EE243 Advanced Electromagnetic Theory

Lec # 24 Imaging as Diffraction

- **Fresnel Zones and Lens**
- **Plane Wave Spectra, Lens Capture and Image**
- **Resolution and Depth of Focus**
- **N-wave imaging**
- **Resolution Enhancement:**
    - **Off axis Illumination**
    - **Phase-Shifting Masks**

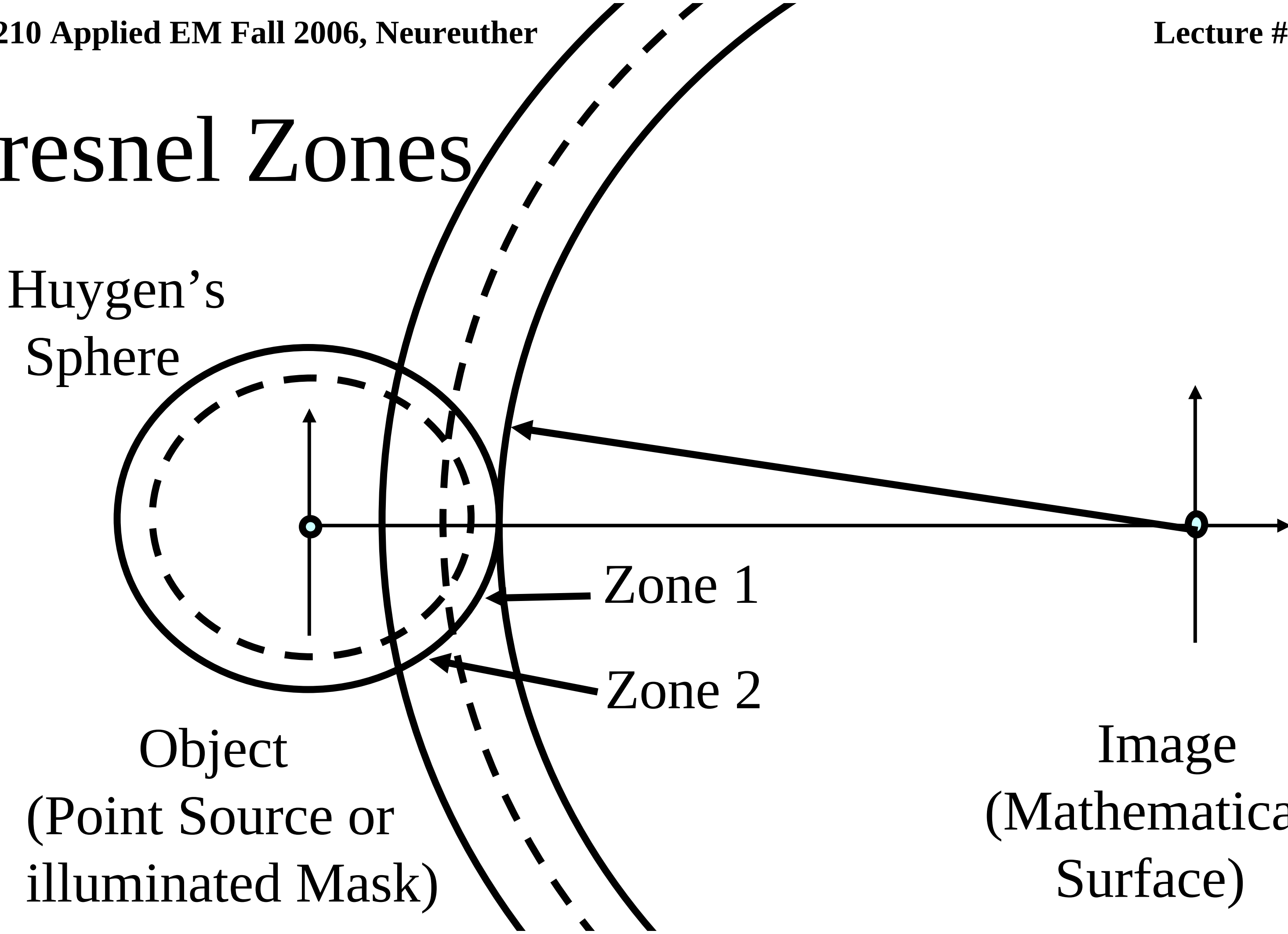
Reading: (This lecture is self contained and is based on excerpts from Born and Wolf Chapters 8 and 9 plus Chapter by Smith in Sheats and Smith)

Overview

Optical imaging is an application of diffraction in which the planewaves that make up the spatial diffraction spectrum are capture and reprocessed.

- The equivalence theorem allows a planewave representation to be built from E and H tangential on a plane.
- Waves that travel at angles captured by the lens are redirected and rephased by the lens to arrive at the focal point with the phase at which they left object plane.
- Resolution enhancement modifies planewave spectrum
- The intra-wave phase is all important
  - Describes image behavior in space about the focal plane
  - Accounts for lens imperfections (Aberrations)
  - Used to develop interferometric instrumentation

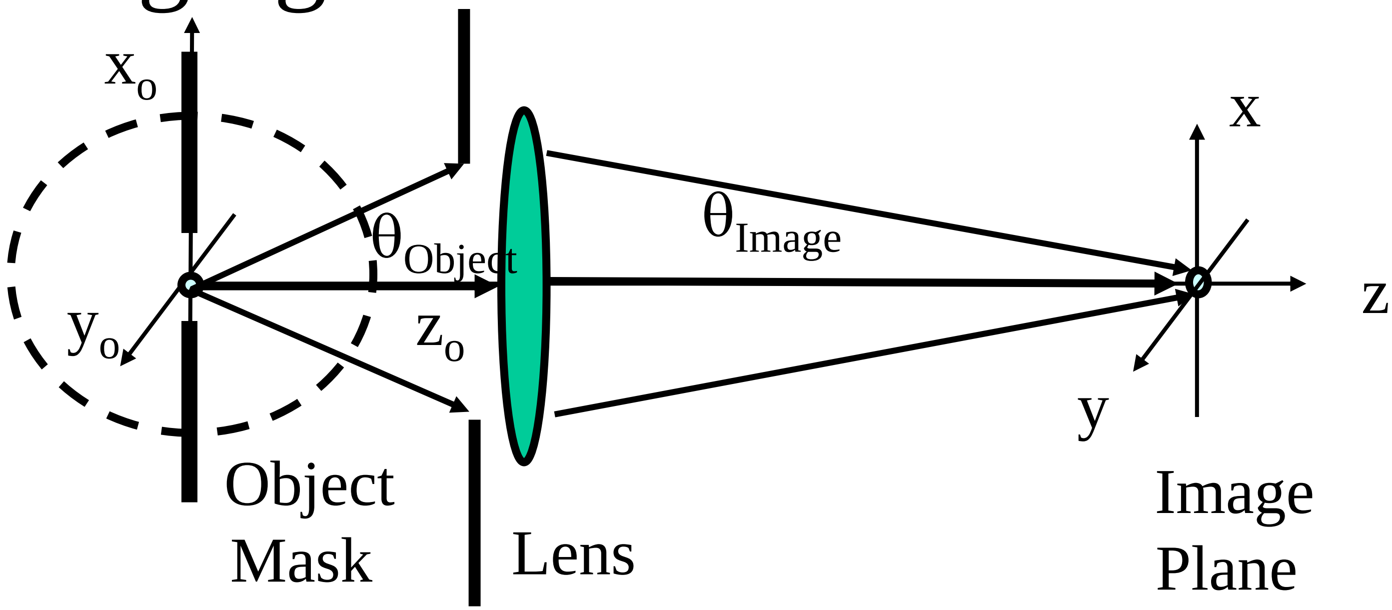
Fresnel Zones



Field at a point can be viewed as adding and subtracting contributions from $\lambda/2$ phase Zones

A Fresnel lens consists of small ring lenses that map ray direction from small to large circle and flip phase of negative zones (so all add)

Imaging as a Form of Diffraction



- Object sends out planewave spatial spectrum (rays)
- Lens is in far field (Fourier Transform) and low pass filters the spatial harmonics
- Lens redirects plane wave directions (change θ_{Object} to θ_{Image}) to converge and to each have its original phase at z_o the image plane z = 0.
- Image is in far field of lens (Fourier Transform) and is thus the inverse transform of spatial spectrum after low pass filtering

Planewave Expansion

$$\overline{E}_s'(\overline{x}) \rightarrow \frac{e^{ikr}}{r}\overline{F}(\overline{k},\overline{k}_0)$$

Jackson 10.7

$$\hat{e}^* \cdot \overline{F}(\overline{k},\overline{k}_0) = \frac{i}{4\pi}\oint_{S_1} e^{i\overline{k}\cdot\overline{x}}\left[\omega\hat{e}^* \cdot (\overline{n}' \times \overline{B}_s) + \hat{e}^* \cdot \left(\overline{k} \times \left(\overline{n}' \times \overline{E}_s\right)\right)\right] da'$$

Example for a mask with period P in x direction.

$$\overline{E}_{TOTAL} = \sum_n \overline{E}_n e^{-j(k_0 \sin(\theta_{no})x_o + k_0 \cos(\theta no)z_o)}$$

$$\overline{E}_{TOTAL} = \sum_{n=-N}^{N} \overline{E}_n A(\theta_{ni}) e^{j\Phi(\theta_{ni})} e^{-j(k_0 \sin(\theta_{ni})x_i + k_0 \cos(\theta ni)z_i)}$$

- Start from **Transverse components** of **E** and **B** on plane
- Make planewave spectrum expansion between mask and lens (assume periodic and switch to e^{jwt})
- Lens then low pass filters and apodizes/phases transmitted spectrum
- Propagation to image plane is thus the Fourier Transform of the filtered/phased spectrum

Electric Field as Sum of Plane Waves

Simplify to (x,z) plane, E in y-direction, A = 1, Φ =0

Mask with period P Bragg Condition Implies

$$\sin(\theta_n) = \frac{n\lambda}{P} \qquad \cos(\theta_n) = \sqrt{1-\left(\frac{n\lambda}{P}\right)^2}$$

