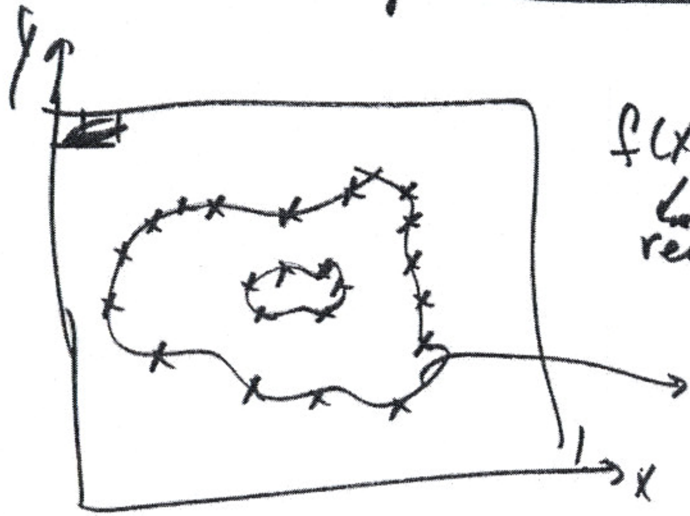
Recon of M-D. Signals from Zero Crossings

Last lecture: 1 bit of Phase of F.T. of signal $\longrightarrow$ reconstruct it

Curtis & Oppenheim
~ 1983-5



$f(x, y)$ = continuous space 2-D signal
real variables.

