Image Restoration

Frequency domain techniques

Periodic noise.

what if interference Pattern is not "clean"?

Sources of periodic interference patterns:
Coupling + amplification of low level signals in electro optical scanners's electronic circuitry.

Approach ① First isolate principal contributions (spikes) of the interference pattern.

② subtract a variable weighted portion of the pattern from the corrupt image.

objective: i.e. e.g. minimize local variance of a processed image.

$g(x,y) \longleftrightarrow G(\omega_1, \omega_2)$ observed degraded signal.

$f(x,y) \longleftrightarrow F(\omega_1, \omega_2) \rightarrow$ clean signal. original undegraded.

$\hat{f}(x,y) \longleftrightarrow \hat{F}(\omega_1, \omega_2) \rightarrow$ processed version of g approximate f.

$$f \rightarrow \boxed{\text{degr.}} \rightarrow g \rightarrow \underset{??}{\blacksquare} \rightarrow \hat{f}$$

Step ① put a notch filter $H(\omega_1, \omega_2)$ at location of each spike.

Find filter

$$N(\omega_1, \omega_2) = H(\omega_1, \omega_2)\, G(\omega_1, \omega_2)$$

noise.

$$\eta(x, y) = F^{-1}\{ H(\omega_1, \omega_2)\, G(\omega_1, \omega_2) \}$$

noise in space domain.

Step 2

$$\hat{f}(x,y) = g(x,y) - \underbrace{w(x,y)}_{\text{weight fn.}}\ \eta(x,y)$$

<u>Optimization</u> : choose local weights $w(x,y)$ to minimize local variance of $\hat{f}$ at (x,y)

---

neighborhood $(2a+1) \times (2b+1)$

local variance over this neighborhood:

$$\sigma^2_{xy} = \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{+a} \sum_{t=-b}^{+b} \left[\hat{f}(x+s, y+t) - \bar{\hat{f}}(x,y) \right]^2$$

$\bar{\hat{f}}$ = local mean = $\bar{\hat{f}}(x,y)$

$$= \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{+a} \sum_{t=-b}^{+b} \hat{f}(x+s, y+t)$$

plug in $\hat{f} = g - w\eta$ into

assume $w(x,y)$ is constant over $[2a+1] \times [2b+1]$ region.

$$w(x+s, y+t) \simeq w(x,y)$$

$$s,t \in [-a, +a] \times [-b, +b]$$

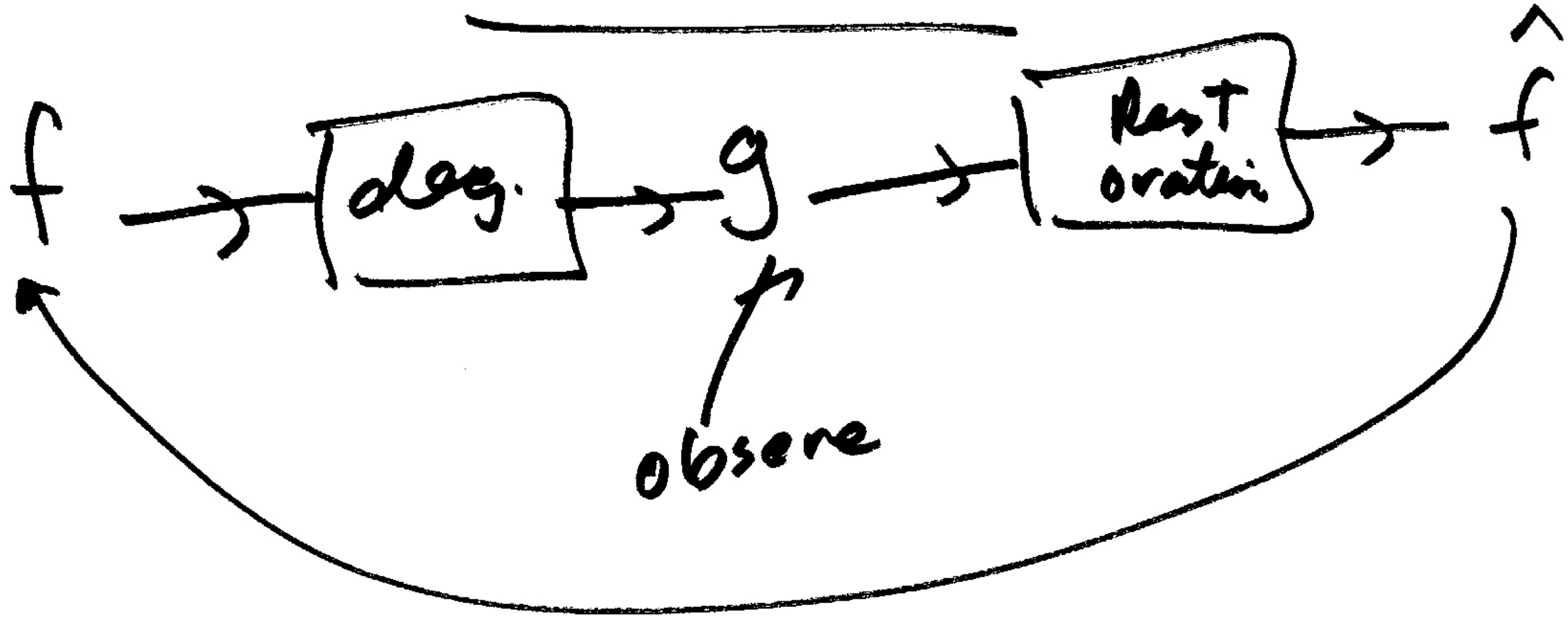
$$\sigma^2_{xy} = \frac{1}{(2a+1)(2b+1)} \sum_{-a}^{+a} \sum_{-b}^{+b} \left\{ \left[g(x+s, y+t) - w(x,y)\,\eta(x+s, y+t) \right] - \left[\bar{g}(x,y) - w(x,y)\,\bar{\eta}(x,y) \right] \right\}^2$$