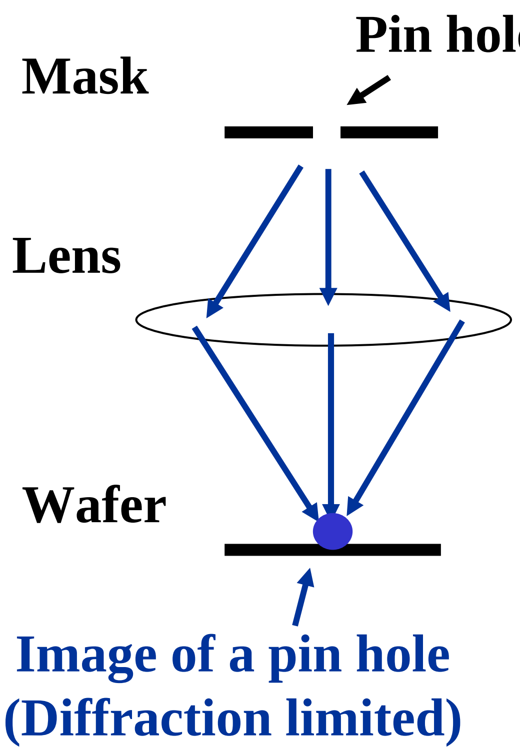
$$\overline{k_n} = k_{x_n}\hat{x} + k_{z\,n}\hat{z} = k_0 \sin(\theta_{x_n})\hat{x} + k_0 \cos(\theta_{z_n})\hat{z}$$

$$E_{TOTAL} = \sum_n E_n e^{-j(k_0 \sin(\theta_n)x + k_0 \cos(\theta n)z)} = \sum_n E_n e^{-j(\overline{k}_n \cdot \overline{x})}$$

Three wave case for on-axis illumination of mask with period P

$$E_{TOTAL} = E_{-1}e^{-j(-\frac{2\pi}{P}x+\frac{2\pi}{\lambda}\cos(\theta_{-1})z)} + E_0 e^{-j(0\cdot\frac{2\pi}{P}x+\frac{2\pi}{\lambda}\cos(\theta_0)z)} + E_{+1}e^{-j(\frac{2\pi}{P}x+\frac{2\pi}{\lambda}\cos(\theta_{+1})z)}$$

Optical System Point Spread Function



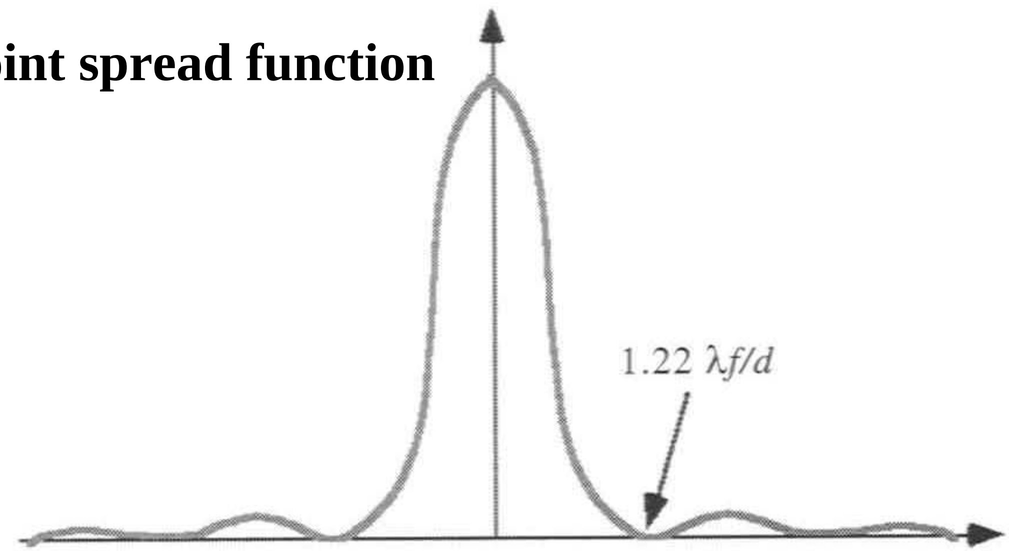
Relationship for electric fields

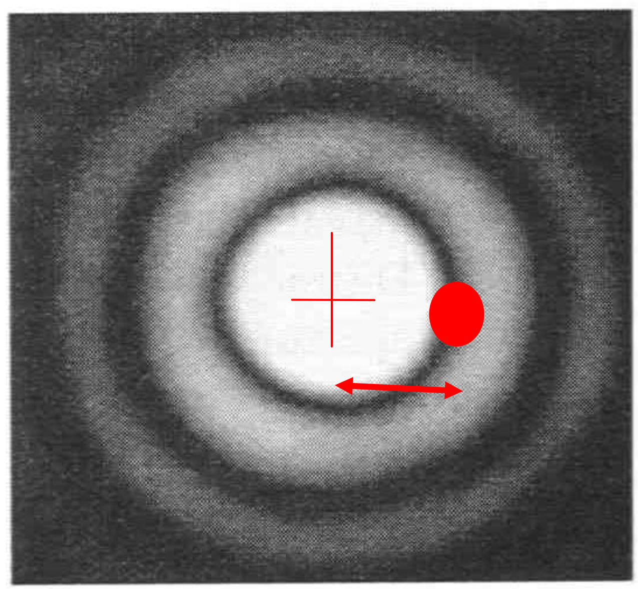
- The small pinhole due to its size diffracts uniformly over all angles.
- This diffraction uniformly fills the lens pupil.
- The lens re-phases the remaining emerging rays so that they re-converge at the wafer with the same relative phases and uniform magnitude.
- The electric field at the waver is thus the inverse Fourier transform of a disk = Airy Function.
- The intensity is the time average of the square of the electric field = (Airy function)^2
- The pattern shape is independent of the shape of the pin hole with diameter 1.22λ/NA.
- The peak E is proportional to pin hole area the peak I is proportional to Area^2 or (dimension)^4.

Resolution in Projection Printing

f = focal distance
d = lens diameter

Point spread function





Minimum separation of a star to be visible.

Null position

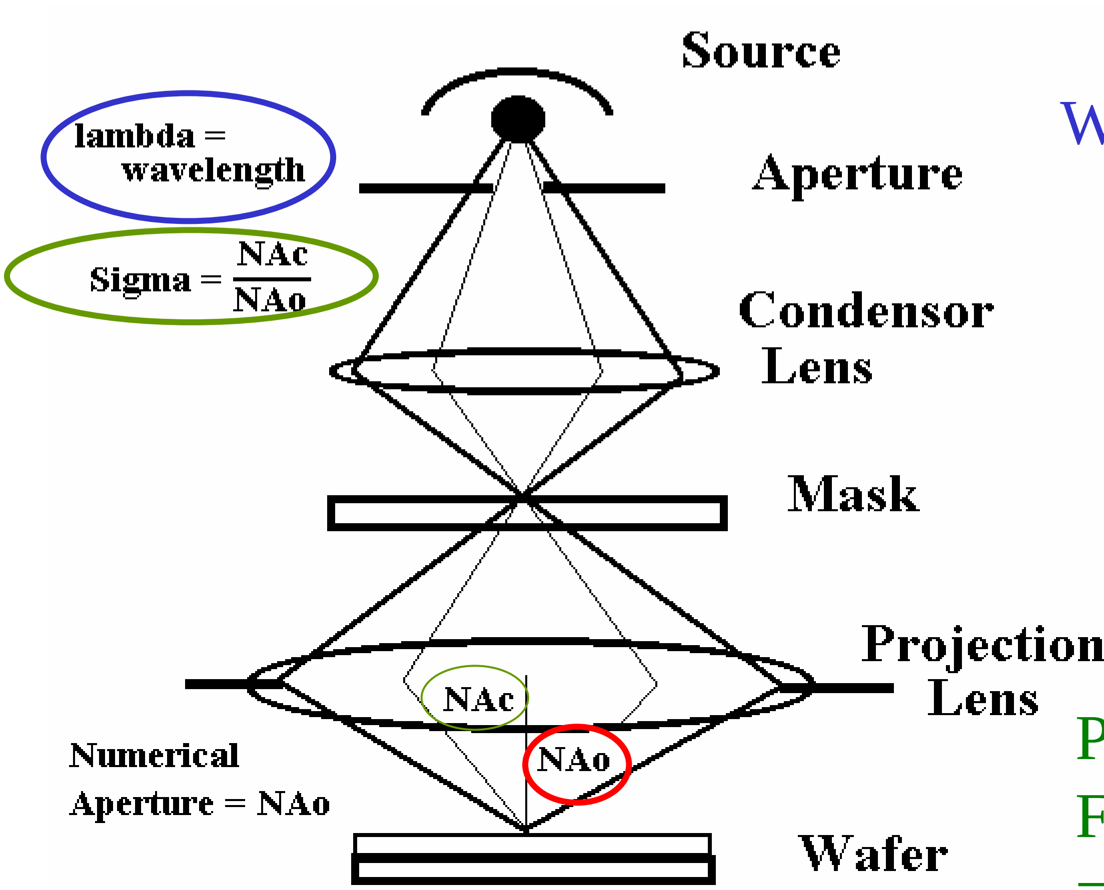
$$1.22\lambda\left(\frac{f}{d}\right) = 0.61\lambda\left(\frac{f}{\frac{d}{2}}\right) = 0.61\frac{\lambda}{NA}$$

F# = f/d

PDG Fig. Ch 5

Optical Projection Printing Parameters

#0 Key Parameters: λ, NA, σ



Wavelength λ = 248 nm)