$\rightarrow$ contour represents "Level crossing" of signal

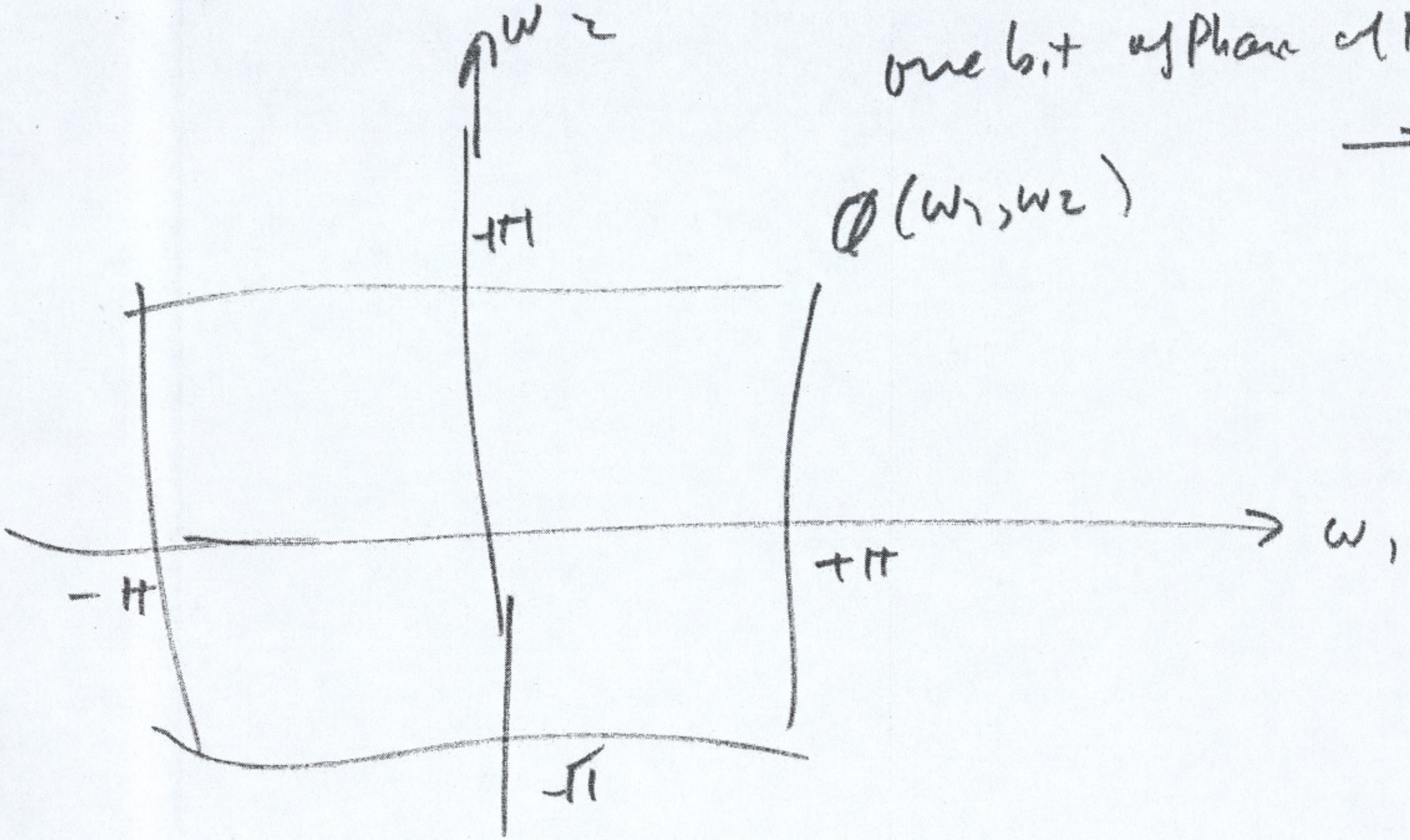
$\Longrightarrow$ dual of the problem

$$X(\omega_1, \omega_2) = |\quad| e^{j\phi(\omega_1, \omega_2)}$$

[illegible]$_x(n_1, n_2)$

one bit of Phase of F.T.

→ Van Hove.



$$f(x,y) = \sum_{k_1=-N}^{+N} \sum_{k_2=-N}^{+N} F(k_1,k_2)\, e^{j2\pi k_1 x}\, e^{j2\pi k_2 y}$$

continuous space signal

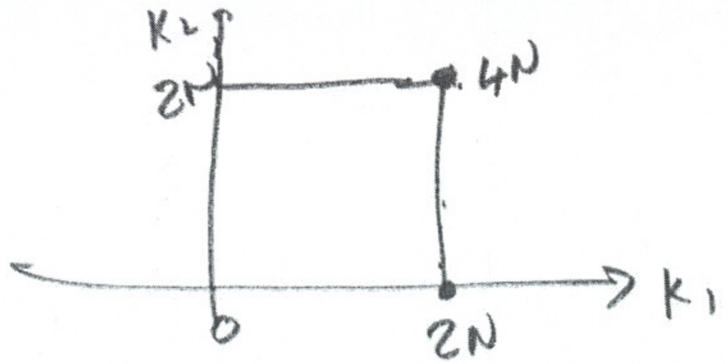
Fourier series coefficients.

is BLP = Bandlimited & Periodic

Change of variables $e^{j2\pi x} = W \quad e^{j2\pi y} = Z$

$$f(x,y) = \sum_{k_1=-N}^{+N} \sum_{k_2=-N}^{+N} F(k_1,k_2)\, W^{k_1} Z^{k_2}$$

polynomial of deg 2N in W
and deg 2N in Z



Bezout's thm: If 2 2D irreducible polynomials of

(max deg in w, z) ← degree r and s, then at most

they have rs common zeros.

$\Longrightarrow$ $16N^2$ or more zeros of $f(x,y)$ $\Longrightarrow$

Bezout $\longrightarrow$ uniquely reconstruct it

$\Longrightarrow$ Can $F(K_1,K_2)$ uniquely determined.

Sample $f(x,a)$ at its $16N^2$ zero crossing.

