Image Coding and Compression

- Why Compression?
  - Two hour SD movie

$$30\frac{\text{frames}}{\text{sec}} \times (720 \times 480)\frac{\text{pixels}}{\text{frame}} \times 3\frac{\text{bytes}}{\text{pixel}} = 31.1\,\text{MB/s}$$

$$31.1\,\text{MB/s} \times (60^2)\frac{\text{sec}}{\text{hr}} \times 2\,\text{hrs} \approx 224\,\text{GB}$$

28 Dual-Layer DVDs

What to Code?

- Image intensities
  - RGB, YCrCb
- Transform coefficients
- Model parameters

Image Coding Model

- Object: Reduce the data size without sacrificing the image quality
- Compression Ratio

Example: Image Quantization

Fidelity Criteria

- RMS

$$e_{\mathrm{rms}} = \left[\frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left[\hat{f}(x,y) - f(x,y) \right]^2 \right]^{1/2}$$

- SNR

$$\mathrm{SNR}_{\mathrm{ms}} = \frac{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \hat{f}(x,y)^2}{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left[\hat{f}(x,y) - f(x,y) \right]^2}$$

- PSNR

$$\frac{\max p_i}{e_{\mathrm{rms}}}$$

Rating Scale

TABLE 8.3
Rating scale of the Television Allocations Study Organization. (Frendendall and Behrend.)

Value	Rating	Description
1	Excellent	An image of extremely high quality, as good as you could desire.
2	Fine	An image of high quality, providing enjoyable viewing. Interference is not objectionable.
3	Passable	An image of acceptable quality. Interference is not objectionable.
4	Marginal	An image of poor quality; you wish you could improve it. Interference is somewhat objectionable.
5	Inferior	A very poor image, but you could watch it. Objectionable interference is definitely present.
6	Unusable	An image so bad that you could not watch it.

Other Metrics

- P-PSNR
- VSNR
- SSIM

(a) (b) (c) (d)

(e) (f) (g) (h) (i)

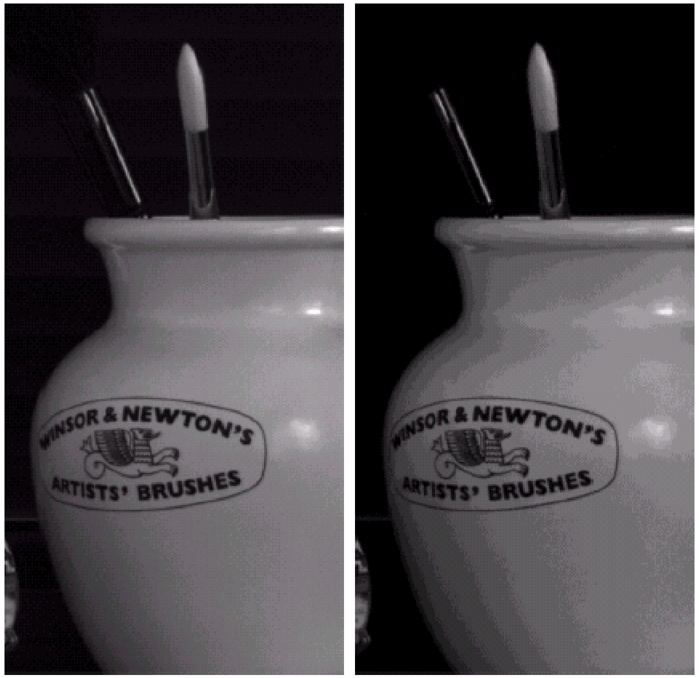
Distortion	PSNR	P-PSNR	SSIM	CWSSIM	WCWSSIM
(b) JPEG	24.7	33.7	0.67	0.61	0.71
(c) Blur	24.8	33.5	0.75	0.46	0.65
(d) Noise	24.8	41.3	0.46	0.66	0.81
(e) Mean Shift	24.8	48.1	0.97	1.00	1.00
(f) Contrast	24.8	37.8	0.99	0.99	1.00
(g) Rotation	23.4	32.2	0.78	0.89	0.93
(h) Zoom	21.4	29.8	0.70	0.87	0.91
(i) Spatial Shift	21.7	30.8	0.68	0.90	0.93

