Dictionary Techniques

- Huffman and arithmetic coding assumed i.i.d. sources
- Most sources are correlated
- Main idea: 1 Build a list of commonly occuring patterns
  2 Transmit index in the list.
- Exploits fact that certain patterns recur frequently.
- Consider 2 cases
  1. Static Dictionary
  2. Dynamic Dictionary

<u>Static Dictionary:</u>

- Suppose Five letter alphabet source:

$$A = \{a, b, c, d, r\}$$

- Using statistics of source, build dictionary

ad	ac	ab	r	d	c	b	a	Entry
111	110	101	100	011	010	001	000	Code

- Encode a b r a c a d a b r a

1. Read ab ---------> 101
2. Read ra ---------> not in dictionary
3. Read r ---------> 100
4. Read ac ---------> ~~0~~ 110
5. . . . .

101 100 110 111 101 101 000
ab r ac ad ab r a

- Opposite of Huffman coding:

Ziv

Adaptive Dictionary:

- ~~Zir~~ Lempel 1977 + 1978

- LZ1 -----> 1977 LZ2 -----> 1978

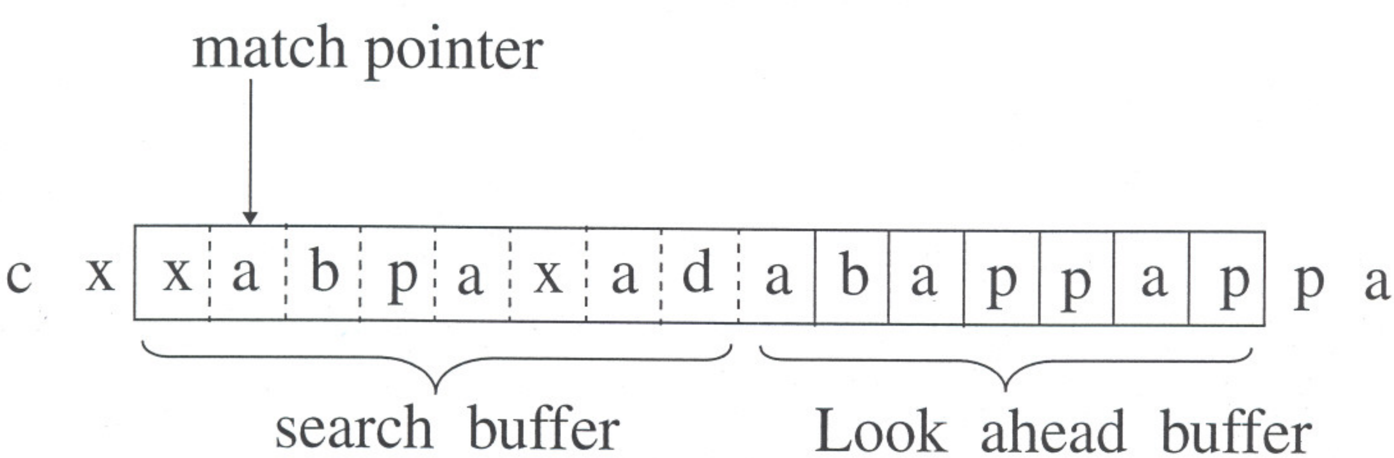
- LZ1 discussed Here.

- Basic idea:

  Dictionary portion of previously encoded sequence

- Sliding window:

  1) search buffer
  2) lookahead buffer



Example

. . . c a b r a c a d a b r a r r a r r a d . . .

- Window = 13
- look ahead buffer = 6
- Search buffer = 7

. . . . | c a b r a c a | d a b r a r | r a . . .

0 ← zero

1. No match to d ---> < Ø, 0, C(d) >

. . . . . c | a b r a c a d | a b r a r r | a r . . .

(letter o)

0

2. Match for a :

$$\begin{cases} Ø = 2 \text{ ---> } l = 1 \\ Ø = 4 \text{ ---> } l = 1 \\ \boxed{Ø = 7 \text{ ---> } l = \cancel{1}} \; 4 \end{cases}$$

< 7, 4, C (r) >

| a d a b r a r | r a r r a d |

t

3. Mach for r :

$$\begin{cases} 0 = 1 \dashrightarrow l = 1 \\ 0 = 3 \dashrightarrow l = 5 \end{cases}$$

$$< 3,\ 5,\ C\,(d) >$$

exceeds the
boundary between
search and
look ahead buffer

Encoding steps

1. Move search pointer back until match in search buffer

2. Offset $\triangleq$ Distance of pointer from look ahead buffer

3. Do consecutive symbols of pointer match also?

4. Search the search buffer for the longest match

5. length of match $\triangleq$ # of consecutive symbol match.

