EE243 Advanced Electromagnetic Theory

Lec # 19 Coupled Mode Theory (Cont.)

- **Waveguide Deformations**
- **Coupling Polarization (Et, Ez)**
- **Coupled-Wave Solutions**
  - **Directional Coupler**
  - **Contra-Directional Coupler**
  - **Periodic Couplers**

Reading: Kogelnik 2.6

Overview

The derivation of coupled modes will be expanded to represent the Δ polarization contribution from other modes and then solutions derived for prototypical applications.

- Coupling can be factored to Et and Ez and misalignment of k-vectors can be included
- Solutions for
  - Directional Couplers
  - Contra Directional Couplers (Distributed Brag Reflectors)
  - General Periodic Couplers including stop bands

Waveguide Deformations

Kogelnik 2.6.16 17,18

Kogelnik 2.6

$$\overline{P}_t = \Delta\varepsilon \overline{E}_t = \Delta\varepsilon \sum (a_v + b_v)\overline{E}_{tv}$$

$$j\omega(\varepsilon + \Delta\varepsilon)\overline{E}_z = \nabla_t \times \overline{H}_t$$

$$P_z = \Delta\varepsilon E_z = \frac{\Delta\varepsilon}{\varepsilon + \Delta\varepsilon}\frac{1}{j\omega}\nabla_t \times \overline{H}_t$$

$$P_z = \frac{\Delta\varepsilon}{\varepsilon + \Delta\varepsilon}\frac{1}{j\omega}\sum (a_v - b_v)\nabla_t \times \overline{H}_t$$

$$P_z = \frac{\Delta\varepsilon}{\varepsilon + \Delta\varepsilon}\sum (a_v - b_v)E_{zv}$$

- Break up ΔP into Transverse and Longitudinal E field components
- Find Pt and Pv in terms of mode components for Et and Ez contributions

Waveguide Deformations (Cont.)

Kogelnik 2.6

$$K_{vu}^{t} = \omega \iint_{\infty} dxdy \Delta\varepsilon \overline{E}_{tv} \cdot \overline{E}_{tu}^{*}$$

$$K_{vu}^{z} = \omega \iint_{\infty} dxdy \frac{\varepsilon}{\varepsilon + \Delta\varepsilon} \Delta\varepsilon \overline{E}_{zv} \cdot \overline{E}_{zu}^{*}$$

$$A_u' = -j \sum \begin{Bmatrix} A_v \left(K_{vu}^{t} + K_{vu}^{z}\right) e^{-j(\beta_v - \beta_u)z} \\ + B_v \left(K_{vu}^{t} - K_{vu}^{z}\right) e^{j(\beta_v + \beta_u)z} \end{Bmatrix}$$

$$B_u' = j \sum \begin{Bmatrix} A_v \left(K_{vu}^{t} - K_{vu}^{z}\right) e^{-j(\beta_v + \beta_u)z} \\ + B_v \left(K_{vu}^{t} + K_{vu}^{z}\right) e^{j(\beta_v - \beta_u)z} \end{Bmatrix}$$

- Substiture Pt and Pz contributions
- Introduce definitions of transverse and longitudinal K’s and rewrite