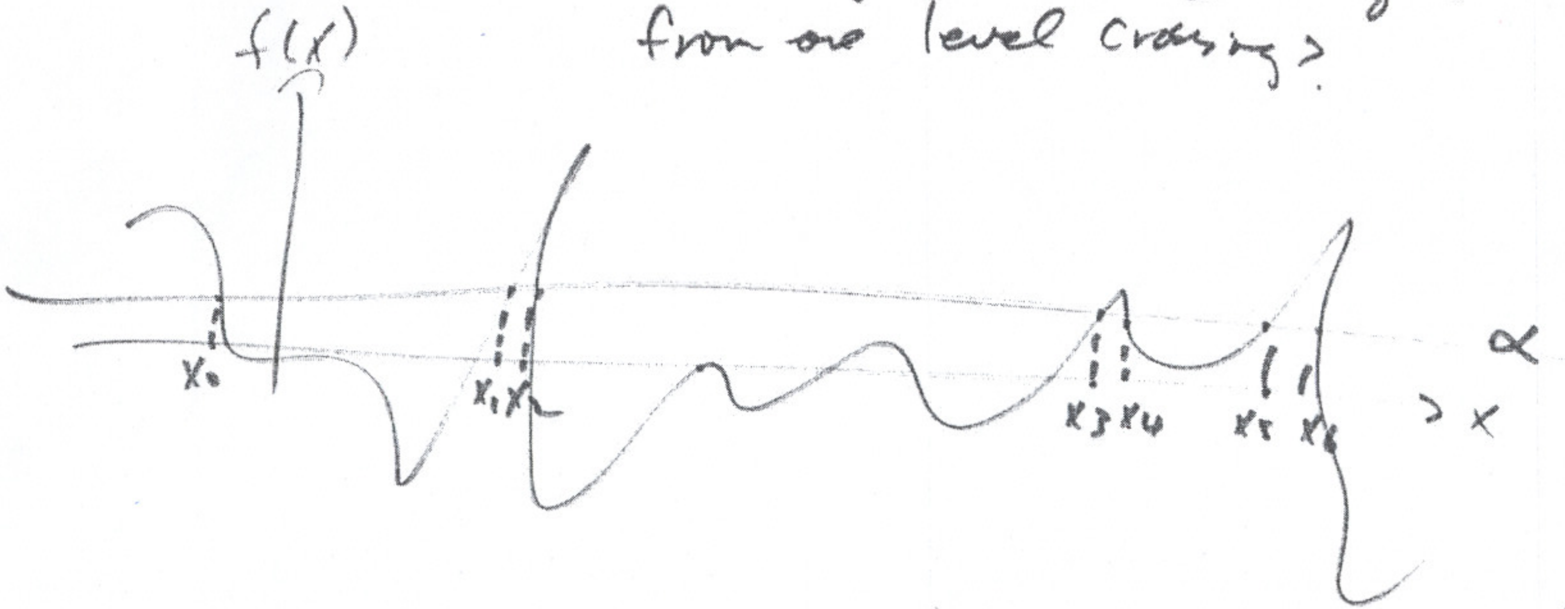
(x_0,y_0)
(x_1,y_1)
(x_2,y_2)
$\vdots$
(x_{16N^2}, y_{16N^2})