6. Send $< o, l, c\,)$

   o = offset

   l = match length

   C $\cancel{c}$ = codeword of symbol in LA buffer, following match

Example: < 7, 2, codeword for a >

Adaptive Dictionary

- Why send C ~~c~~?
    - just in case no match
- Total # of bits:

W → capital W

$$\lceil \log_2 S \rceil + \lceil \log_2 \cancel{w} \rceil + \lceil \log_2 A \rceil$$

S = size of search buffer

W = Size of window (search + LA)

A = size of source alphabet

What to Code (Classification of Image Coding Systems)

1. Waveform Coder (code the intensity)
   - PCM (Pulse Code Modulation) and its improvements
   - DM (Delta Modulation)
   - DPCM (Differential Pulse Code Modulation)
   - Two-channel Coder

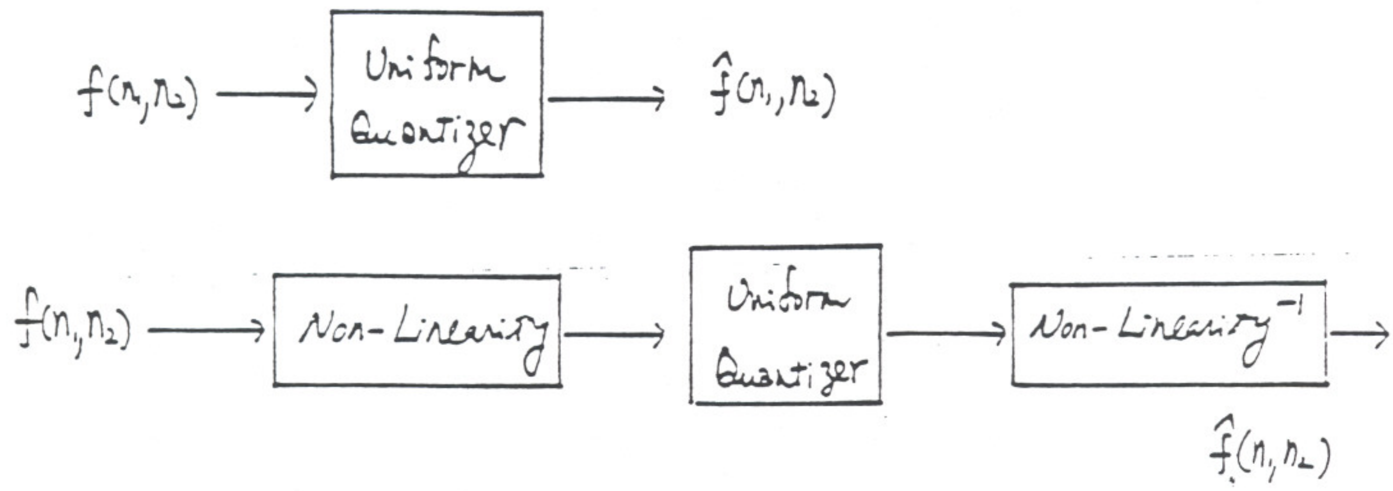
2. Transform Coder (code transform coefficients of an image)
   - Karhunen-Loeve Transform
   - Discrete Fourier Transform
   - Discrete Cosine Transform