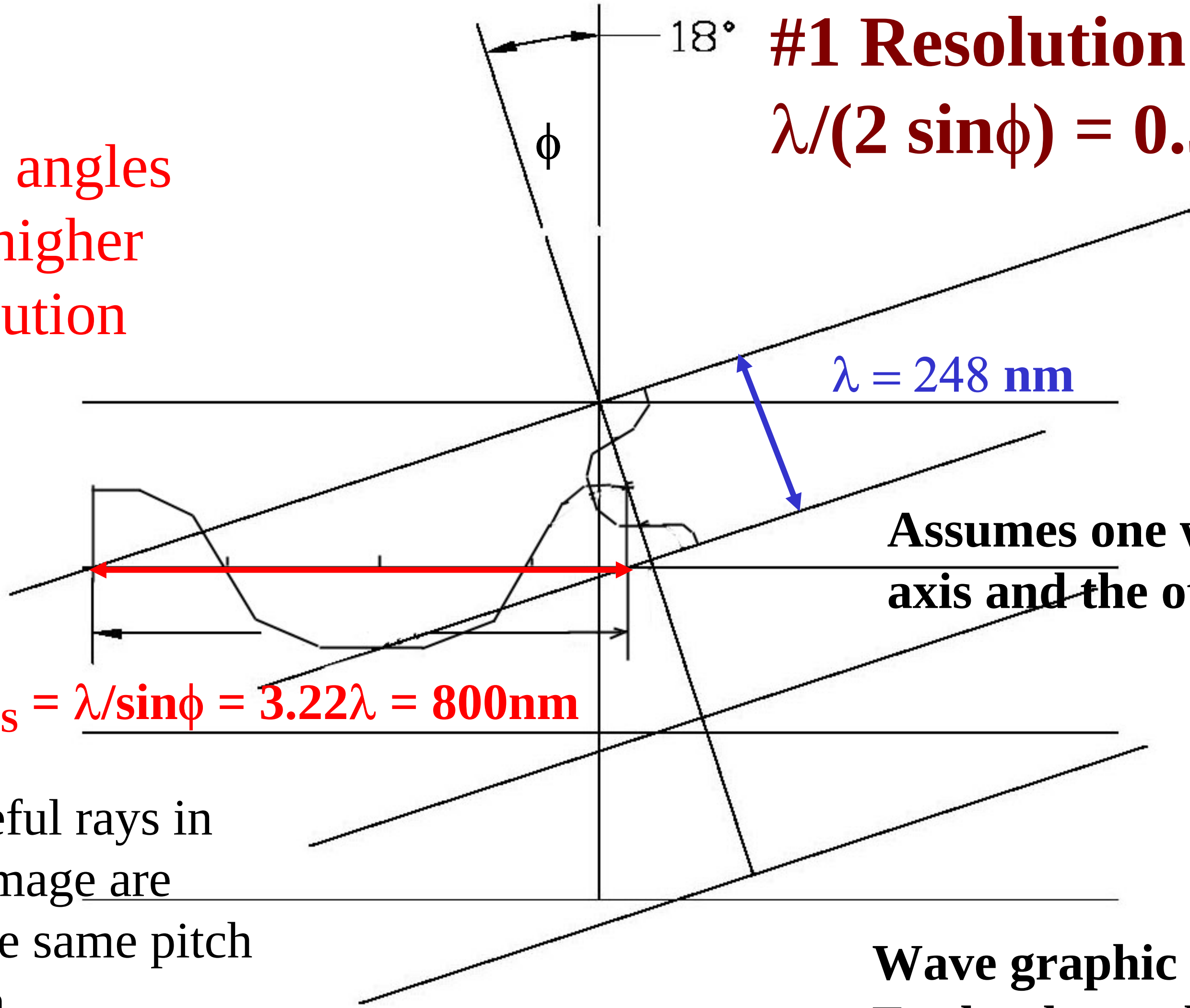
Numerical Aperture
NA = sin (θ) = 0.5

Partial Coherence Factor σ = (NAc/NAo) = 0.3

Resolution ~ Transverse Variation

#1 Resolution = P/2 = $\lambda/(2 \sin\phi) = 0.5(\lambda/NA)$)

Larger angles give higher resolution



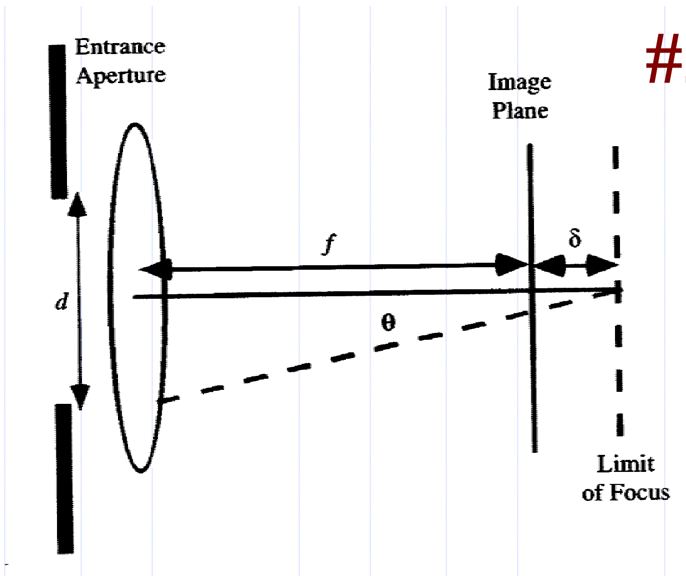
$\lambda_{TRANS} = \lambda/\sin\phi = 3.22\lambda = 800$nm

Assumes one wave is on-axis and the other off-axis

The most useful rays in forming an image are those with the same pitch as the pattern

Wave graphic by Ongi Englander and Kien Lam

Depth of Focus in Projection Printing



#2 Depth of Focus = $\lambda/(2NA^2)$

$$\lambda/4 = \delta - \delta\cos\Theta$$

$$\lambda/4 = \delta\left[1 - \left(1 - \frac{\Theta^2}{2}\right)\right] \cong \delta\frac{\Theta^2}{2}$$

$$\Theta \cong \sin\Theta = \frac{d}{2f} = NA$$

$$\therefore\ DOF = \delta = \pm\frac{\lambda}{2(NA)^2} = \pm k_2\frac{\lambda}{(NA)^2}$$

Result must be modified for

a) **High NA, and**

b) **Two waves at arbitrary angles.**

PDG Fig. Ch 5

Parameters for Microlab Projection Printers

Working Resolution

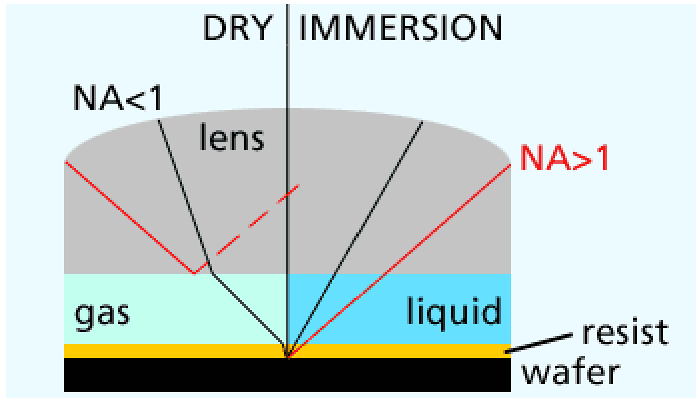
Tool	λ nm	NA	σ	k_1	θ_{LEN} deg	θ_{ILL} deg	$k_1\lambda/NA$ nm	$\lambda/(4NA)$ nm	TFR nm	M
Canon-gh	436 405	0.28	0.7	0.8	16	11	1250	390	5500	4
GCA-g	436	0.28	0.7	0.8	16	11	1250	390	5500	10
GCA-i	365	0.32	0.5	0.8	19	13	900	285	3500	10
ASML-DUV	248	0.5	0.25	0.7	30	7.2	350	125	990	5

TFR = Total focus range = 2 x Rayleigh Depth of Focus = 2DOF

M is the demagnification factor

$$L_{LINEWIDTH} = k_1 \frac{\lambda}{NA} \qquad DOF = k_2 \frac{\lambda}{2(NA)^2}$$

Immersion Lithography



- Concept
  - Imaging in a liquid medium with refractive index n offers an a factor of n reduction in resolution
  - n_{WATER} @ 193 nm = 1.44 to 1.46
  - n_{FUTURE} @ 193nm = 1.2 to 1.5?
- Implementation: Drop and Drag
  - Dispense water from front side of lens, use the surface tension to make the drop follow the lens, and suck in the liquid on the back of the lens.