Waveguide Deformations: Physics

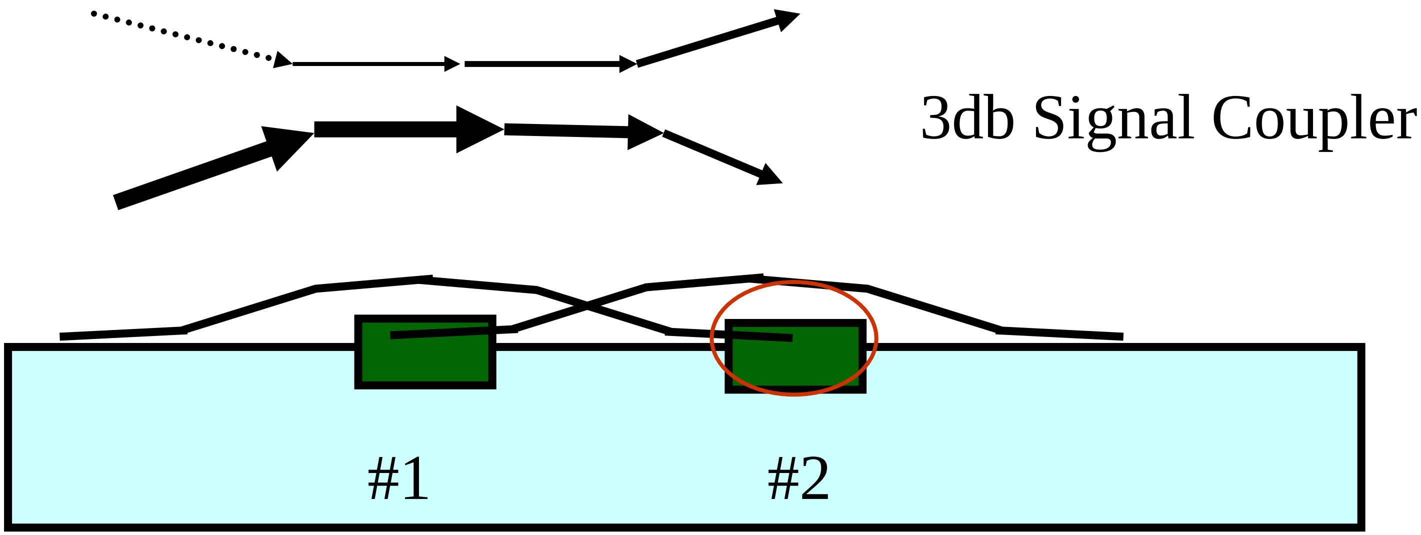
$$A_u' = -j\sum \left\{ \begin{aligned} & A_v\left(K_{vu}^t + K_{vu}^z\right)e^{-j(\beta_v-\beta_u)z} \\ & + B_v\left(K_{vu}^t - K_{vu}^z\right)e^{j(\beta_v+\beta_u)z} \end{aligned} \right\}$$

Kogelnik 2.6

$$B_u' = j\sum \left\{ \begin{aligned} & A_v\left(K_{vu}^t - K_{vu}^z\right)e^{-j(\beta_v+\beta_u)z} \\ & + B_v\left(K_{vu}^t + K_{vu}^z\right)e^{j(\beta_v-\beta_u)z} \end{aligned} \right\}$$

- These equations show the complete coupling of every mode to every other mode
- Criteria to pick out physics
  - Nearly synchronous (left side is slowly varying so can neglect combinations of u and v with large oscillation
  - Small Δε so mode nature is preserved (Alt. Super nodes)
- Direction
  - Co-directional (Au-Av) strong Kt+Kz
  - Contra-directional (Au -> Bv) weaker Kt-kz

Directional Coupler



- Waves on adjacent guides spillover laterally and induce additional polarization when they encounter dielectric material not in included in their normal structure definition.
- This additional polarization acts a source that excites the modes on the second structure.
- This can be used to design 3db directional couplers.
- Modes on the composite structure (super modes) can also be used.

Coupled-Wave Solutions: Co-Directional

Kogelnik 2.6.25-31

$$A' = -j\kappa B e^{-2j\delta z}$$

$$B' = -j\kappa A e^{2j\delta z}$$

$$A = Re^{-j\delta z}$$

$$B = Se^{j\delta z}$$

$$R' - j\delta R = -j\kappa S$$

$$S' + j\delta S = -j\kappa R$$

$$R(0) = 1$$

$$S(0) = 0$$

- Select subset of terms
- Remove residual lack of synchronization
- Coupled Equations
- Matrix solution
- Boundary conditions

$$S(z) = -j\kappa \sin\left(z\sqrt{\kappa^2 + \delta^2}\right) / \sqrt{\kappa^2 + \delta^2}$$

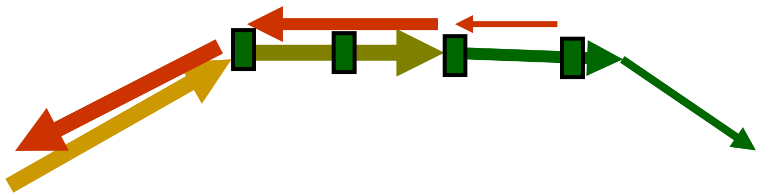
$$R(z) = \cos\left(z\sqrt{\kappa^2 + \delta^2}\right) + j\delta \sin\left(z\sqrt{\kappa^2 + \delta^2}\right) / \sqrt{\kappa^2 + \delta^2}$$