3. Image Model Coder
   - Auto-regressive Model for texture
   - Modelling of a restricted class of images

Note: Each of the above can be made to be adaptive

Waveform Coder

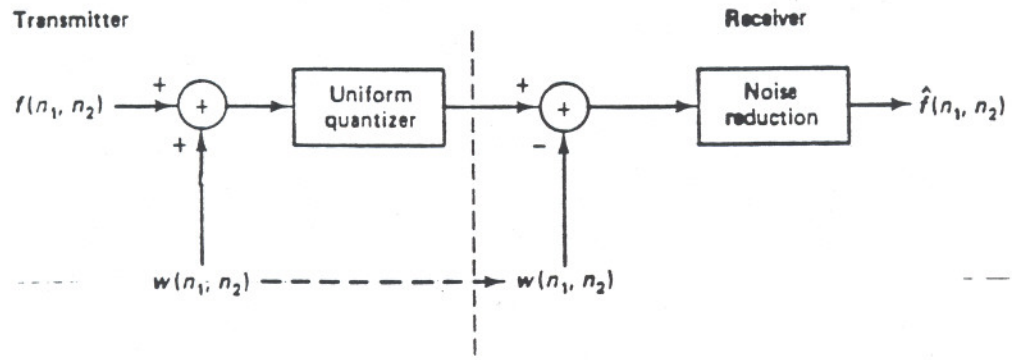
PCM Coding



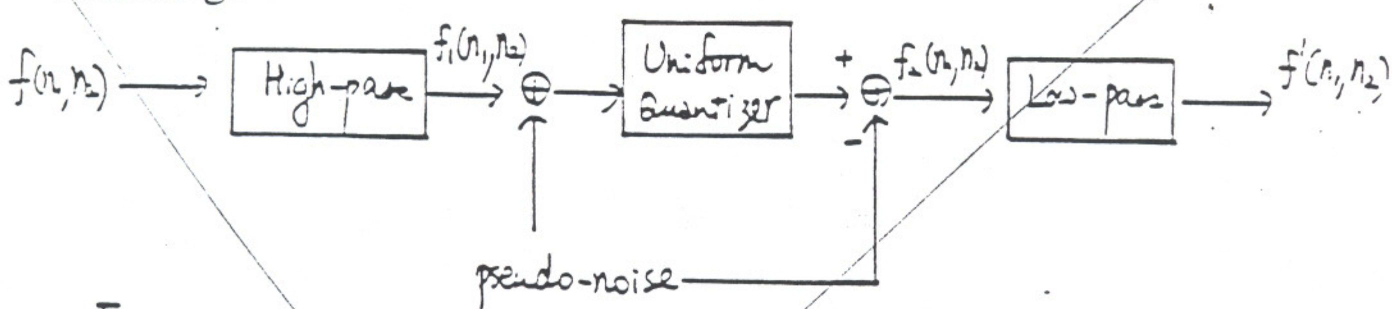
- very simple
- typically requires over 5-6 bits/pixel for good quality
- false contours for low-bit rate case

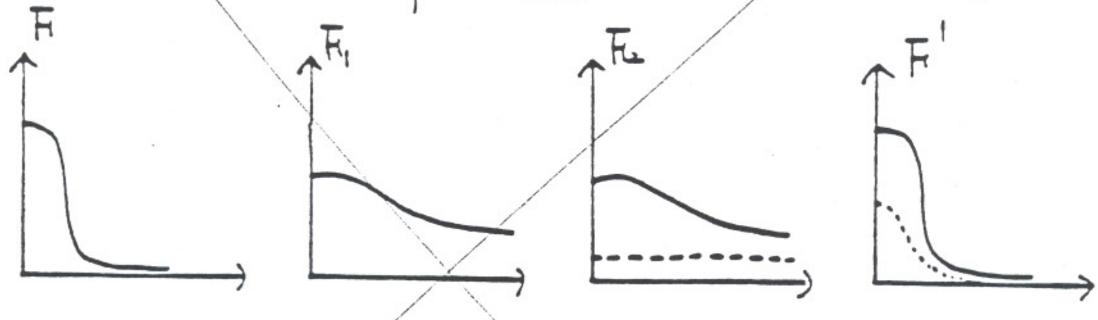
Improvements of PCM (cont.)