EUV Projection (X-Ray) Lithography

13.5 nm Sn

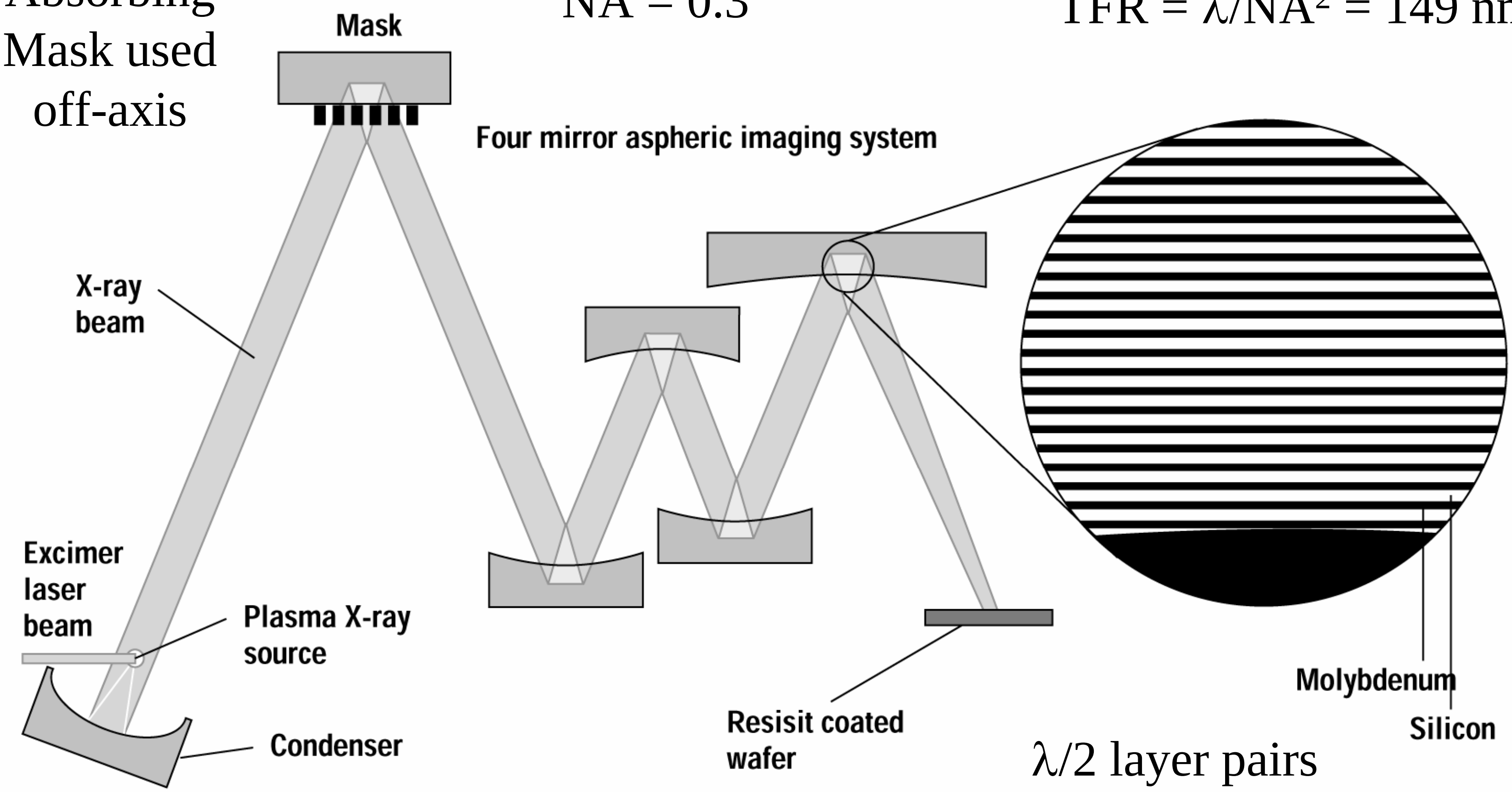
13.4 nm wavelength

NA = 0.3

$0.7\lambda/NA = 31$ nm

$TFR = \lambda/NA^2 = 149$ nm

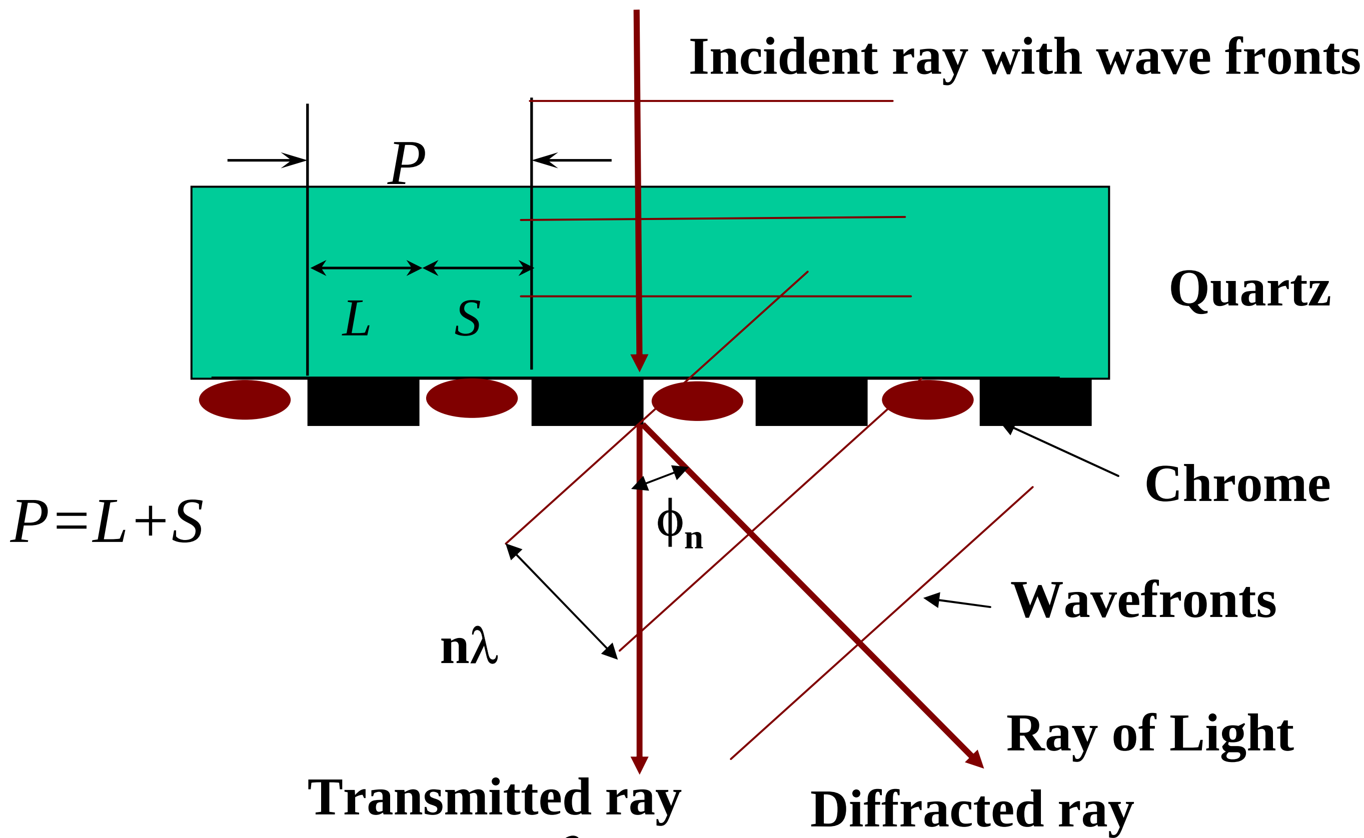
Absorbing Mask used off-axis



λ/2 layer pairs

Figure 9.24 An x-ray projection lithography system using x-ray mirrors and a reflective mask *(after Zorpette, reprinted by permission, © 1992 IEEE).*

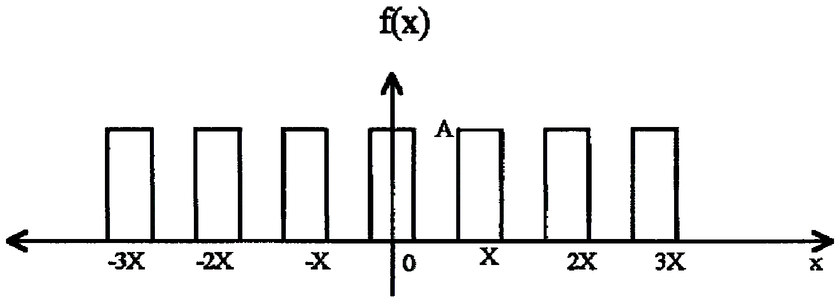
Bragg Condition



$$P \sin \varphi_n = n\lambda$$

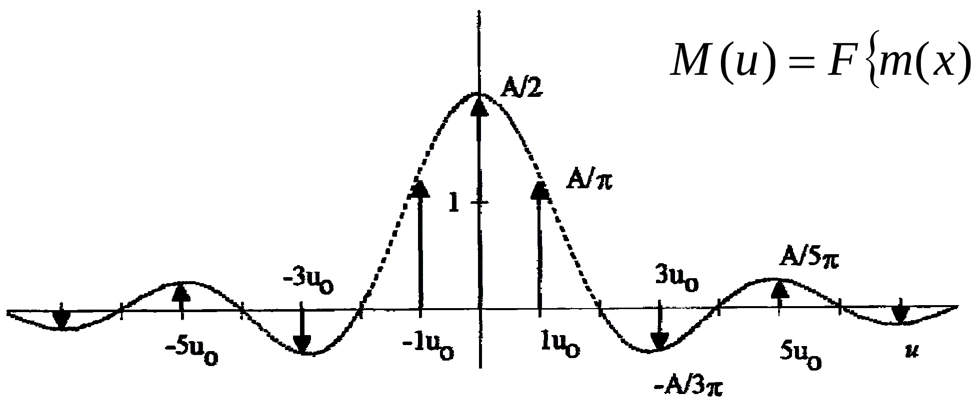
#3 The Bragg condition sets the diffraction angles

Electric Field Spectrum M(u) from E(x)



$$m(x) = A \cdot rect\left(\frac{x}{P/2}\right) * comb\left(\frac{x}{P}\right)$$

Figure 18 A periodic rectangular wave, representing dense mask features.



$$M(u) = F\{m(x)\} = F\left\{A \cdot rect\left(\frac{x}{P/2}\right)\right\} \cdot F\left\{comb\left(\frac{x}{P}\right)\right\}$$

$$M(u) = \frac{A}{2} \operatorname{sin}c\left(\frac{u}{2u_0}\right) \sum_{n=-\infty}^{\infty} \delta(u - u_0)$$

Figure 19 The amplitude spectrum of a rectangular wave, A/2 sinc($u/2u_0$). This is equivalent to the discrete orders of the coherent Fraunhofer diffraction pattern.

