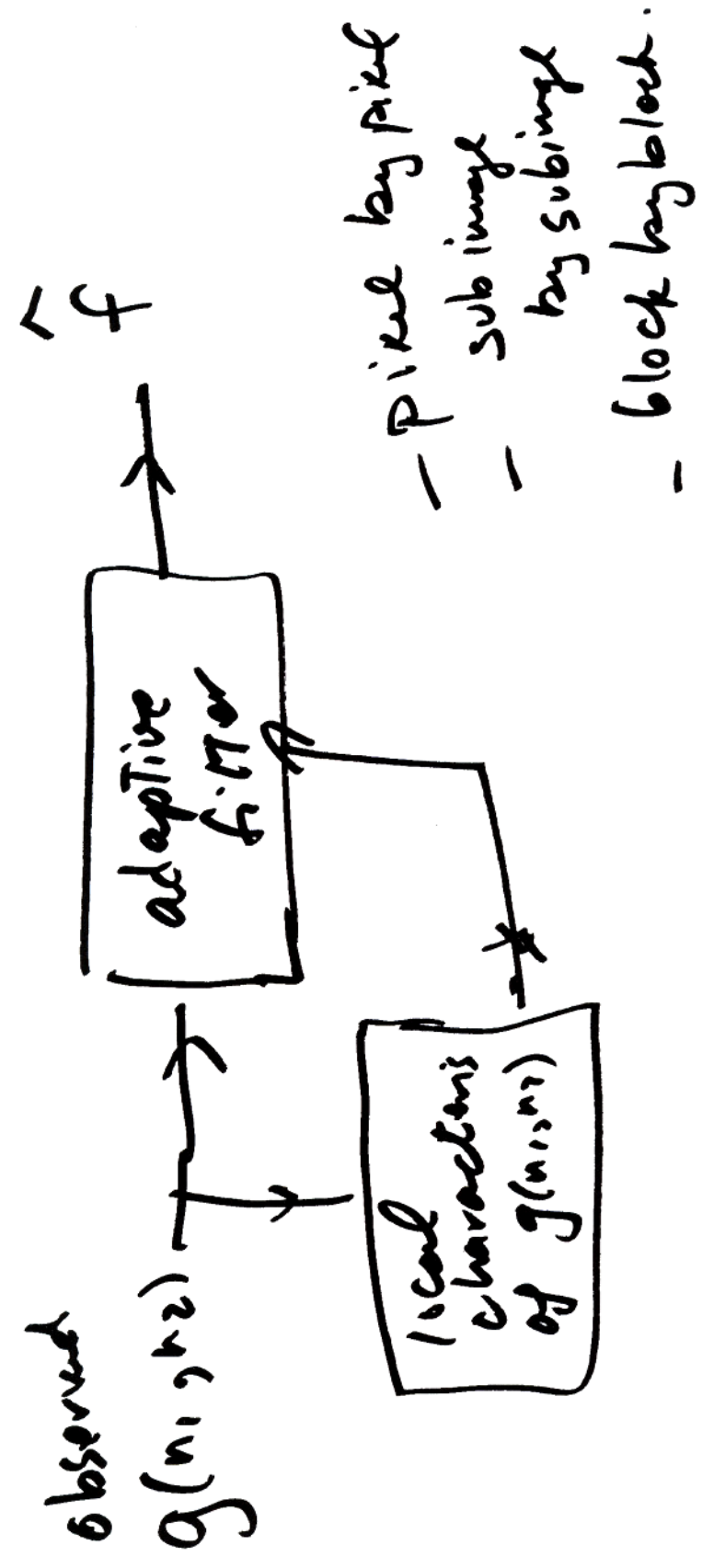
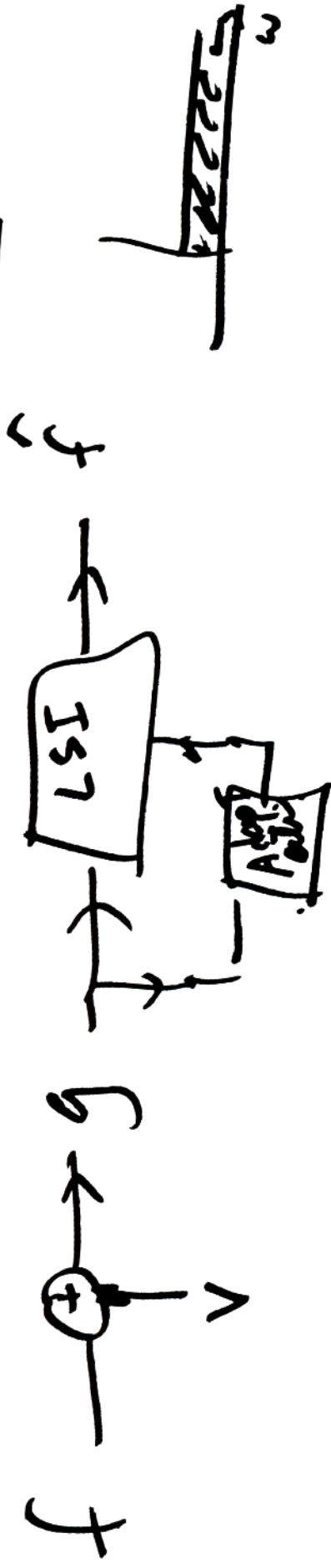