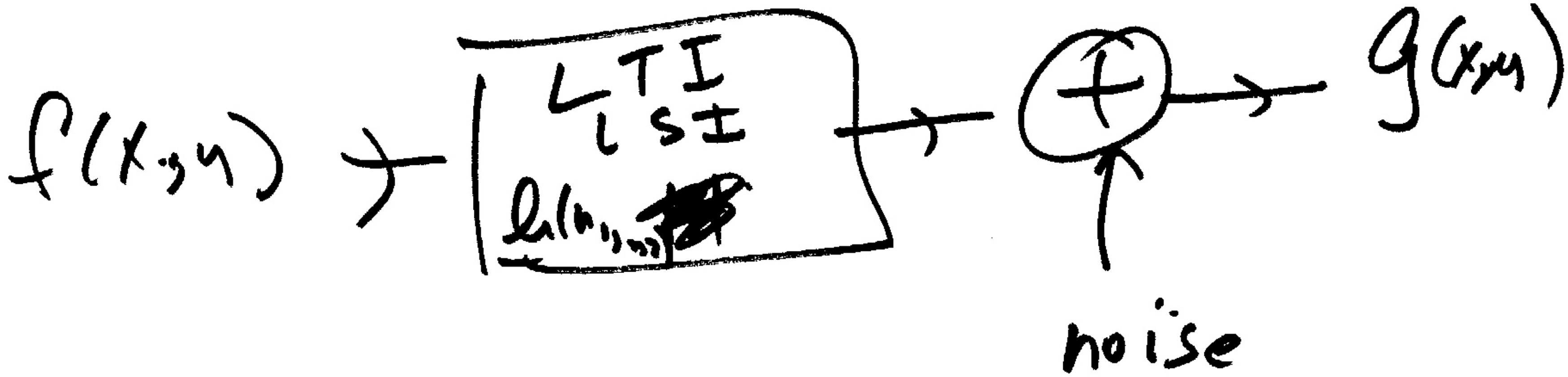
$$\frac{\partial \sigma^2_{xy}}{\partial w(x,y)} = 0 \Rightarrow$$

optimal weight

$$w(x,y) = \frac{\overline{g(x,y)\,\eta(x,y)} - \bar{g}(x,y)\,\bar{\eta}(x,y)}{\overline{\eta^2}(x,y) - \bar{\eta}^2(x,y)}$$