$$S(z) = -j\kappa \sin(\kappa z)$$

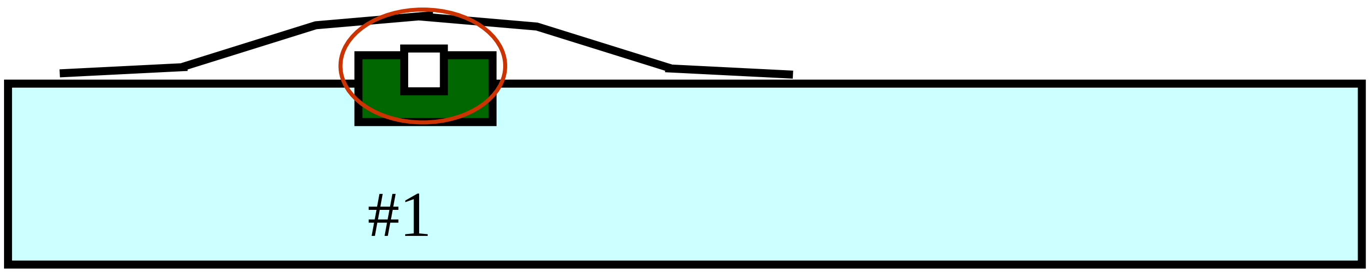
$$R(z) = \cos(\kappa z)$$

- Simplification for synchronous case

Stop Band (Photonic Crystal)



Periodic inserts spaced to coherently add for red



- Wave on guide encounters periodic insert not in included in their normal structure definition.
- This additional polarization acts a source that excites reverse modes in the same structure
- This can be used to design a frequency (color) dependent stop band.
- This is a 1-D form of a photonic crystal

Coupled-Wave Solutions: Contra-Directional

Kogelnik 2.6.33-40

$$A' = -j\kappa B e^{2j\delta z}$$

$$B' = j\kappa A e^{-2j\delta z}$$

$$A = \mathrm{R}e^{j\delta z}$$

$$B = Se^{-j\delta z}$$

$$R' + j\delta R = -j\kappa S$$

$$S' - j\delta S = +j\kappa R$$

$$R(0) = 1$$

$$S(L) = 0$$

- Select subset of terms
- Remove residual lack of synchronization
- Coupled Equations
- Matrix solution
- Boundary conditions

$$S(0) = -j\kappa / \left[\sqrt{\kappa^2 - \delta^2}\right] \coth\left(z\sqrt{\kappa^2 - \delta^2} + j\delta\right)$$

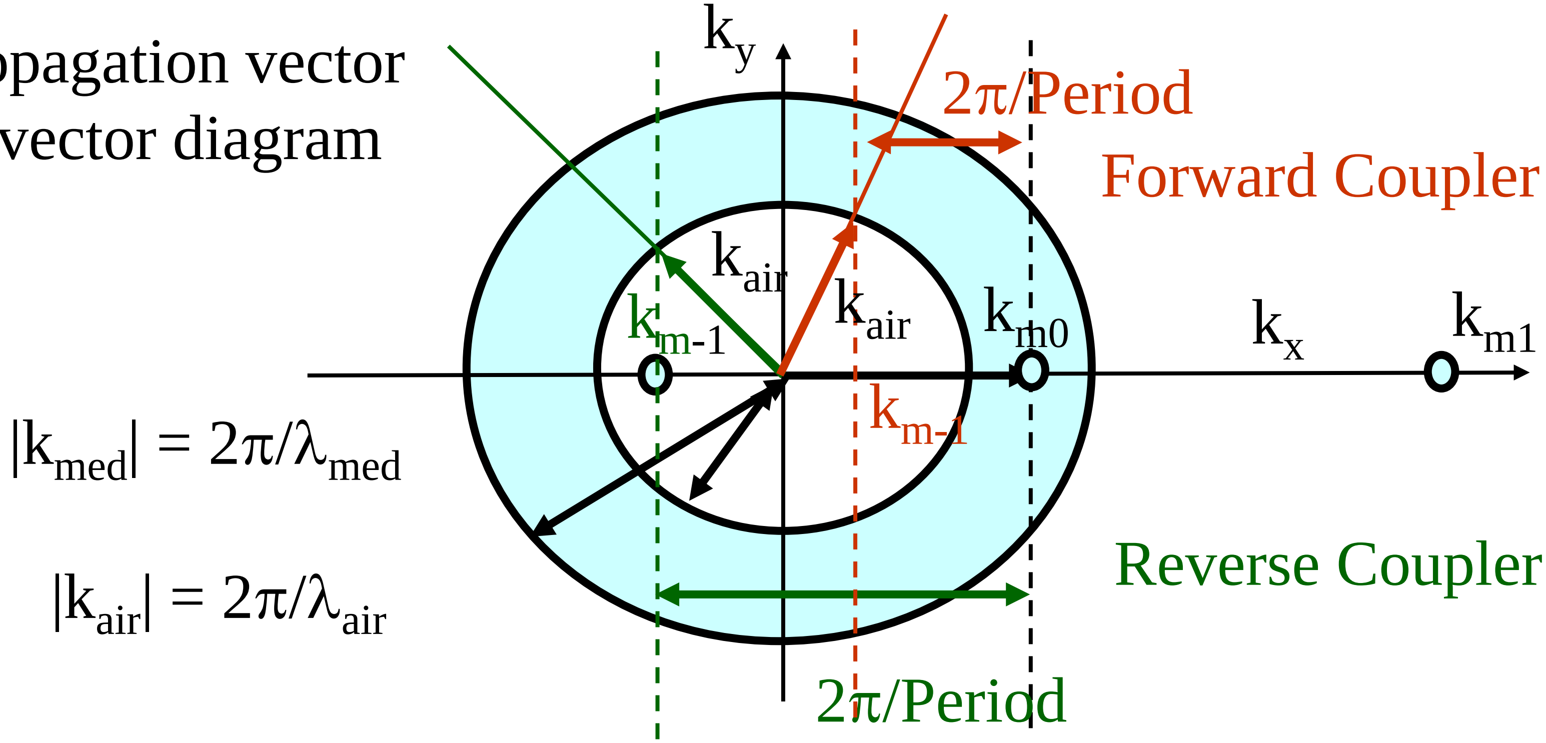
$$R(z) = \sqrt{\kappa^2 - \delta^2} / \left[\sqrt{\kappa^2 - \delta^2} \cosh\left(z\sqrt{\kappa^2 - \delta^2}\right) + j\delta \sinh\left(z\sqrt{\kappa^2 - \delta^2}\right)\right]$$