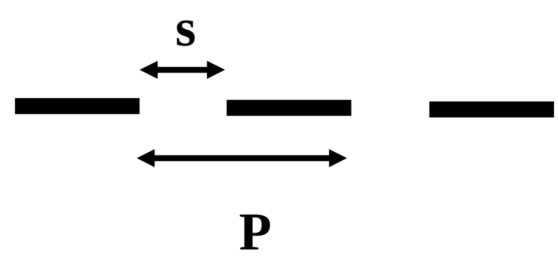
u_0 = 1/P

Values are ½, $1/\pi$, $1/3\pi$, $1/5\pi$

Sheats and Smith

Electric Field: Sinusoids

Binary Mask with period P and opening space s



When filtered to three waves (0, +1, and -1)

$$E(x) = E_0 + 2E_1 \cos\left(\frac{2\pi x}{P}\right)$$

$$E_n = \frac{A}{2}\frac{\sin\left[n\pi\left(s/P\right)\right]}{\left[n\pi\left(s/P\right)\right]}$$

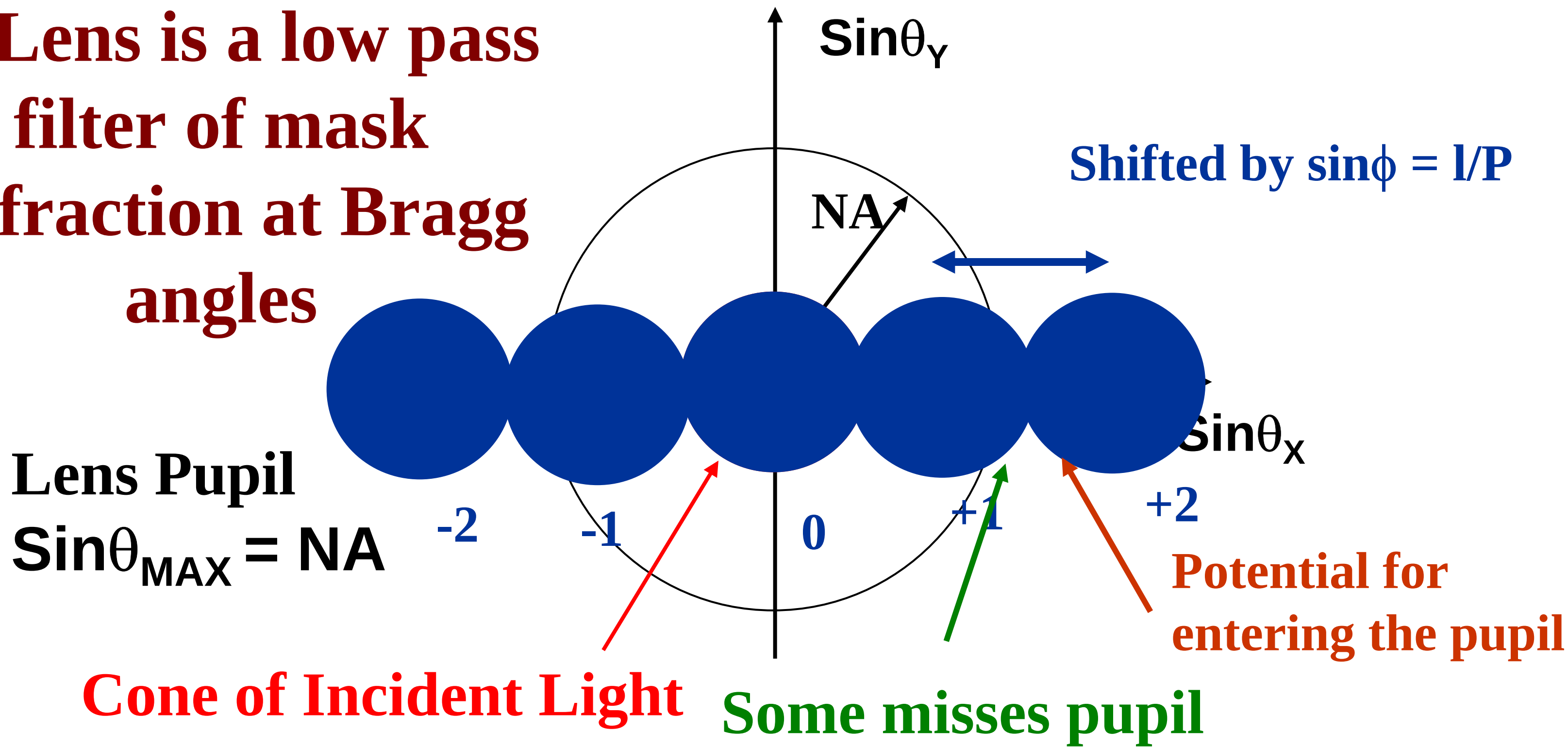
$k_{x1} = 2\pi/P$

When s = P/2

$$E(x) = 0.5 + \left(\frac{2}{\pi}\right)\cos\left(\frac{2\pi x}{P}\right)$$

Pupil Wave Traffic: Partial Coherence

#4 Lens is a low pass filter of mask diffraction at Bragg angles



Diffracted Orders from a mask with period P

Intensity as Square of Electric Field

The energy carried by a wave and the work done on a material are proportional to the time average of the square of the electric field.

Thus the intensity is proportional to E^2 when the field is real and EE* when phasors are used and E is complex.

Intensity = EE* gives

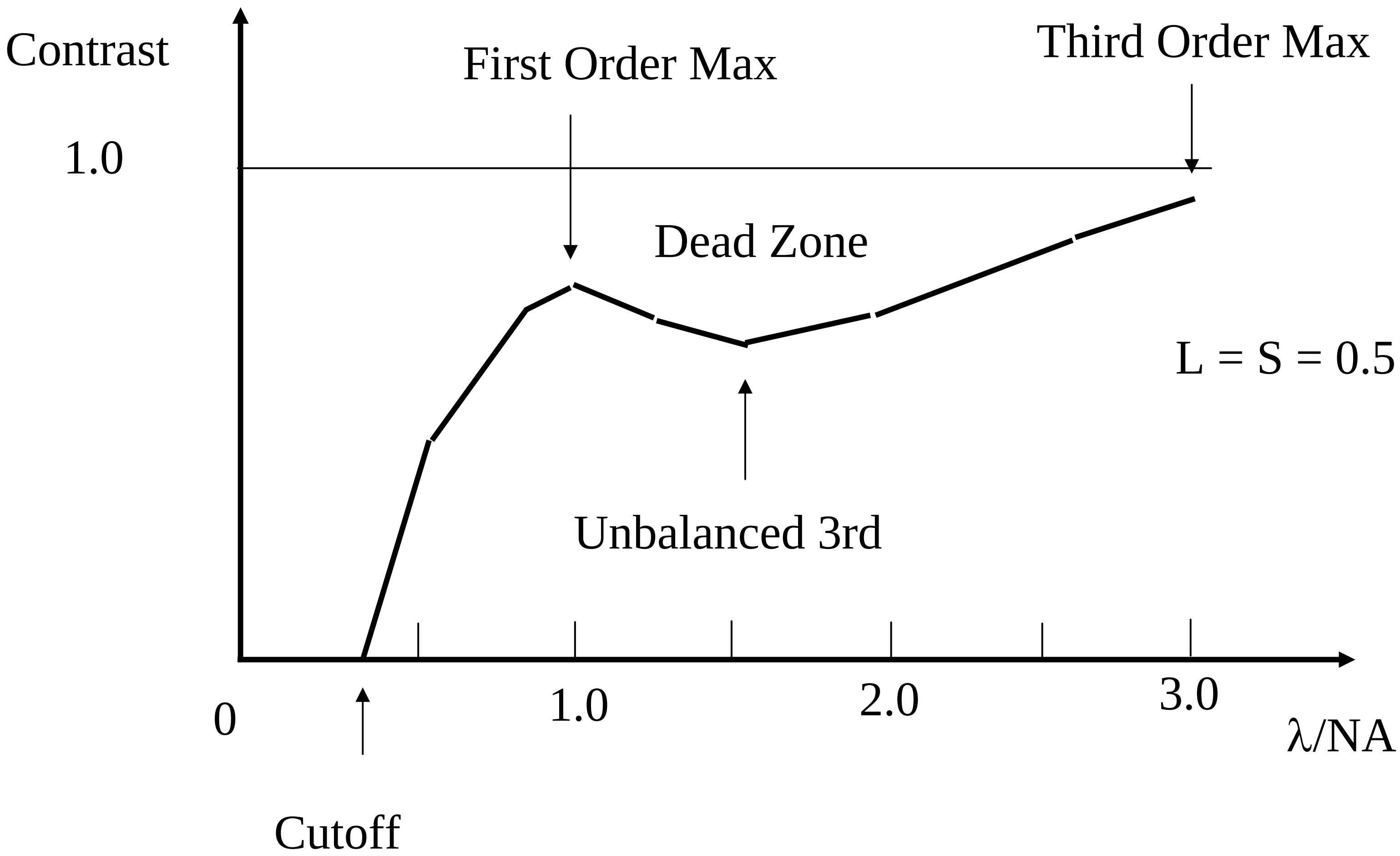
$$I(x) = EE^* = E_0^2 + 2E_0E_1\cos\left(\frac{2\pi x}{P}\right) + 4E_1^2\cos^2\left(\frac{2\pi x}{P}\right)$$

Since the Fourier transform converges to the average at a discontinuity, the electric field at the mask edge will be about 0.5, and the intensity at a mask edge will be about 0.25.

#5 The intensity at a mask edge is only 30% of the clear field intensity regardless of feature type and size.