Restoration

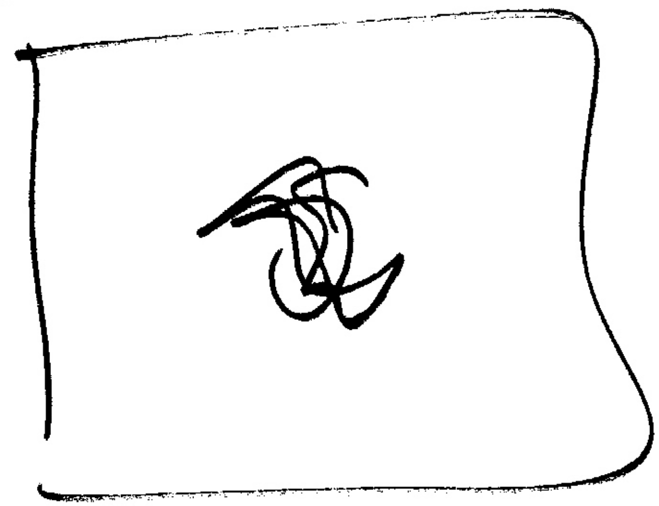




Estimating the degradation fn

① ~~[illegible]~~ Observation

go to parts of g
that you have
a priori knowledge.



see how it looks.
compare to what it should have looked like.
$\Longrightarrow$ make some intelligent guess about
h and noise

② Experimentation : put a known signal
into your system to calibrate

3 Modeling

Stanley + Hufnagel.

Atmospheric turbulence

$$H(\omega_1, \omega_2) = e^{-K(\omega_1^2 + \omega_2^2)^{5/6}}$$

Motion Blur $\leftarrow$

Blur
- → motion Blur
- → out of focus Blur
- → Atmospheric Turbulence.

<u>Motion Blur</u>. Assume. scene translates @ constant velocity $\nu_{relative}$ under an angle ϕ radian w.r.t. horizontal axis

- Exposure Time $[0, t_{exposure}]$

- Length of motion $= L = \nu_{relative} \cdot t_{exposure}$.

PSF

$$d_{L,\phi}(x,y) = \begin{cases} \frac{1}{L} & \text{if } \sqrt{x^2+y^2} \leq \frac{L}{2}, \quad \frac{x}{y} = -\tan\phi \\ 0 & \text{otherwise.} \end{cases}$$

Out of focus Blur

$$d_R(x,y) = \begin{cases} \frac{1}{\pi R^2} & \text{if } \sqrt{x^2+y^2} \le R^2 \\ 0 & \text{otherwise.} \end{cases}$$

R = parameter for how much out of focus.

$$d_R(n_1,n_2) = \begin{cases} \frac{1}{C} & \text{if } \sqrt{n_1^2+n_2^2} \le R^2 \\ 0 & \text{otherwise} \end{cases}$$

Atmospheric Blur

$$d_{\sigma_G}(x,y) = c \exp\left\{ - \frac{x^2+y^2}{2\sigma_G^2} \right\}$$

Motion Blur modeling

image $f(x,y)$

$x_0(t)$ Time varying component of motion along x.

$y_0(t)$ " " " " " " y.

T = duration of exposure.

$g(x,y)$ = observed or captured signal.

$$g(x,y) = \int_0^T f(x - x_0(t), y - y_0(t))\, dt$$

$$\text{F.T.}\{g(x,y)\} = G(\omega_x, \omega_y) = \int\int_{-\infty}^{+\infty} g(x,y)\, e^{-j2\pi(\omega_x x + \omega_y y)}\, dx\, dy.$$

$$G(w_x,w_y)=\int_0^T\left(\iint_{-\infty}^{+\infty} f(x-x_0(t),y-y_0(t))\,e^{-j2\pi(\omega_x x+\omega_y y)}\,dx\,dy\right)dt$$

$$=\int_0^T F(\omega_x,\omega_y)\,e^{-j2\pi(\omega_x x_0(t)+\omega_y y_0(t))}\,dt.$$

$$\boxed{G(\omega_x,\omega_y)=F(\omega_x,\omega_y)\underbrace{\int_0^T e^{-j2\pi(\omega_x x_0(t)+\omega_y y_0(t))}\,dt}_{H(\omega_x,\omega_y)}}$$

- $x_0(t)=0 \quad y_0(t)=0 \Longrightarrow g(x,y)=T\,f(x,y)$
- $x_0(t)=\frac{at}{T}$ = constant speed along x direction $y_0(t)=0$