2. Roberts' Pseudo-Noise Technique with Noise Reduction:

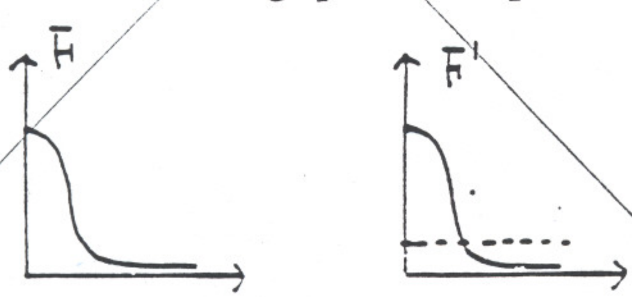


3. Roberts' Pseudo-Noise Technique and Highpass/Lowpass Filtering:



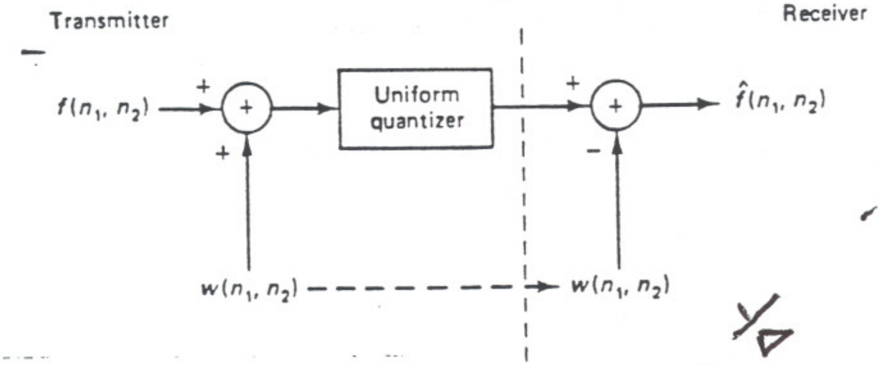


If there were not Highpass/Lowpass Filtering:

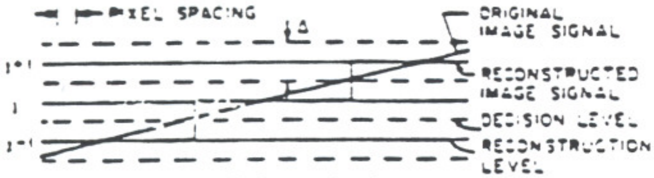


Improvements of PCM

1. Roberts' Pseudo-Noise Technique:



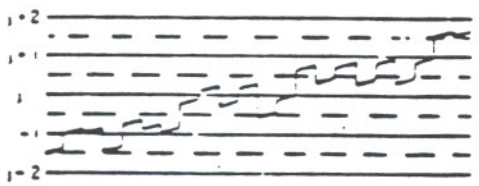
$$p(w) = \begin{cases} \frac{1}{\Delta} & -\frac{\Delta}{2} \le w \le \frac{\Delta}{2} \\ 0 & \text{otherwise} \end{cases}$$



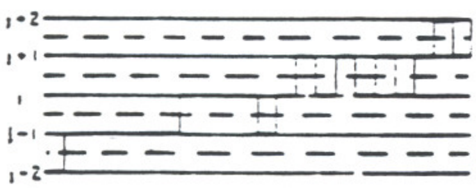
(a) Nominal Quantization

- false contours disappear — replaced by additive random noise

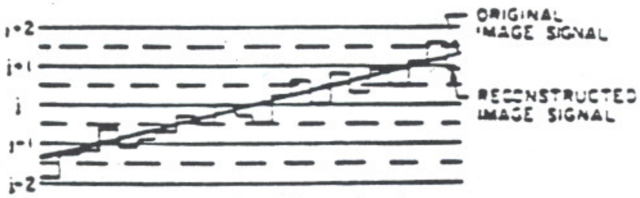
(b) One bit Pseudonoise



(c) Original Image Signal Plus Noise



(d) Quantized Image Signal Plus Noise



(e) Pseudonoise Quantization

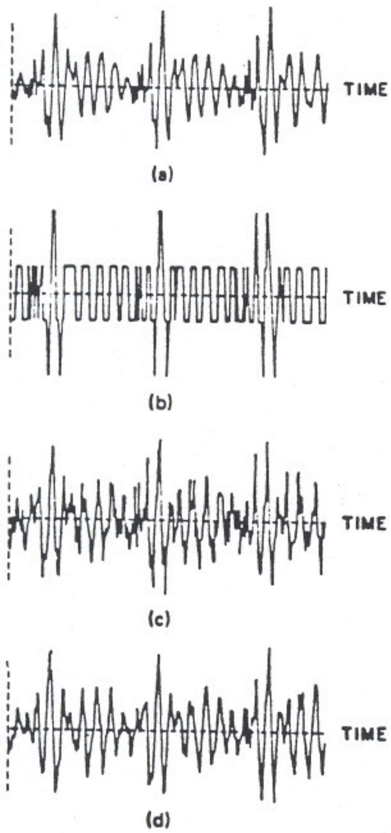


Figure 10.21 Example of quantization noise reduction in PCM speech coding. (a) Segment of noise-free voiced speech; (b) PCM-coded speech at 2 bits/sample; (c) PCM-coded speech at 2 bits/sample by Roberts's pseudonoise technique; (d) PCM-coded speech at 2 bits/sample with quantization noise reduction.