Contrast vs. Period

$$C = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$



Electric Field and Intensity: Defocus

$$E_{TOTAL} = E_{-1} e^{-j(-\frac{2\pi}{P}x + \frac{2\pi}{\lambda}\cos(\theta_{+1})z)} + E_0 e^{-j(0\cdot\frac{2\pi}{P}x + \frac{2\pi}{\lambda}\cos(\theta_0)z)} + E_{+1} e^{-j(\frac{2\pi}{P}x + \frac{2\pi}{\lambda}\cos(\theta_{+1})z)}$$

Note that $\cos(\theta_{-1}) = \cos(\theta_{+1})$ and $\cos(\theta_0) = 1$

$$\cos(\theta_n) = \sqrt{1 - \left(\frac{n\lambda}{P}\right)^2}$$

And that $E_{-1} = E_{+1}$ for a binary (real transmission function) mask

$$E_{TOTAL} = +E_0 e^{-j(\frac{2\pi}{\lambda}z)} + 2E_{+1}\cos(\frac{2\pi x}{P}) e^{-j(+\frac{2\pi}{\lambda}\cos(\theta_{+1})z)}$$

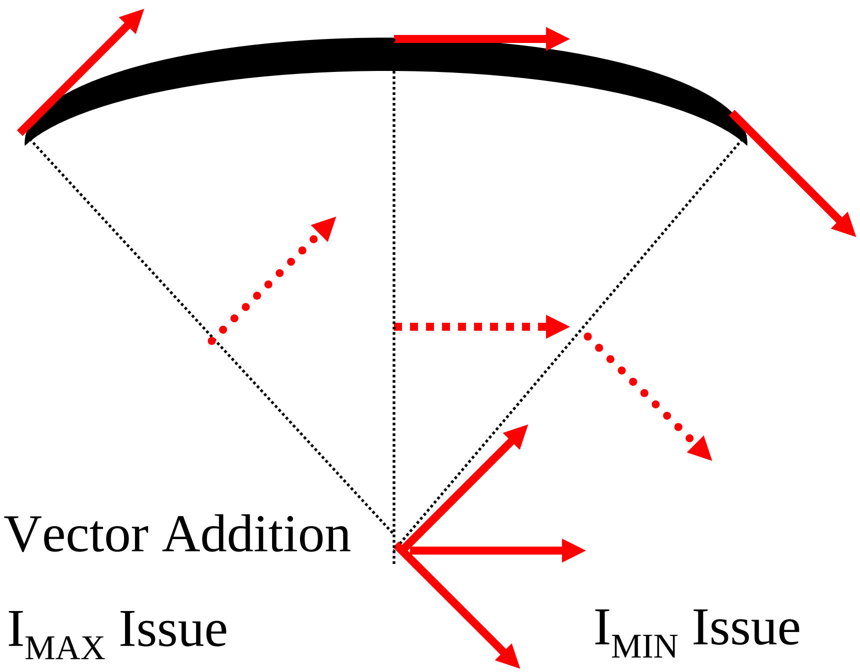
The Rayleigh defocus z value to give $\lambda/4$ is

$$z = \frac{\lambda}{4\left|1 - \cos(\theta_{+1})\right|}$$

As expected the on-axis and off-axis parts are 90º out of phase.

Polarization Effects at High NA

Parallel Orientation

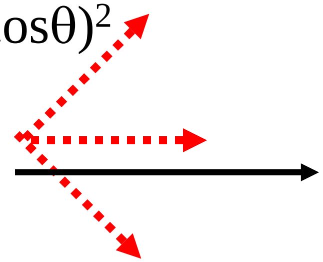


Vector Addition

I_{MAX} Issue

$E_X \sim \cos\theta$

$I_{MAX} \sim (\cos\theta)^2$



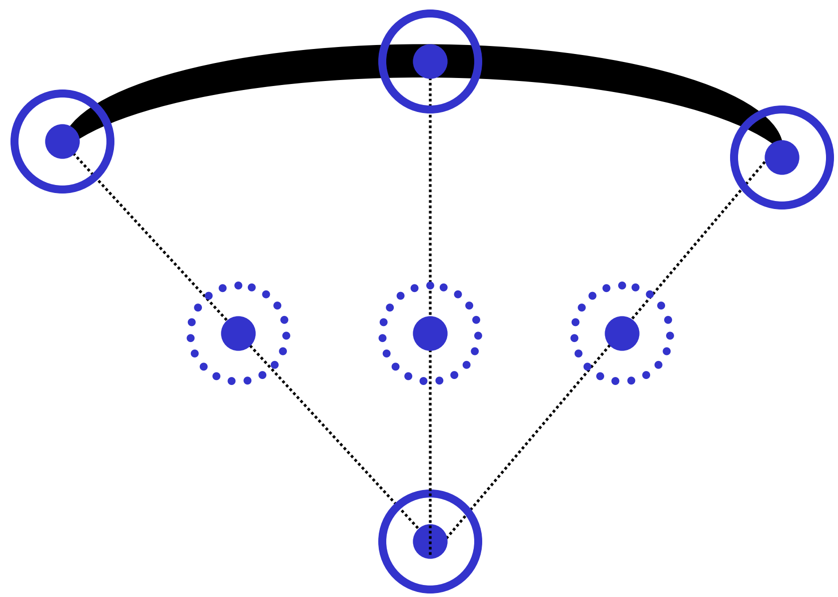
I_{MIN} Issue

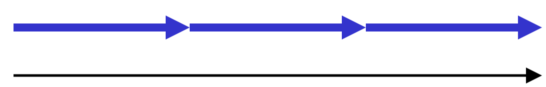
$E_Z \sim \sin\theta$

$I_{MIN} \sim (\sin\theta)^2$

$\vec{E}_{total}$

Perpendicular Orientation

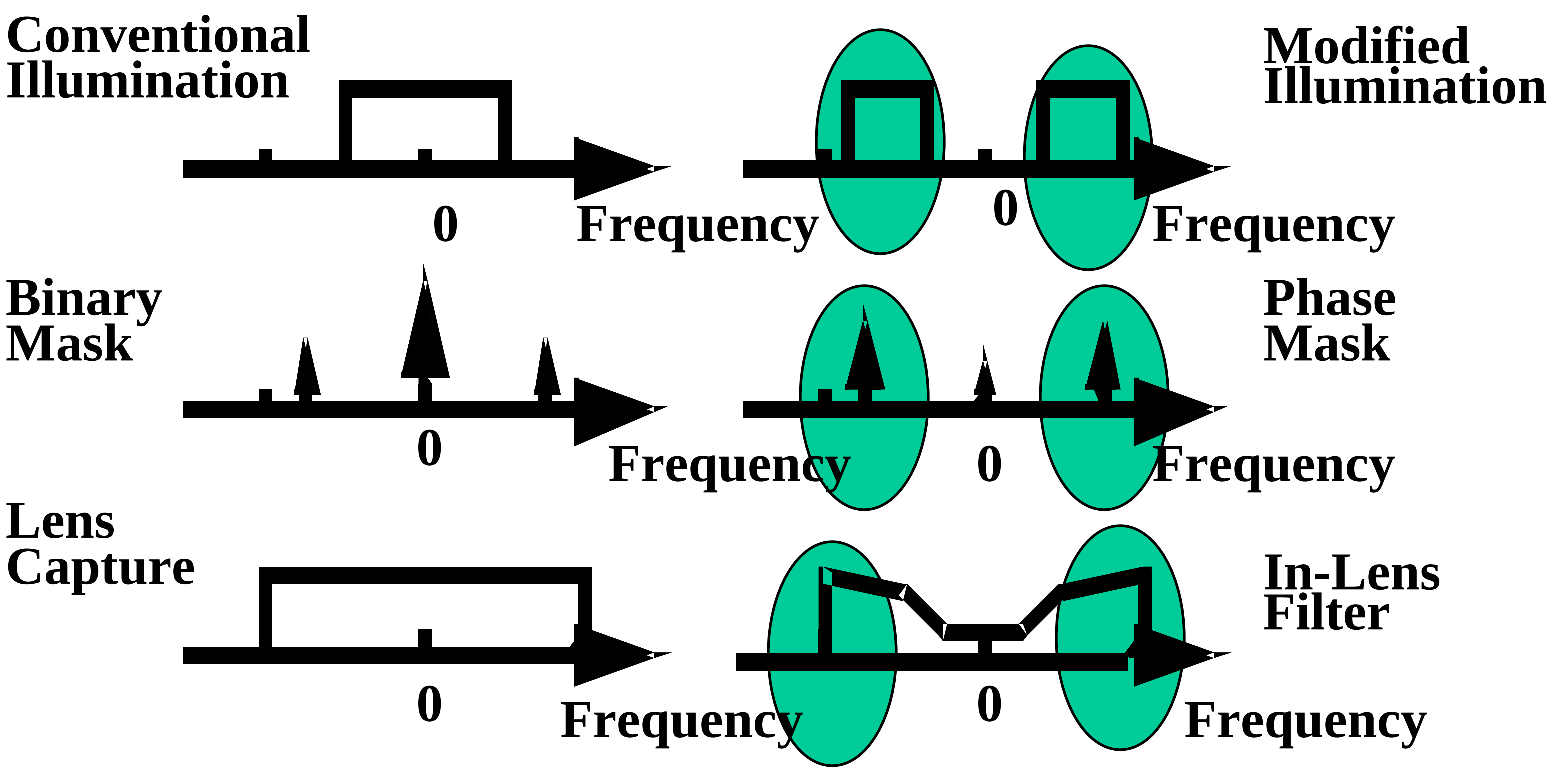




$\vec{E}_{total}$

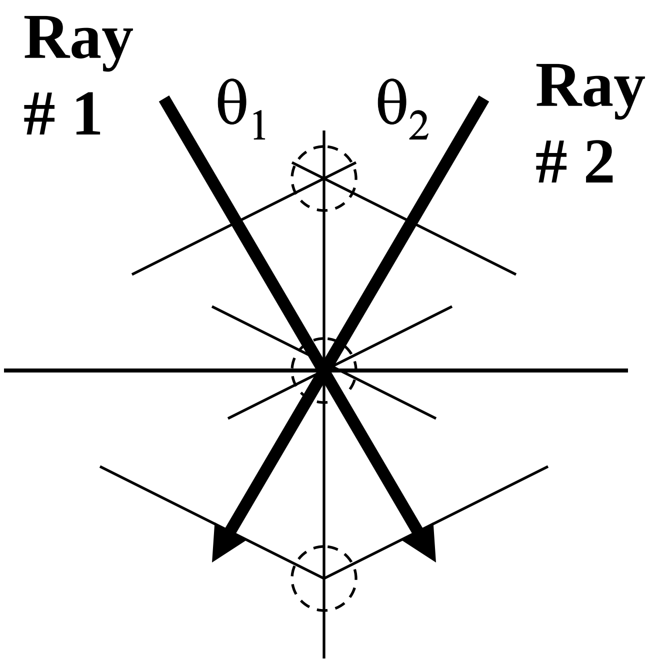
Resolution Enhancement Techniques

Resolution Enhancement Emphasizes High Frequencies



Bokor, Neureuther, Oldham, Circuits and Devices, 1996

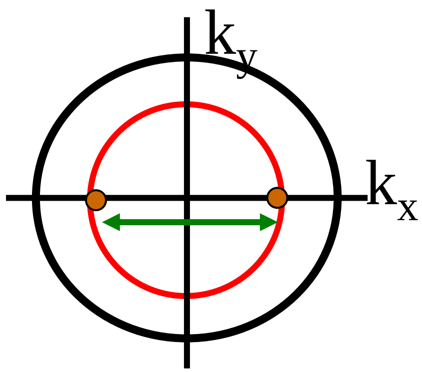
Two Ray Infinite DOF



When $\theta_1 = \theta_2$ the contributions from Ray #1 and Ray #2 track each exactly with axial distance and an INFINITE depth of focus is produced.