$$S(0) = -j \tanh(\kappa L)$$

$$R(l) = 1/\cosh(\kappa L)$$

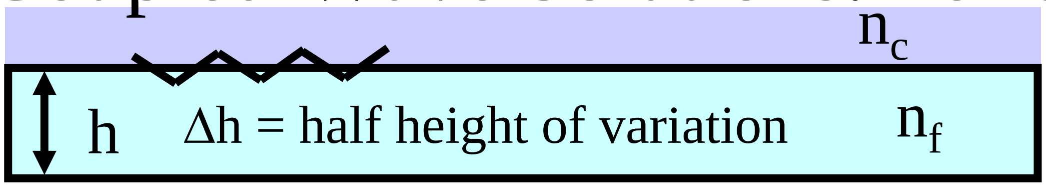
- Simplification for synchronous case

Periodic Wave Vectors



- The mode k-vector is larger than k_O and smaller than k_G
- The periodic coupling creates new k-vectors spaced by 2π/Period
- The new k-vectors within the k_0 circle correspond to radiation waves
- Forward coupling uses a larger period than backward coupling.

Coupled-Wave Solutions: Periodic Waveguides



Kogelnik 2.6.41-48

$$h(z) = h_0 + \Delta h \cos(Kz)$$

$$K = 2\pi / \Lambda$$

$$\Delta\varepsilon = \varepsilon_0\left(n_f^2 - n_c^2\right)$$

$$\Delta\varepsilon = -\varepsilon_0\left(n_f^2 - n_c^2\right)$$

$$K^1_{u,-u} = \omega \int_{-\infty}^{+\infty} dx \Delta\varepsilon E_y^2$$

$$K^1_{u,-u} \approx \omega E_c^2 \int_{-\infty}^{+\infty} dx \Delta\varepsilon$$

$$K^1_{u,-u} \approx \omega \varepsilon_0 E_c^2 \left(n_f^2 - n_c^2\right) \Delta h \left(e^{jKz} + e^{-jKz)}\right)$$

$$K^1_{u,-u} \approx \frac{\pi}{\lambda} \frac{\Delta h}{h_{eff}} \frac{n_f^2 - N^2}{N} \left(e^{jKz} + e^{-jKz)}\right)$$

- Film n_f plus cover n_c
- Sinusoidal height
- Period Λ and k-vector K
- Two Δε
- Ec is mode field at surface
- Δε z-variation produces (k-vector shift)
- N is the effective index

Periodic Waveguide Coupling Physics

Kogelnik 2.6

- Does not depend on substrate as ε difference and Ec exactly compensate
- Forward coupling strongest for 2δ = 2βu – K => K = $4\pi/\lambda_0$ and Period = λ/2

$$\delta = \beta_u - \beta_0 = \Delta\beta \approx \frac{d\beta}{d\omega}\Delta\omega$$

- Backward coupling is more complicated with higher harmonics due to discontinuities in fields at the boundary
- Generalize to cross-directional terms if Δε has off axis elements.