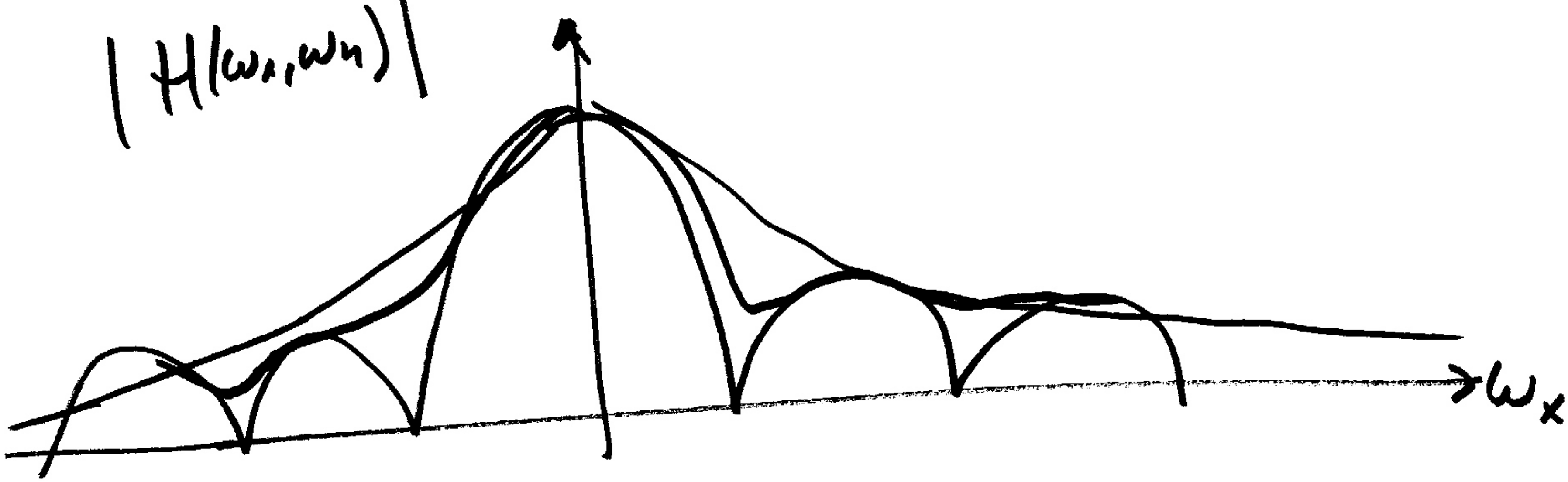
$$\boxed{G(\omega_x,\omega_y)=F(\omega_x,\omega_y)\,H(\omega_x,\omega_y)}$$

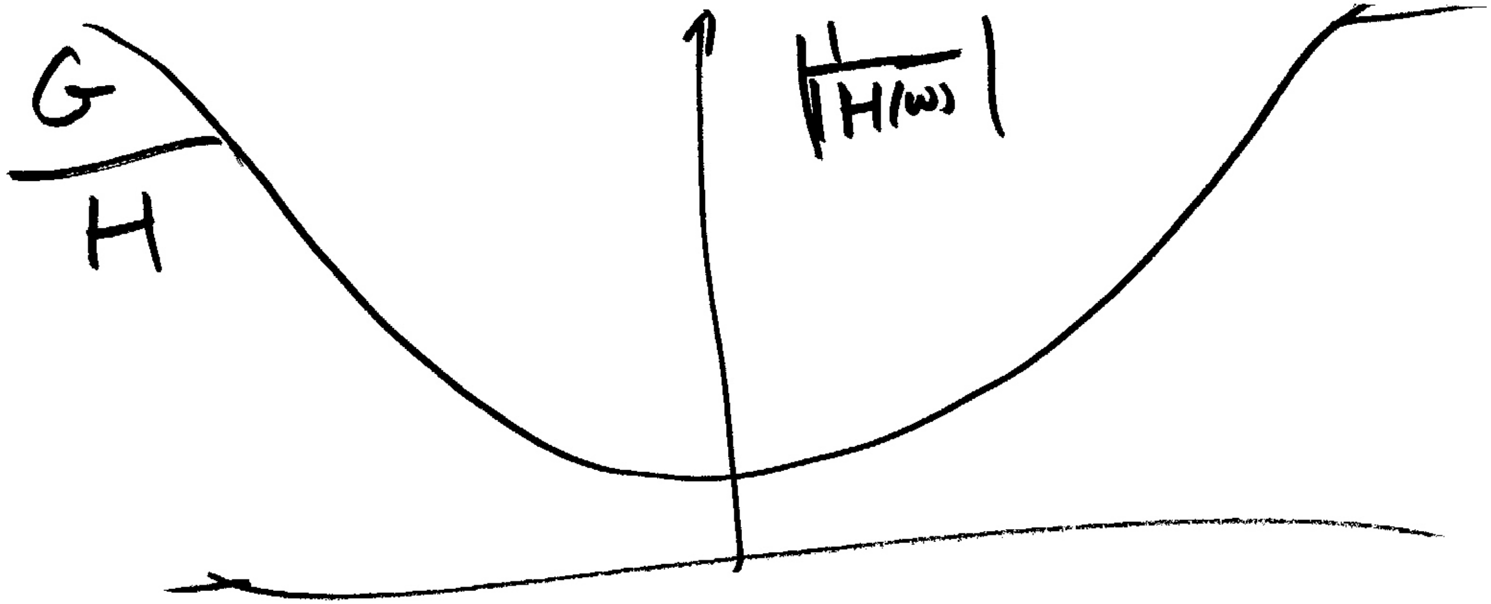
$$H(\omega_x,\omega_y) = \int_0^T e^{-j2\pi(\omega_x\, x_1(t))}\, dt$$