Image Quantization and Reconstruction

- Quantization is the "lossy" part
  - Achieves the most compression efficiency
- Scalar Quantization
- Vector Quantization

Uniform Quantization

- Equal spacing of the reconstruction levels
- False contours



Improved Gray-Scale (IGS)

- Treat the LSBs as noise and use them to remove the false contours

Pixel	Gray Level	Sum	IGS Code
$i-1$	N/A	0000 0000	N/A
i	0110 1100	0110 1100	0110
$i+1$	1000 1011	1001 0111	1001
$i+2$	1000 0111	1000 1110	1000
$i+3$	1111 0100	1111 0100	1111

Non-Uniform Quantization

- Optimal Quantizer
  - Minimizing RMS by choosing the decision levels and reconstruction levels

- Lloyd-Max Quantizer

Bits	Uniform r_i	Uniform d_i	Gaussian r_i	Gaussian d_i	Laplacian r_i	Laplacian d_i
1	−0.5000	−1.0000	−0.7979	−∞	−0.7071	−∞
	0.5000	0.0000	0.7979	0.0000	0.7071	0.0000
		1.0000		∞		∞
2	−0.7500	−1.0000	−1.5104	−∞	−1.8340	−∞
	−0.2500	−0.5000	−0.4528	−0.9816	−0.4198	−1.1269
	0.2500	0.0000	0.4528	0.0000	0.4198	0.0000
	0.7500	0.5000	1.5104	0.9816	1.8340	1.1269
		1.0000		∞		∞
3	−0.8750	−1.0000	−2.1519	−∞	−3.0867	−∞
	−0.6250	−0.7500	−1.3439	−1.7479	−1.6725	−2.3796
	−0.3750	−0.5000	−0.7560	−1.0500	−0.8330	−1.2527
	−0.1250	−0.2500	−0.2451	−0.5005	−0.2334	−0.5332
	0.1250	0.0000	0.2451	0.0000	0.2334	0.0000
	0.3750	0.2500	0.7560	0.5005	0.8330	0.5332
	0.6250	0.5000	1.3439	1.0500	1.6725	1.2527
	0.8750	0.7500	2.1519	1.7479	3.0867	2.3769
		1.0000		∞		∞
4	−0.9375	−1.0000	−2.7326	−∞	−4.4311	−∞
	−0.8125	−0.8750	−2.0690	−2.4008	−3.0169	−3.7240
	−0.6875	−0.7500	−1.6180	−1.8435	−2.1773	−2.5971
	−0.5625	−0.6250	−1.2562	−1.4371	−1.5778	−1.8776
	−0.4375	−0.5000	−0.9423	−1.0993	−1.1110	−1.3444
	−0.3125	−0.3750	−0.6568	−0.7995	−0.7287	−0.9198
	−0.1875	−0.2500	−0.3880	−0.5224	−0.4048	−0.5667
	−0.0625	−0.1250	−0.1284	−0.2582	−0.1240	−0.2664
	0.0625	0.0000	0.1284	0.0000	0.1240	0.0000
	0.1875	0.1250	0.3880	0.2582	0.4048	0.2644
	0.3125	0.2500	0.6568	0.5224	0.7287	0.5667
	0.4375	0.3750	0.9423	0.7995	1.1110	0.9198
	0.5625	0.5000	1.2562	1.0993	1.5778	1.3444
	0.6875	0.6250	1.6180	1.4371	2.1773	1.8776
	0.8125	0.7500	2.0690	1.8435	3.0169	2.5971
	0.9375	0.8750	2.7326	2.4008	4.4311	3.7240
		1.0000		∞		∞