level crossing = α

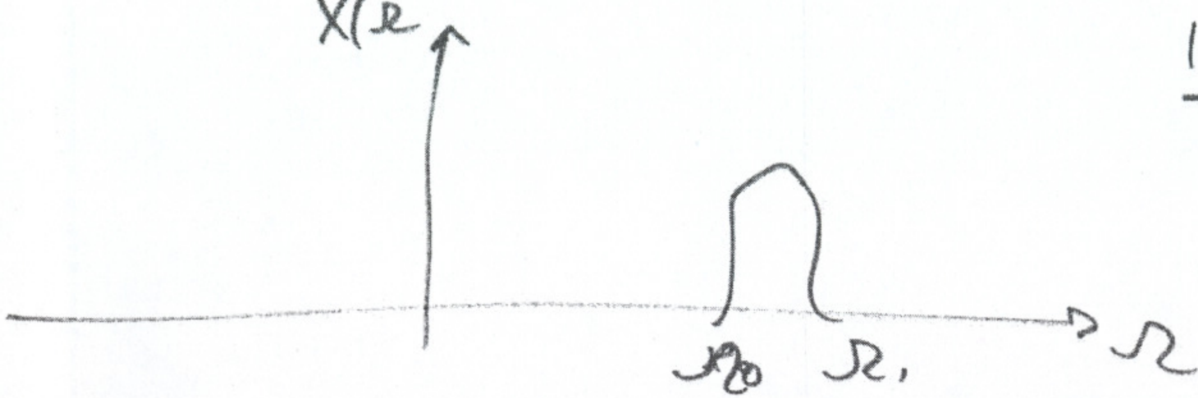
$$16N^2\updownarrow\begin{bmatrix}\alpha\\ \alpha\\ \alpha\\ \vdots\\ \alpha\end{bmatrix} = \begin{array}{r} (x_0,y_0)\rightarrow \\ (x_1,y_1)\rightarrow \\ \\ \\ \end{array} \underbrace{\left[\begin{array}{cc|c} e^{j2\pi 0x_0} & e^{j2\pi 0\cdot y_0} & e^{j2\pi} \cdots \\ e^{j2\pi a x_1} & e^{j2\pi 0 y_1} & \cdots \\ \vdots & & \\ \vdots & & \cdots \end{array}\right]}_{4N^2} \begin{bmatrix}F(0,0)\\ F(0,1)\\ \vdots\\ F(2N,2N)\end{bmatrix}$$

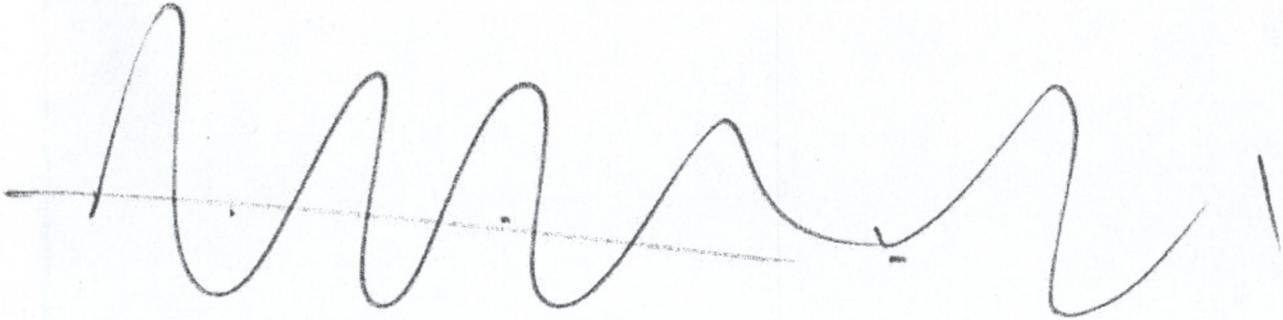
To get reasonable recon ⟶
need to know the location of zero crossings
To very high precision
or else ⟶ garbage out.
⟹ Problem is ill conditioned

<u>1D case</u> : Can you recon 2D signal from one level crossing?



Logan thm : Band pass 1-D signals That are Band limited under certain condition can be reconstructed from level crossin 1950-1960.