Adaptive Wiener Filter:

Basic idea: no image stationary.



One possible Adaptive filter



- Assume, noise v is white, zero mean, variance σ_v

$$P_v(w_1, w_2) = \sigma_v^2$$

- Assume signal satisfies the following model:

$$f(n_1, n_2) = m_f + \sigma_f w(n_1, n_2)$$

- w is a white noise, zero mean, unit variance process.

\Rightarrow Wiener filter $H(w_1, w_2) =$
assuming zero mean input.

$$\frac{P_f(w_1, w_2)}{P_f(w_1, w_2) + P_v(w_1, w_2)}$$

$$H(w_1, w_2) = \frac{\sigma_f^2}{\sigma_f^2 + \sigma_v^2}$$

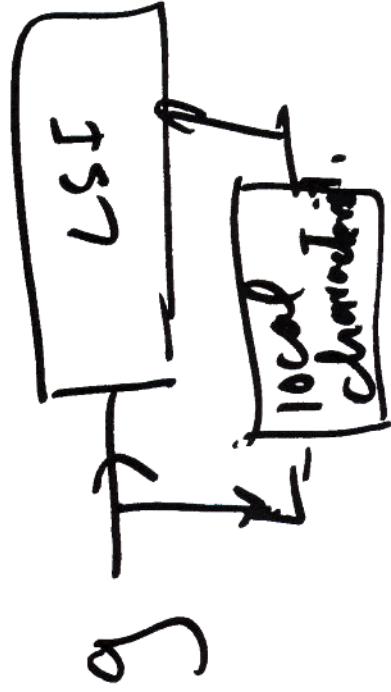
$$\Rightarrow h(n_1, n_2) = \frac{\sigma_f^2}{\sigma_f^2 + \sigma_v^2} \delta(n_1, n_2)$$

~~Handwritten scribbles~~

* Taking care of The mean

$$\hat{f}(n_1, n_2) = m_f + (g(n_1, n_2) - m_f) * h(n_1, n_2)$$

$$\hat{f}(n_1, n_2) = m_f + (g(n_1, n_2) - m_f) * \frac{\sigma_f^2}{\sigma_f^2 + \sigma_v^2} \delta(n_1, n_2)$$



~~mf~~ m_f σ_f^2 are both frs of (n_1, n_2)

$$f(u_1, v_1, z_1) = m_f(u_1, v_1) + (g(u_1, v_1) - m_f(u_1, v_1)) \frac{\sigma_f(u_1, v_1)}{\sigma_f(u_1, v_1) + \sigma_v^2}$$

$$f(u_1, v_1, z_1) = m_f(u_1, v_1) + (g(u_1, v_1) - m_f(u_1, v_1)) \frac{\sigma_f^2}{\sigma_f^2 + \sigma_v^2}$$

2 cases: (1) $\sigma_f^2 \ll \sigma_v^2 \Rightarrow \hat{f} \approx m_f(u_1, v_1)$

flat parts of image for (u_1, v_1, z_1) or

regions where image is "flat"
 \Rightarrow output is just mean.