$$= \int_0^T e^{-j2\pi \frac{\omega_x a t}{T}}\, dt$$

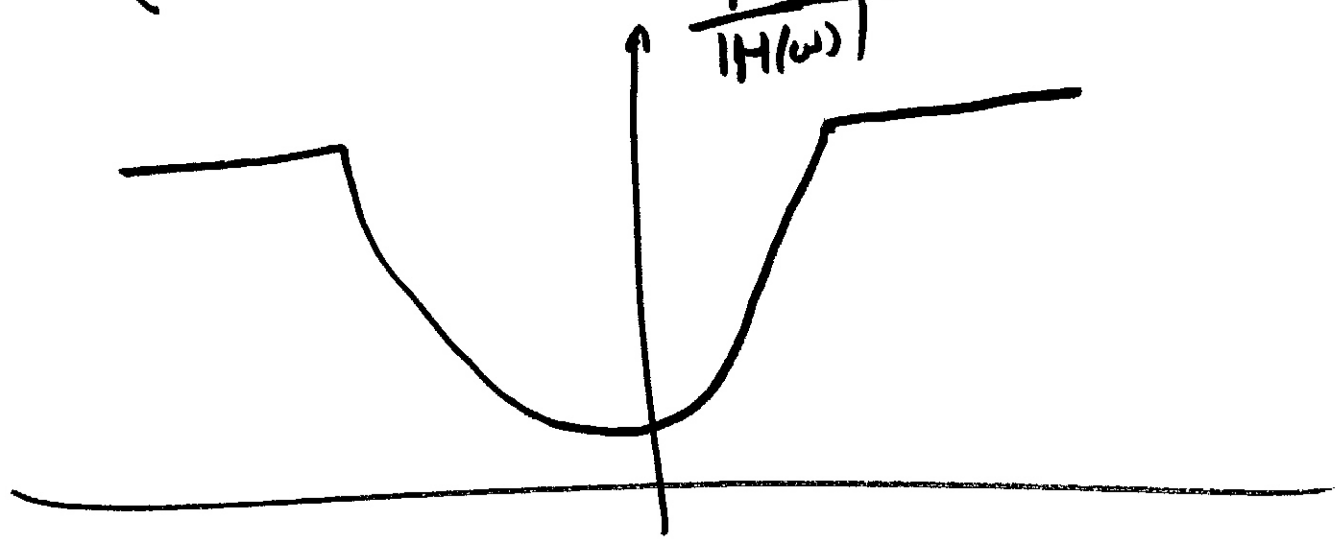
$$H(\omega_x,\omega_y) = \frac{T}{\pi\omega_x a} \sin(\pi\omega_x a)\, e^{-j\pi\omega_x a}$$

$|H(\omega_x,\omega_y)|$





$$G \quad F(\omega_x, \omega_y) = F(\omega_x, \omega_y)\, H(\omega_x, \omega_y) + \underline{\underline{\text{Noise}}}$$



Techniques. for deconvolution

1. Blur fn is known ←
2. is unknown ← Blind deconvolution.

3 classes of Restoration / Deconvolution Algs:

1. inverse filtering.
2. Least square filters → weiner, CLS.
3. Iterative filters.