$$Period = Pitch = \frac{2\pi}{\Delta k_{Transverse}}$$

$$\Delta k_{Transverse} = 2k_0 \sin(\theta)$$



$$Pitch = \frac{\lambda}{2\sin(\theta)} \xrightarrow{\sin(\theta)=NA} \frac{\lambda}{2NA}$$

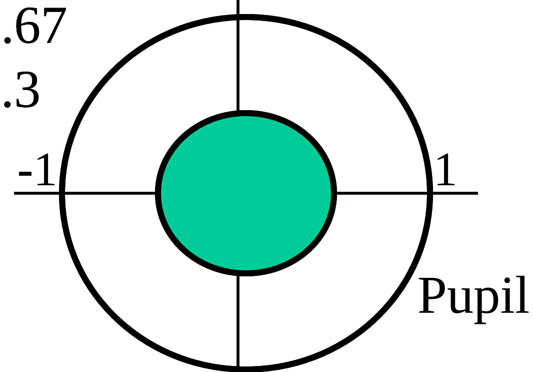
Doubled Resolution! With infinite DOF

Illumination Schemes

Top Hat – General Shapes

$k_1 = 0.67$

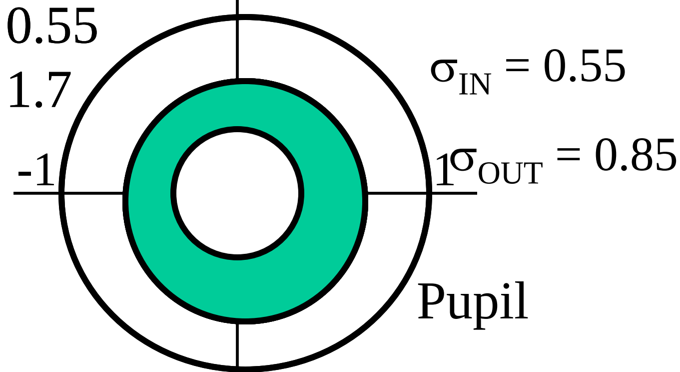
$k_2 = 1.3$



Annular – DOF, Contacts

$k_1 = 0.55$

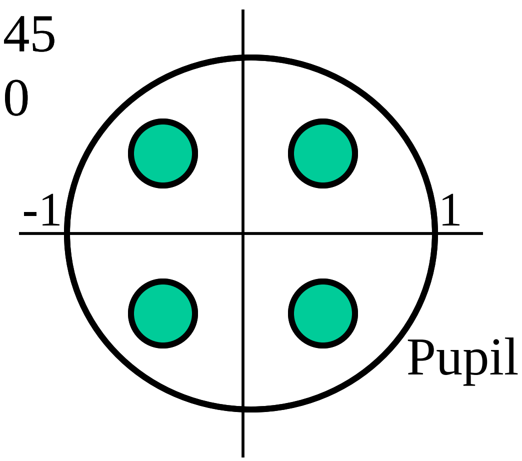
$k_2 = 1.7$



Quadruple – H,V lines, DOF

$k_1 = 0.45$

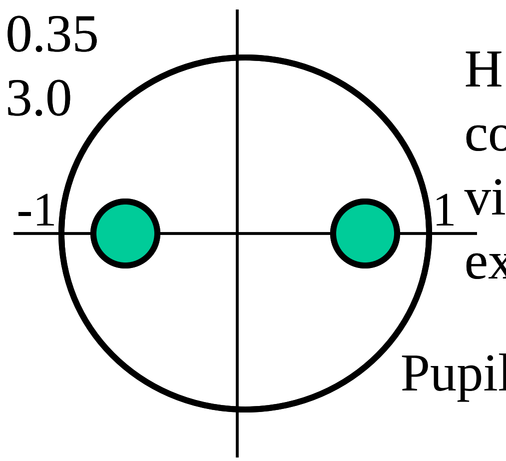
$k_2 = 2.0$



Dipole – V lines, DOF

$k_1 = 0.35$

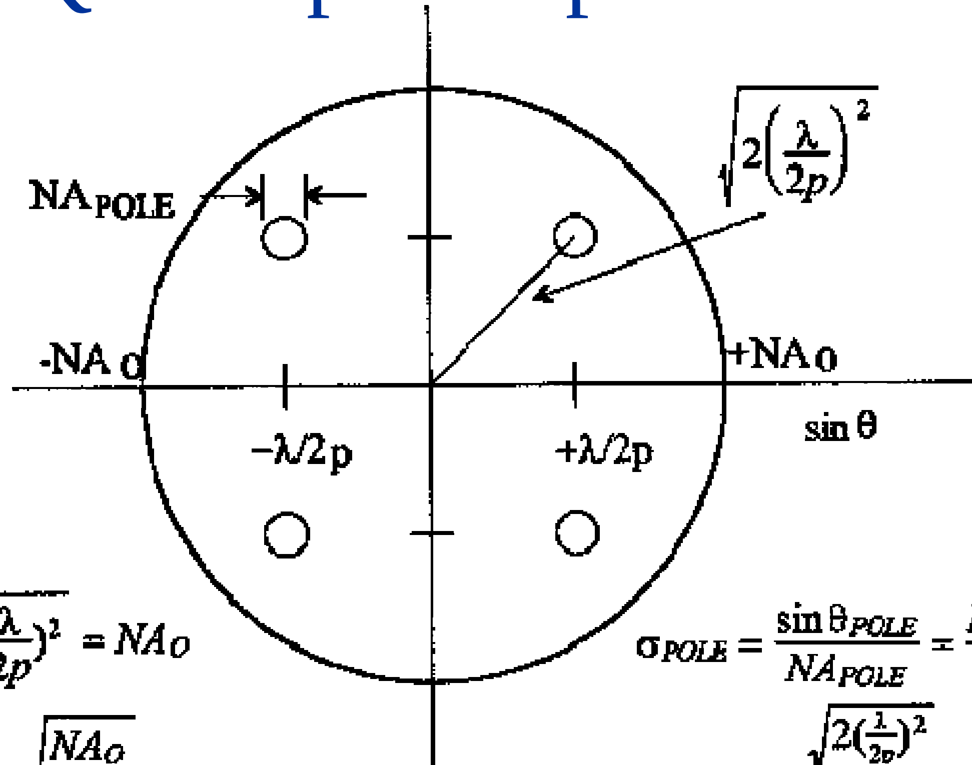
$k_2 = 3.0$



H lines and contacts formed via a double exposure

- The k1 factor is inversely proportional to the lateral separation of the illumination $k_1 = 1/(2 \text{ x separation})$

Quadrapole Optimization



If $\sqrt{2(\frac{\lambda}{2p})^2} = NA_O$

$$\frac{\lambda}{2p} = \sqrt{\frac{NA_O}{2}}$$

$$R_{min} = \frac{\lambda}{4\sqrt{\frac{NA_O^2}{2}}}$$

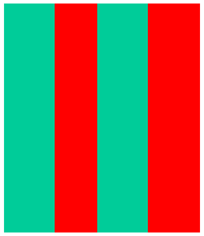
$$\sigma_{POLE} = \frac{\sin\theta_{POLE}}{NA_{POLE}} = \frac{NA_{POLE}}{NA_O}$$

$$\sigma_{CENTER} = \frac{\sqrt{2(\frac{\lambda}{2p})^2}}{NA_O}$$

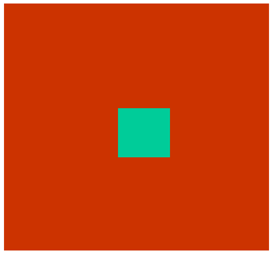
Dipole illumination can print 0.707 smaller lines but lines only.

Figure 56 Optimal quadrupole illumination with diagonal poles. Pole size and position can be specified in relative sigma values, σ_{pole} and σ_{center}. Minimum resolution (R_{min}) is also derived.

Phase-Shifting Mask Types



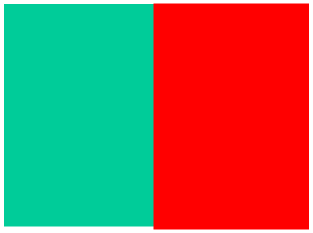
Alternating (Strong)



Attenuating (Weak)

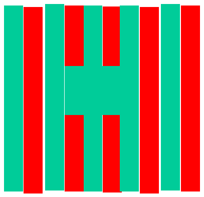
Used for Contacts

6% to 10% gives slope improvement of 30%. Sidelobe issue.



Phase Edge

Requires second trim mask exposure.