(2) $\sigma_f^2 \gg \sigma_v^2 \Rightarrow \hat{f} \approx g(u_1, v_1)$

Textured part of image

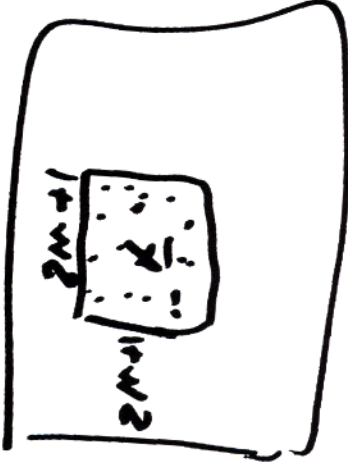
How to estimate m_f ?

$$f \rightarrow g$$

$$m_g = m_v + m_f \quad \cdot m_v = 0$$

$$\hat{m}_g = \hat{m}_f = \hat{m}_g = \text{[scribbled out]}$$

$$\hat{m}_f = m_g = \frac{1}{(2M+1)^2} \sum_{k_1=1}^{n_1+M} \sum_{k_2=1}^{n_2+M} g(k_1, k_2)$$



plug this into \mathbb{R}

$$\hat{f} = g$$

* h'

$$h'(n_1, n_2) =$$

$$\left\{ \frac{g^2 + \frac{6v^2}{(2M+1)^2}}{6f^2 + 6v^2} \quad \frac{6v^2}{(2M+1)^2} \right\} \quad 0$$

$$n_1 = n_2 = 0$$

$$|n_1| \leq M$$

$$|n_2| \leq M$$

except $n_1 = n_2 = 0$

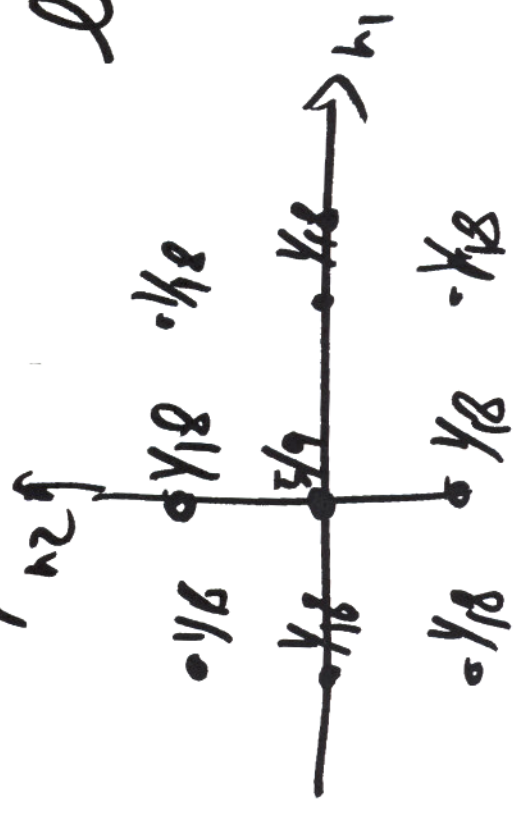
Otherwise

3 cases ① $\sigma_f^2 \gg \sigma_v^2 \Rightarrow \sigma_{h_1}^2 \approx \sigma_f^2$