Figure 10.22 Example of quantization noise reduction in PCM image coding. (a) Original image of 512 × 512 pixels; (b) PCM-coded image at 2 bits/pixel; (c) PCM-coded image at 2 bits/pixel by Roberts's pseudonoise technique; (d) PCM-coded image at 2 bits/pixel with quantization noise reduction.

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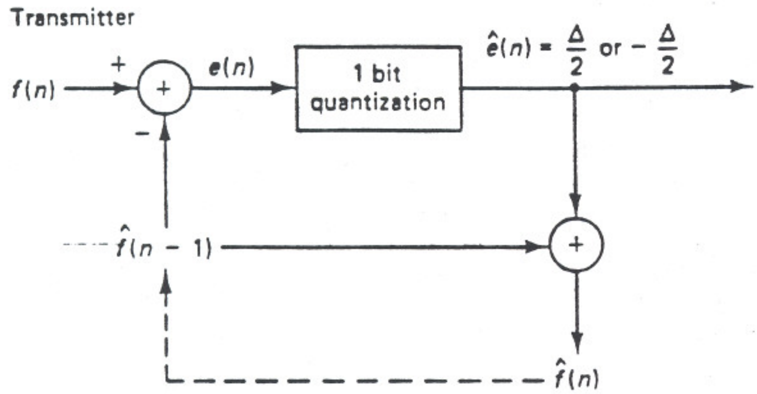
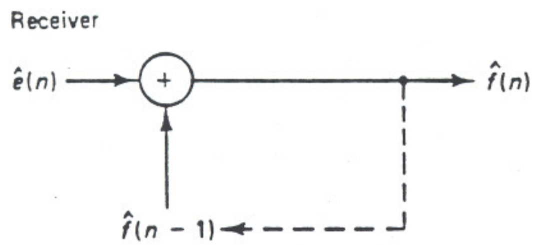
Delta Modulation (DM)

$f(n_1, n_2)$: signal, $\hat{f}(n_1, n_2)$: coded signal

Transmitter



Receiver



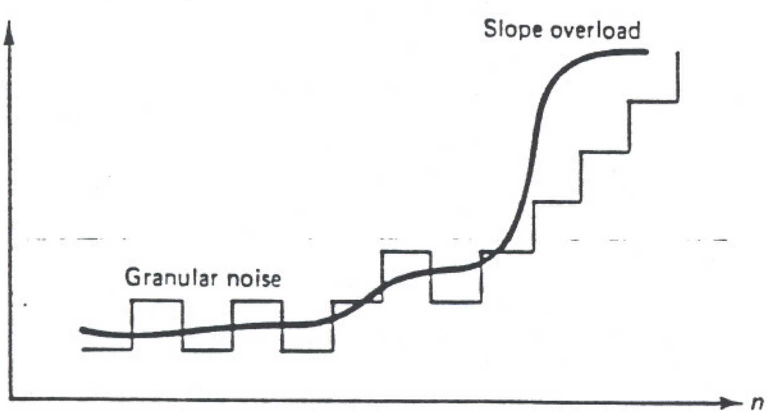


Figure 10.26 Granular noise and slope-overload distortion in delta modulation.

- needs over 2-3 bits/pixel to get good quality

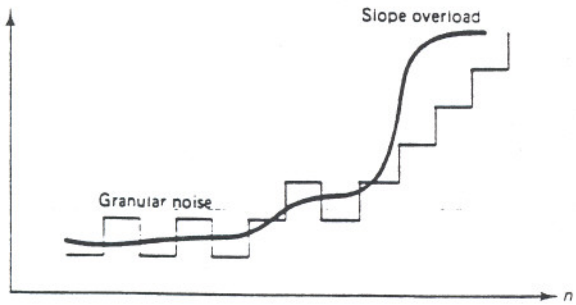


Figure 10.26 Granular noise and slope-overload distortion in delta modulation.

quirements, and the step size Δ is chosen through some compromise between the two requirements.