Multiple level crossing

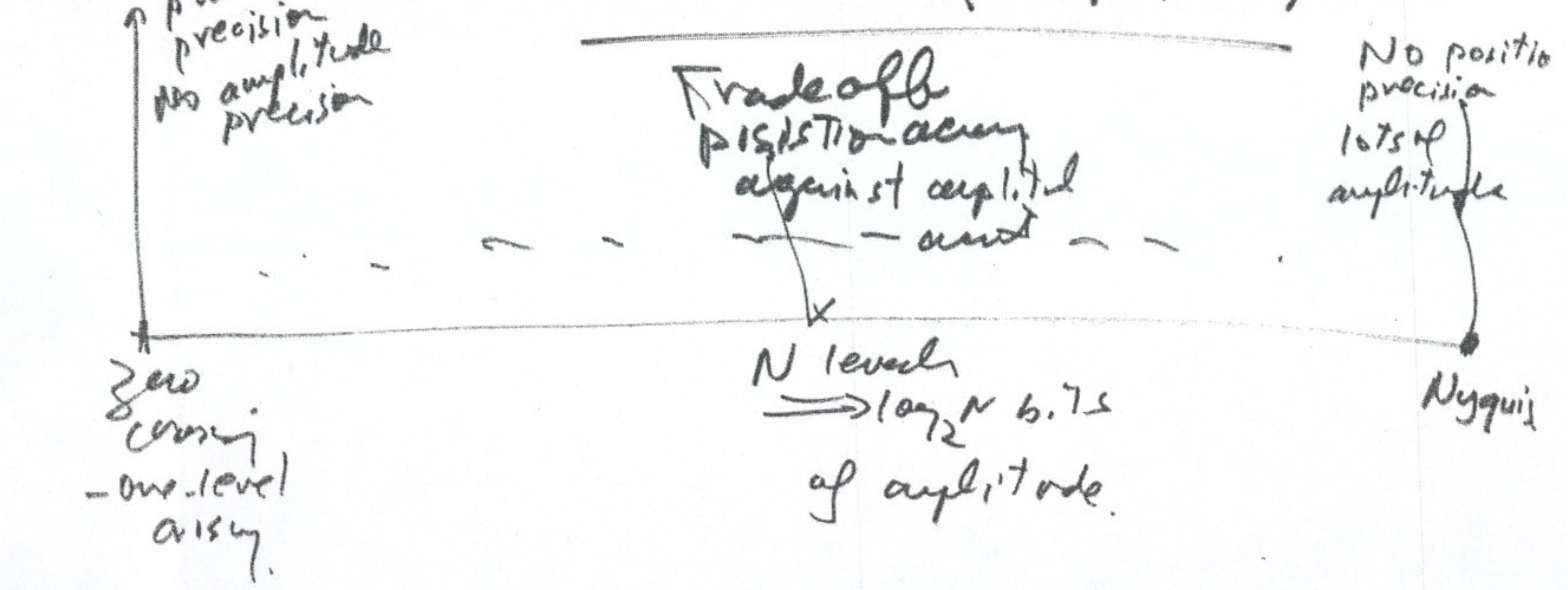
⟹ relax The precision requisits for The location of The sample

① Semi-implicit.

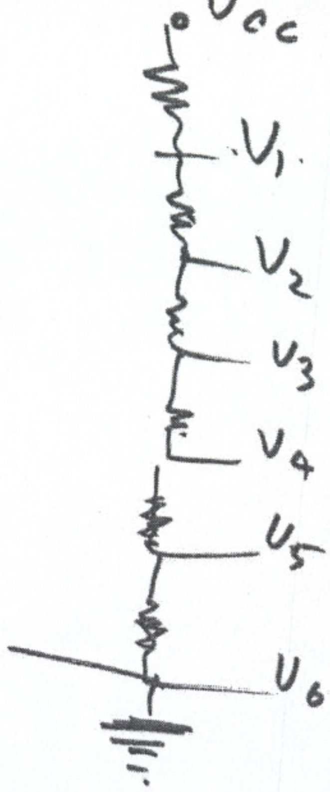
lines of rational slopes.