Chromeless (Only 0 order)

Attenuating Phase-Shifting Masks

Intensity of 6% comes from an electric field of -0.25

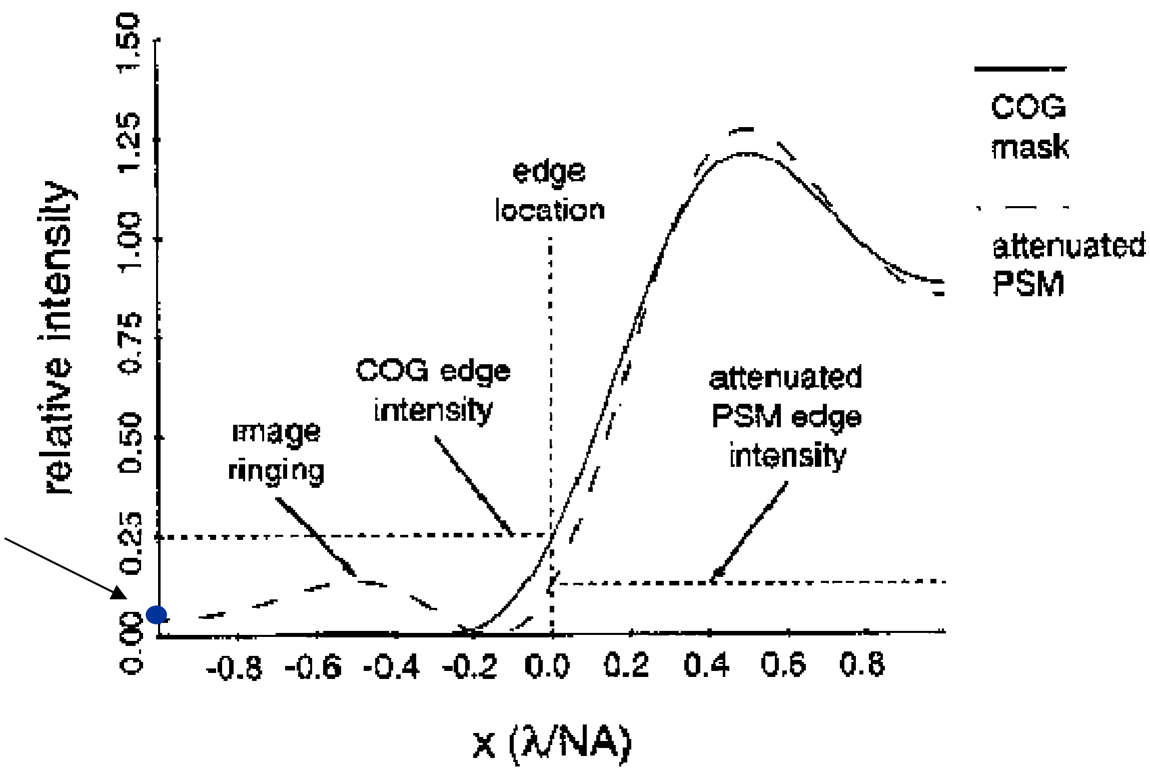


Figure 6.2: Coherent images of an edge.

Going from positive electric fields to negative electric fields increases edge slope and creates darker intensity near edge.

Alternating Phase-Shifting Mask

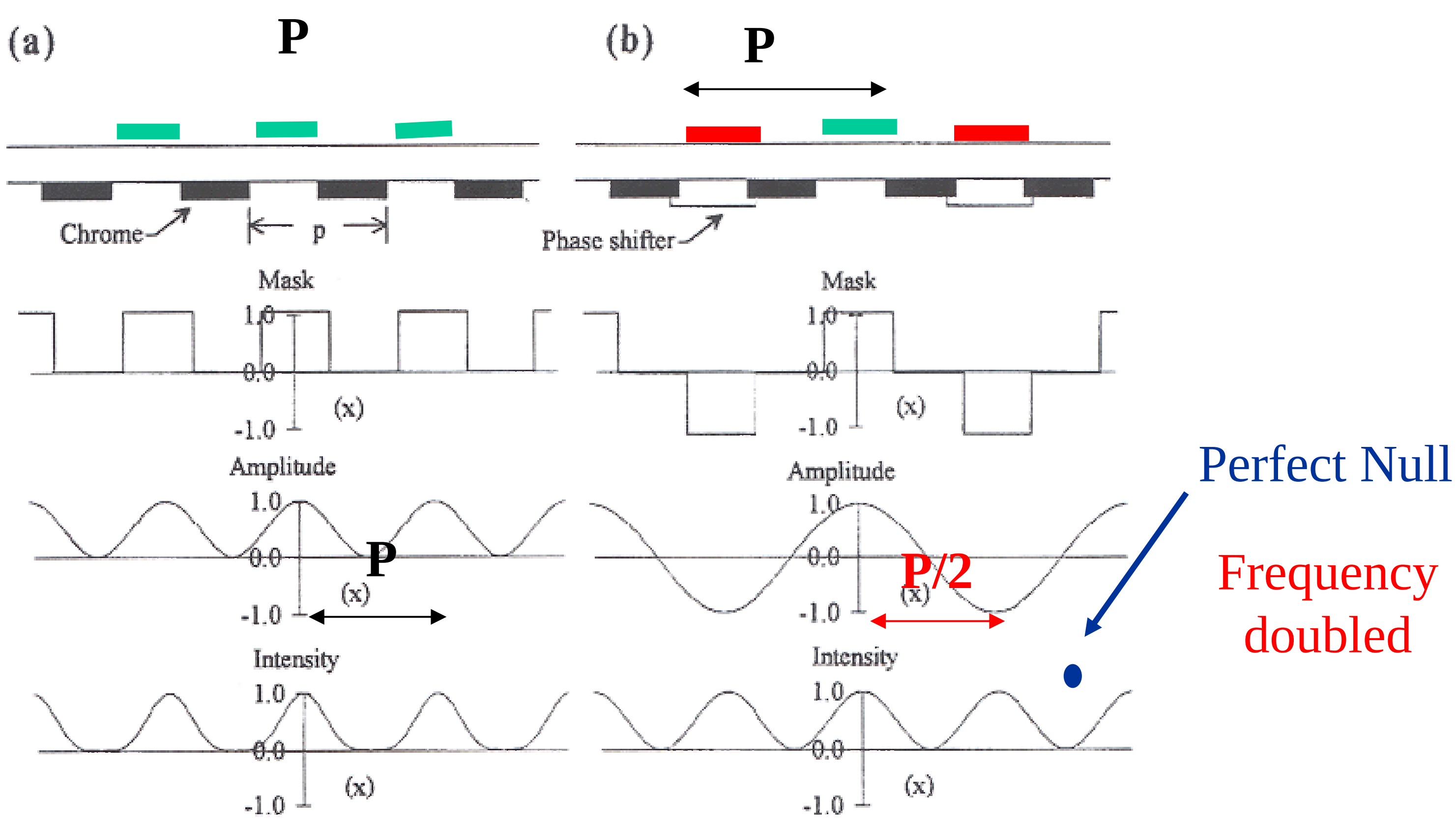


Figure 64 Schematic of (a), a conventional binary mask (b) an alternating phase shift mask. The mask electric field, image amplitude, and image intensity is shown for each.

Sheats and Smith

Phase-Shifting Mask: Electric Fields

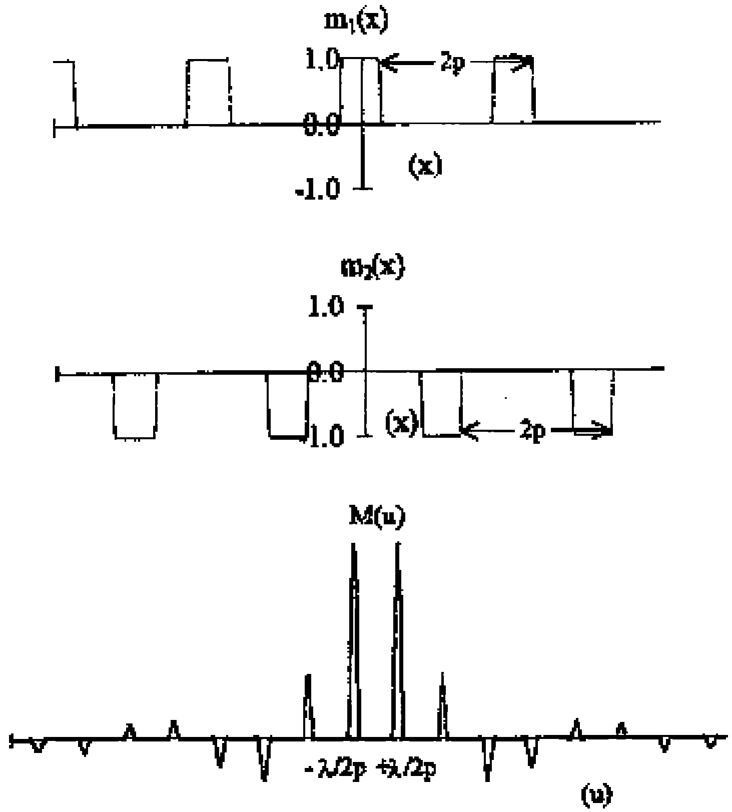


Figure 65 Spatial frequency distribution m(*u*) resulting from coherent illumination of an alternating phase shift mask, as decomposed into $m_1(x)$ and $m_2(x)$.

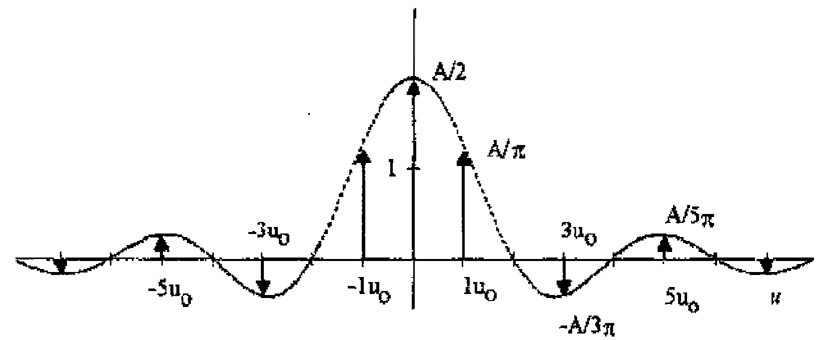


Figure 19 The amplitude spectrum of a rectangular wave, A/2 sinc($u/2u_0$). This is equivalent to the discrete orders of the coherent Fraunhofer diffraction pattern.

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