Figure 10.27 illustrates the performance of a DM system. Figures 10.27(a) and (b) show the results of DM with step sizes of $\Delta = 8\%$ and 15%, respectively, of the overall dynamic range of $f(n_1, n_2)$. The original image used is the 512×512-pixel image in Figure 10.22(a). When Δ is small [Figure 10.27(a)], the granular noise is reduced, but the slope overload distortion problem is severe and the resulting image appears blurred. As we increase Δ [Figure 10.27(b)], the slope overload distortion is reduced, but the graininess in the regions where the signal varies slowly is more pronounced.

(a) (b)

Figure 10.27 Example of delta-modulation (DM)-coded image. The original image used is the image in Figure 10.22(a). (a) DM-coded image with $\Delta = 8\%$ of the overall dynamic range, NMSE = 14.8%, SNR = 8.3 dB; (b) DM-coded image with $\Delta = 15\%$, NMSE = 9.7%, SNR = 10.1 dB.

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Figure 10.28 DM-coded image at 2 bits/pixel. The original image used is the image in Figure 10.22(a). NMSE = 2.4%, SNR = 16.2 dB.

To obtain good quality image reconstruction using DM without significant graininess or slope overload distortion, 3–4 bits/pixel is typically required. A bit rate higher than 1 bit/pixel can be obtained in DM by oversampling the original analog signal relative to the sampling rate used in obtaining $f(n_1, n_2)$. Oversampling reduces the slope of the digital signal $f(n)$ so a smaller Δ can be used without increasing the slope overload distortion. An example of an image coded by DM at 2 bits/pixel is shown in Figure 10.28. To obtain the image in Figure 10.28, the size of the original digital image in Figure 10.22(a) was increased by a factor of two by interpolating the original digital image by a factor of two along the horizontal direction. The interpolated digital image was coded by DM with $\Delta = 12\%$ of the dynamic range of the image and the reconstructed image was undersampled by a factor of two along the horizontal direction. The size of the resulting image is the same as the image in Figure 10.27, but the bit rate in this case is 2 bits/pixel.

10.3.3 Differential Pulse Code Modulation

Differential pulse code modulation (DPCM) can be viewed as a generalization of DM. In DM, the difference signal $e(n) = f(n) - \hat{f}(n-1)$ is quantized. The most recently coded $\hat{f}(n-1)$ can be viewed as a prediction of $f(n)$ and $e(n)$ can be viewed as the error between $f(n)$ and a prediction of $f(n)$. In DCPM, a prediction of the current pixel intensity is obtained from more than one previously coded pixel intensity. In DM, only one bit is used to code $e(n)$. In DPCM, more than one bit can be used in coding the error.

A DPCM system is shown in Figure 10.29. To code the current pixel intensity $f(n_1, n_2)$, $f(n_1, n_2)$ is predicted from previously reconstructed pixel intensities. The predicted value is denoted by $f'(n_1, n_2)$. In the figure, we have assumed that $\hat{f}(n_1 - 1, n_2)$, $\hat{f}(n_1, n_2 - 1)$, $\hat{f}(n_1 - 1, n_2 - 1)$, . . . were reconstructed prior to coding $f(n_1, n_2)$. We are attempting to reduce the variance of

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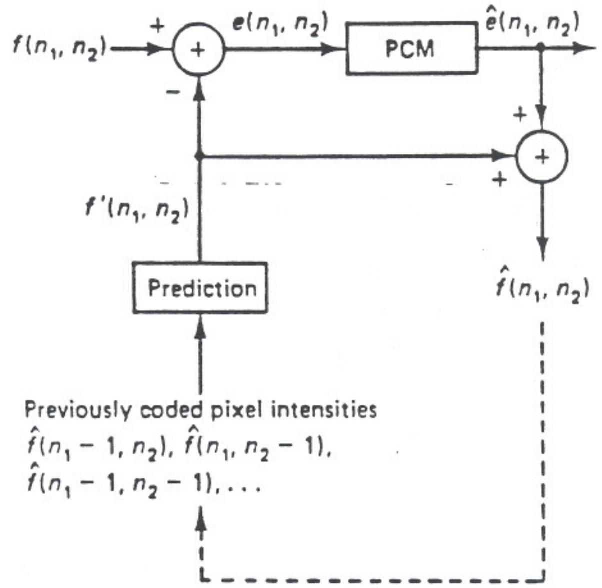
Differential Pulse Code Modulation (DPCM)