Inverse Filtering

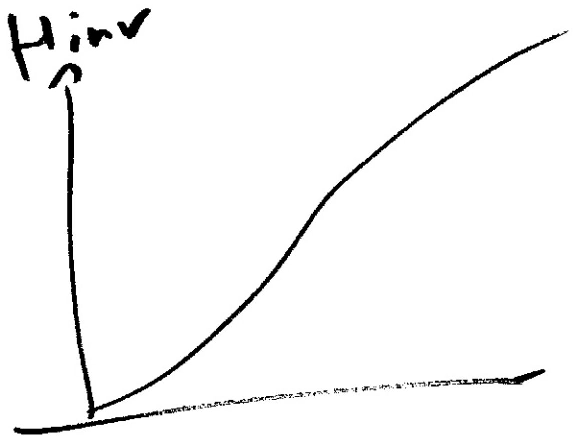
$$h_{inv} * d = \delta(n_1, n_2)$$

$$H_{inv}(w_1, w_2) = \frac{1}{D(w_1, w_2)}$$

$H_{inv}(\omega_1, \omega_2)\, D(\omega_1, \omega_2) = 1.$

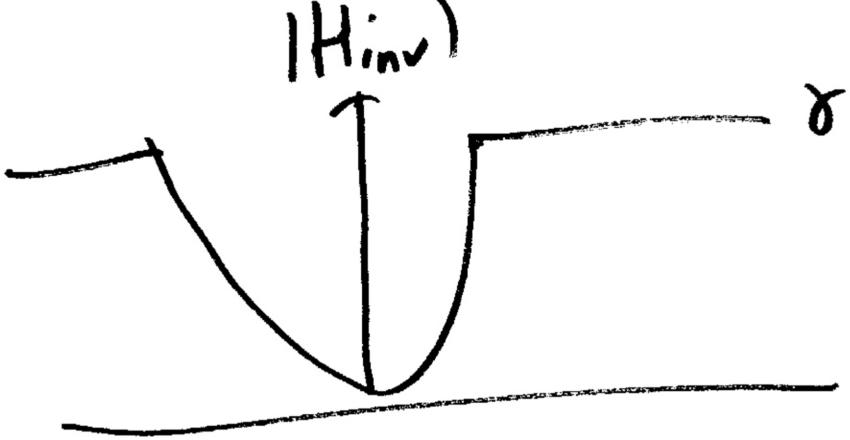
2 problem $\longrightarrow$ (1) noise : H_{inv} is High pass filter.

$\Longrightarrow$ accentuates noise.



(2) $D(\omega_1, \omega_2) \to 0$ everything goes bad

$$H_{inv}(\omega_1, \omega_2) = \begin{cases} \dfrac{1}{D(\omega_1, \omega_2)} & \text{only if } \dfrac{1}{D(\omega_1, \omega_2)} < \gamma \\ \gamma & \text{otherwise} \end{cases}$$



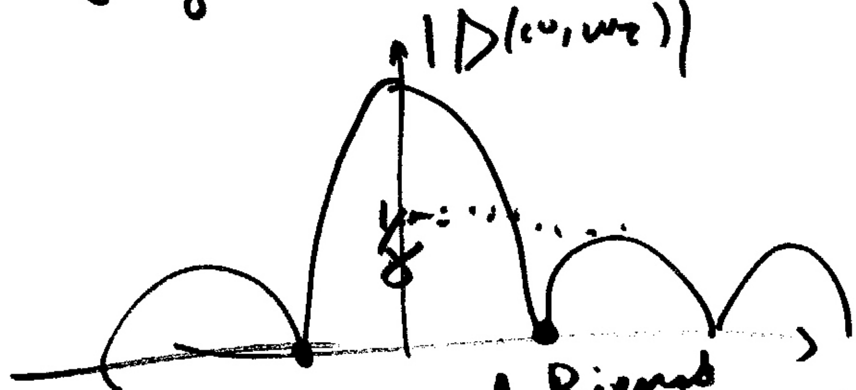


Fig 5 of Biemond
fig 5.29 of Gonz.