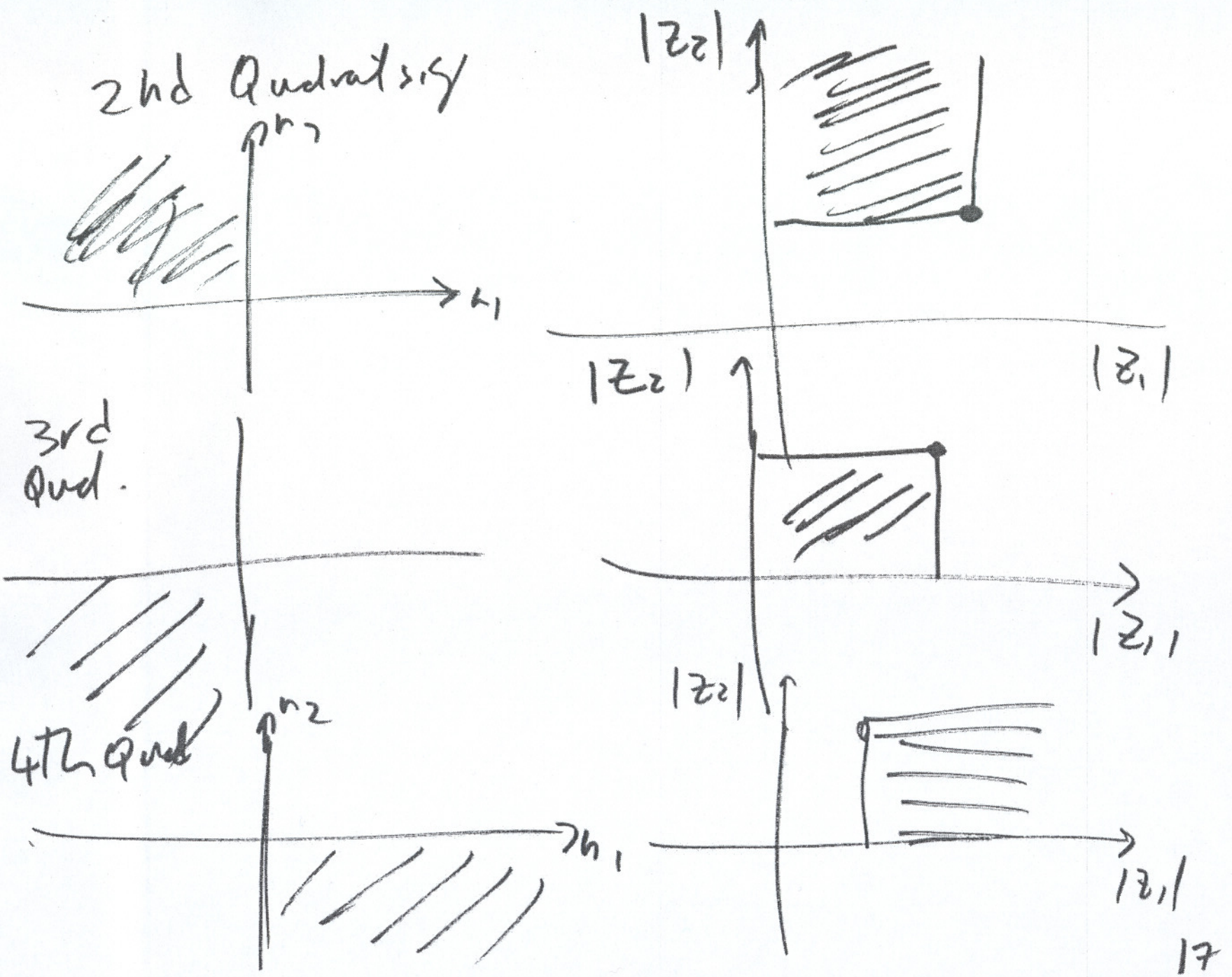


if point $|z_1'|, |z_2'|$ is in ROC

Then $\forall$ $|z_1| > |z_1'|$ and

$|z_2| > |z_2'| \Longrightarrow$ will also be in ROC

Ditto: for 2nd, 3rd, 4th Quadrant sig.


2nd Quadrant
n_2
n_1
3rd Quad.
4th Quad
n_2
n_1
$|z_2|$
$|z_1|$
$|z_2|$
$|z_1|$
$|z_2|$
$|z_1|$

Ex $x(n_1, n_2) = -a^{n_1} b^{n_2} u(-n_1 - 1, n_2 - 1)$