$f(n_1, n_2)$: original image

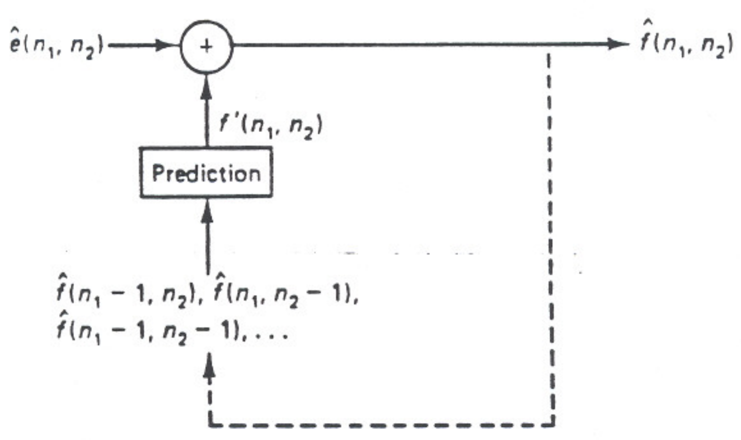
$\hat{f}(n_1, n_2)$: reconstructed image

Transmitter



- the Auto-regressive Model parameters are obtained from the image by solving a linear set of equations or by a Markov process assumption

Receiver



- requires 2-3 bits/pixel for good quality image where R_a is the region of (k_1, k_2) over which $a(k_1, k_2)$ is nonzero. Typically, $f'(n_1, n_2)$ is obtained by linearly combining $\hat{f}(n_1 - 1, n_2)$, $\hat{f}(n_1, n_2 - 1)$, and $\hat{f}(n_1 - 1, n_2 - 1)$. Since the prediction of $f(n_1, n_2)$ is made in order to reduce the variance of $e(n_1, n_2)$, it is reasonable to estimate $a(k_1, k_2)$ by minimizing

$$E[e^2(n_1, n_2)] = E[(f(n_1, n_2) - \sum\sum_{(k_1,k_2) \in R_a} a(k_1, k_2)\hat{f}(n_1 - k_1, n_2 - k_2))^2]. \qquad (10.40)$$

Since $\hat{f}(n_1, n_2)$ is a function of $a(k_1, k_2)$ and depends on the specific quantizer used, solving (10.40) is a nonlinear problem. Since $\hat{f}(n_1, n_2)$ is the quantized version of $f(n_1, n_2)$, and is therefore a reasonable representation of $f(n_1, n_2)$, the prediction coefficients $a(k_1, k_2)$ are estimated by minimizing

$$E\left[(f(n_1, n_2) - \sum\sum_{(k_1,k_2) \in R_a} a(k_1, k_2)f(n_1 - k_1, n_2 - k_2))^2\right]. \qquad (10.41)$$

Since the function in (10.41) minimized is a quadratic form of $a(k_1, k_2)$, solving (10.41) involves solving a linear set of equations in the form of

$$R_f(l_1, l_2) = \sum\sum_{(k_1,k_2) \in R_a} a(k_1, k_2)R_f(l_1 - k_1, l_2 - k_2) \qquad (10.42)$$

where $f(n_1, n_2)$ is assumed to be a stationary random process with the correlation function $R_f(n_1, n_2)$. The linear equations in (10.42) are the same as those used in the estimation of the autoregressive model parameters discussed in Chapters 5 and 6.

Figure 10.30 illustrates the performance of a DPCM system. Figure 10.30 shows the result of a DPCM system at 3 bits/pixel. The original image used is the image in Figure 10.22(a). The PCM system used in Figure 10.30 is a nonuniform quantizer. The prediction coefficients $a(k_1, k_2)$ used to generate the example are

Figure 10.30 Example of differential pulse code modulation (DPCM)-coded image at 3 bits/pixel. Original image used is the image in Figure 10.22(a). NMSE = 2.2%, SNR = 16.6 dB.

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