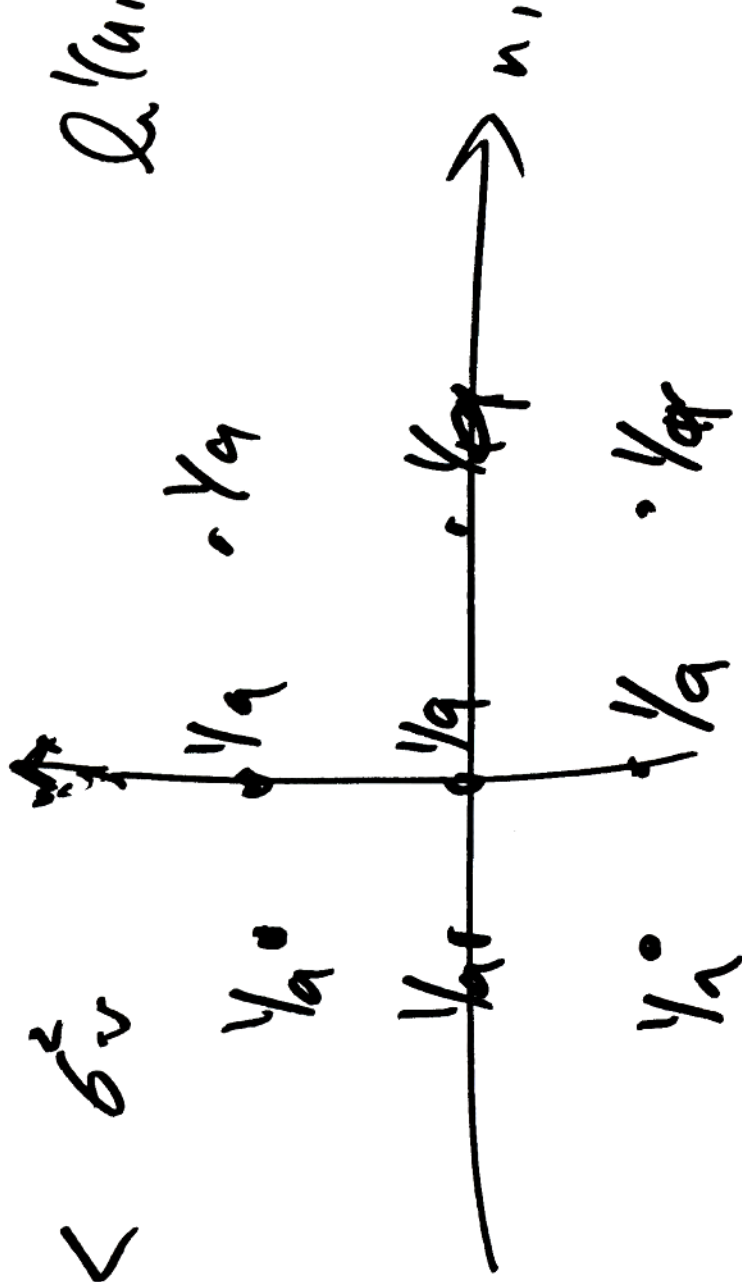
If energy/variance of f is much larger than noise, let the signal through.

② $\sigma_f^2 \approx \sigma_v^2, M=1 \Rightarrow \sigma_{h_1}^2 \approx \sigma_f^2$



③ $\sigma_f^2 \ll \sigma_v^2$

$M=1$



$f \rightarrow g$

f, v are independent

v

$$g = v + f$$

$$\sigma_g^2 = \sigma_v^2 + \sigma_f^2$$

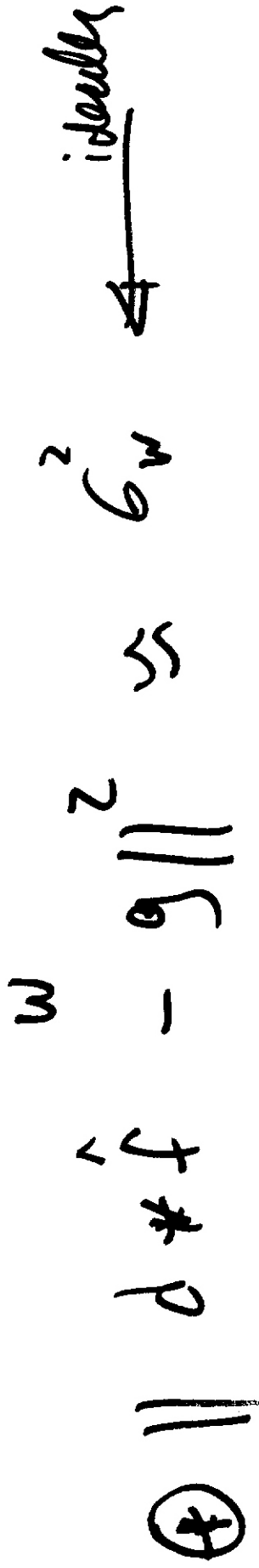
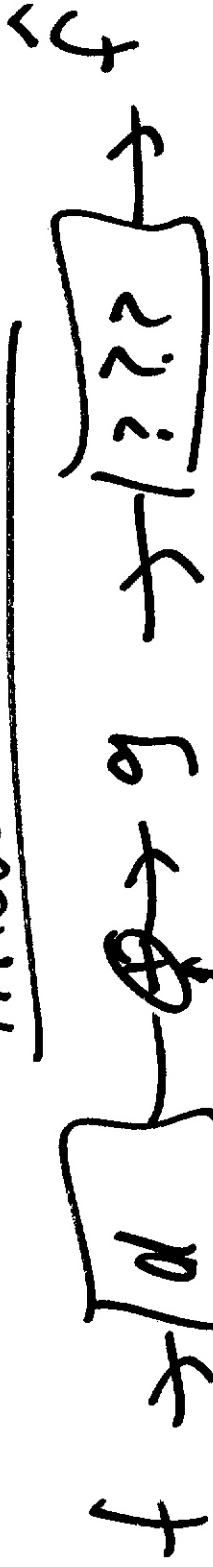
$$\Rightarrow \sigma_f^2 = \sigma_g^2 - \sigma_v^2 \quad ??$$