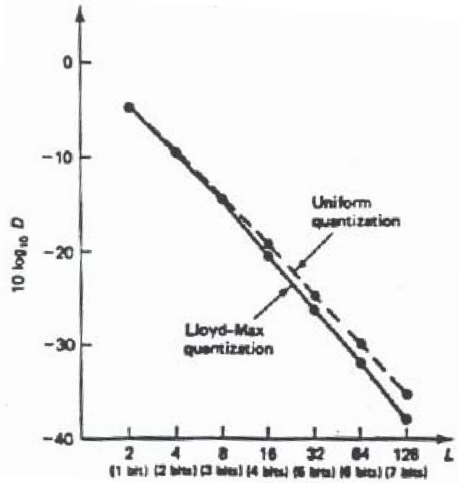
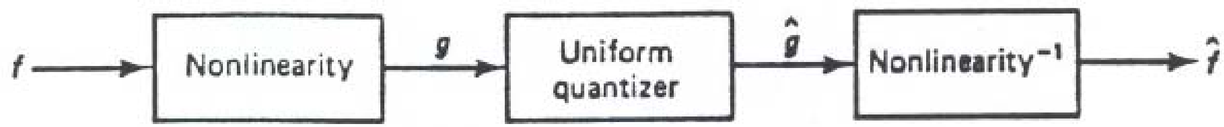


Figure 10.6 Comparison of average distortion $D = E[(\hat{f} - f)^2]$ as a function of L, the number of reconstruction levels, for a uniform quantizer (dotted line) and the Lloyd-Max quantizer (solid line). The vertical axis is $10 \log_{10} D$. The probability density function is assumed to be Gaussian with variance of 1.

Non-Uniform Quantizer by Transformation

- We can "flatten" a non-uniform distribution source by applying transformation

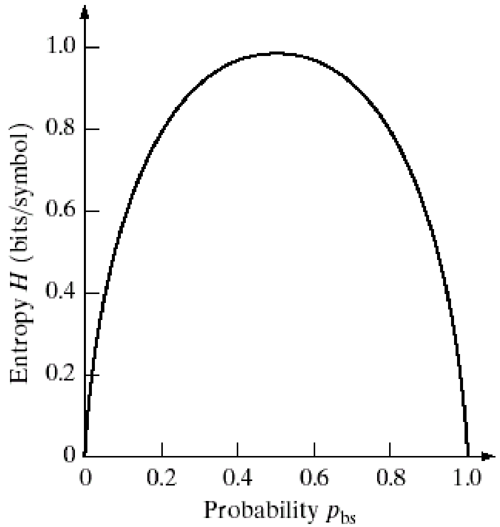


Codeword Assignment: Bit Allocation

- Uniform assignment
- Non-Uniform assignment
  - Assign the codeword according to its probability
  - Uniquely decodable

Holy Grail: Entropy

$$H \equiv -\sum_{i=1}^{L} p_i \log_2 p_i$$



- Entropy just provides the lower bound of the average codeword length, not the actual codeword assignment method

Huffman Coding

FIGURE 8.11 Huffman source reductions.

Original source		Source reduction			
Symbol	Probability	1	2	3	4
a_2	0.4	0.4	0.4	0.4	0.6
a_6	0.3	0.3	0.3	0.3	0.4
a_1	0.1	0.1	0.2	0.3	
a_4	0.1	0.1	0.1		
a_3	0.06	0.1			
a_5	0.04				

FIGURE 8.12 Huffman code assignment procedure.

Original source			Source reduction							
Sym.	Prob.	Code	1		2		3		4	
a_2	0.4	1	0.4	1	0.4	1	0.4	1	0.6	0
a_6	0.3	00	0.3	00	0.3	00	0.3	00	0.4	1
a_1	0.1	011	0.1	011	0.2	010	0.3	01		
a_4	0.1	0100	0.1	0100	0.1	011				
a_3	0.06	01010	0.1	0101						
a_5	0.04	01011								

L_{avg}=2.2   H=2.14

Huffman Coding

$$H(x) \leq L_{avg} \leq H(x) + 1$$

Tighter bound:

$p_{max} < 0.5,$

$$H(x) \leq L_{avg} \leq H(x) + p_{max}$$

$p_{max} \geq 0.5,$

$$H(x) \leq L_{avg} \leq H(x) + p_{max} + 0.086$$

Fail at highly skewed data