② Implicit ← Algebraic geometry

Spectrum of Sampling Techniques



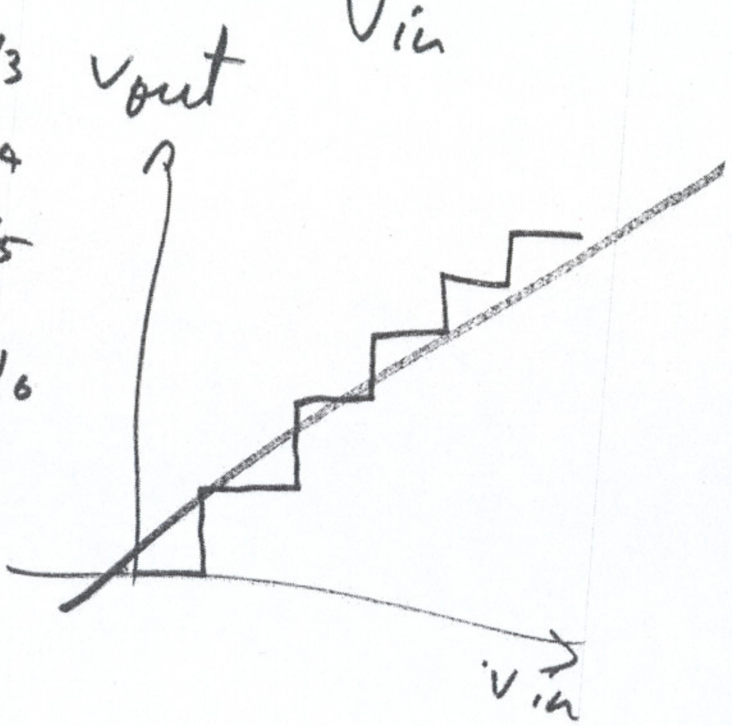
A/D

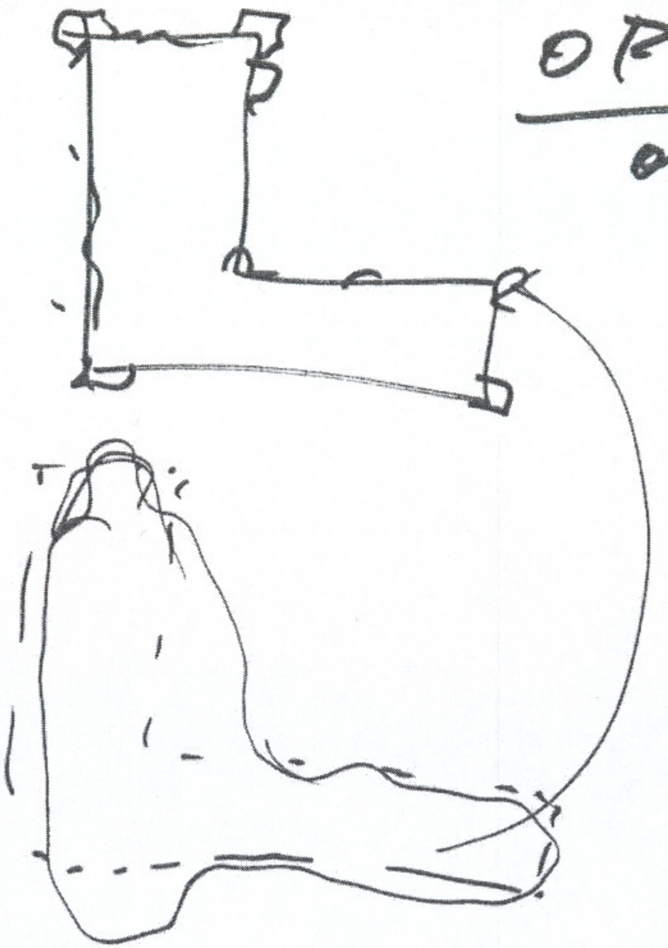


ΣΔ :

Tradeoffs:
sample rate
against
amplitude

Amp : 16.4.
oversample ∴ larger.





OPC

optical prox correction