$$\hat{\sigma}_F^2 = \begin{cases} \hat{\sigma}_g^2 - \hat{\sigma}_v^2 & \text{only if } \hat{\sigma}_g^2 > \hat{\sigma}_v^2 \\ 0 & \text{otherwise.} \end{cases}$$

otherwise.

$$\hat{\sigma}_{g(n_1, n_2)}^2 = \frac{1}{(2M+1)^2} \sum_{k_1=n_1-M}^{n_1+M} \sum_{k_2=n_2-M}^{n_2+M} [g(k_1, k_2) - \hat{m}_g(n_1, n_2)]^2$$

Attention To Weier



Approach find a "smooth" signal
such that Φ is satisfied.

Define $c(h, r, z)$ as high pass filter.

$$\text{minimize } \|c * f\|_2^2$$

subject to Φ

→ Constrained Least Squares.

$$H_{es}(w_1, w_2) = \frac{D^*(w_1, w_2)}{(D^*(w_1, w_2)D(w_1, w_2) + \alpha C^*(w_1, w_2)C(w_1, w_2))}$$

$\alpha =$ identity β so chosen to satisfy α is regularization parameter.

example of $c \rightarrow$ 2D Laplacian operator

$$\begin{array}{c|c} -1 & -1 \\ \hline -1 & 4 \\ \hline -1 & -1 \end{array}$$

Fig 8 \rightarrow Bickard

Figs. 30 & 6/w.

Iterative Filters

Motivation:

- ① Actively control Trade off between ringing + blurring.
- ② Can incorporate a priori constraint about your signal.

$$\hat{f}_{i+1}(u_1, u_2) \leftarrow \hat{f}_i(u_1, u_2) +$$

$$\beta [g(u_1, u_2) - \alpha f_i]$$

Show convergence if $|1 - \beta D(u_1, u_2)| < 1$
($H(u_1, u_2)$)

Assaying $|D(u, w)| \leq 1$

Then: converges.

$0 < \beta < 2$ provided $D(u, w) > 0$

$\lim_{i \rightarrow \infty} f_i = \text{hinv} * g$

Fig 9 based. / Blend.

Apriori knowledge : We know α_i is positive. \rightarrow Apply a projection operation.

$$P[\hat{f}(u_1, v_2)] = \begin{cases} \hat{f}(u_1, v_2) & \text{if } f(u_1, v_2) > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\hat{f}_{i+1}(u_1, v_2) \leftarrow P[\hat{f}_i + \beta(g - d * f_i)]$$

$$\hat{f}_{i+1} \leftarrow P_1 P_2 P_3 [\dots]$$

POCS = Projection Onto Convex Set.

Convex Set $\div S$

$b \in S$.

$a \in S$

is also $\in S$

$\forall \epsilon$

$a \in S + (1-\epsilon)b$ is also Convex.

Example :

set of positive integers is convex.
 $S_1, S_2, S_3 \rightarrow$ convex



want to find any point in the intersection

start at an arbitrary ~~and~~ point, keep
projecting \rightarrow eventually converge

To the intersection.