EE243 Advanced Electromagnetic Theory

Lec # 16 Dielectric Waveguides

- Recap Solutions for Homework Set 6
- Review for the Midterm Examination
- Dispersion Equation Dielectric Slab Waveguides
- Modes in Dielectric Slab Waveguides

Reading: Jackson 8.11 and Harrington 4.7.

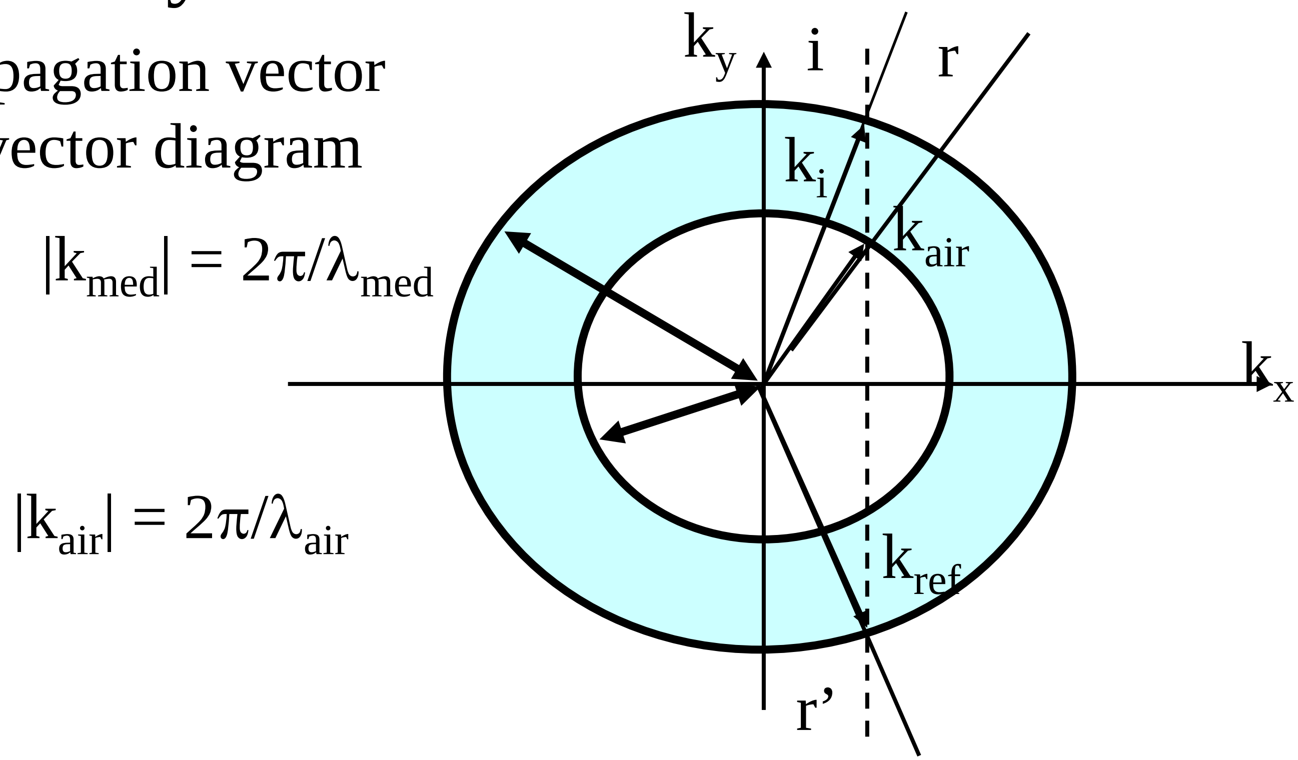
Midterm Exam

Review Th, Oct 19

- In Class Tuesday October 24th
- Covers material through Chapter 7 (Lecture 12)
- Open Book, Open Notes, Bring Calculator, Paper Provided
- Topics
  - Green's functions free space and use in Theorems and concepts with emphasis on statics
  - Separation of variables in rectangular coordinates using N-1 and N method
  - Time-Harmonic ME, planewaves, boundary conditions, and dispersion

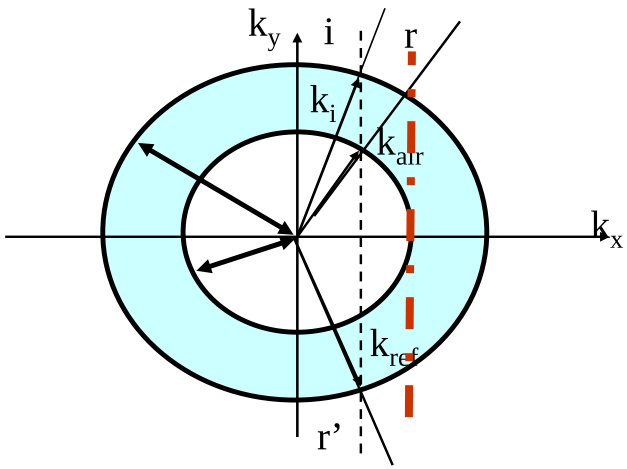
Physical Effects: Wave Direction Change

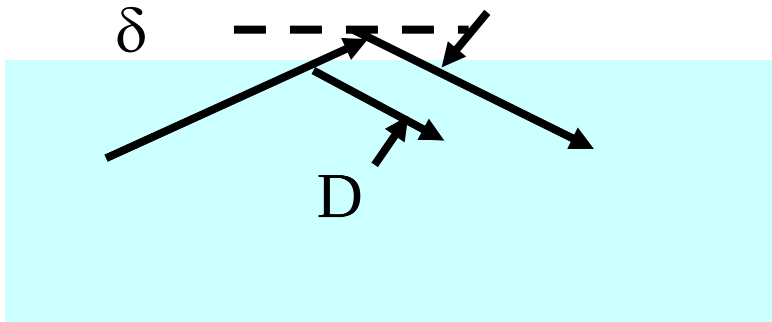
Propagation vector
k-vector diagram



- Draw concentric circles of radius k_{air} and k_{med}
- Incident wave has k vector given (arrow k_1)
- Find the component parallel to the surface (dotted line)
- Force the k-vector in air k_{air} and k-vector reflected k_{ref} to have the same parallel component (lie on dotted line)
- Choose point on the circle to give these new k-vectors (arrows) the correct length for the wave equation in the media that they are in

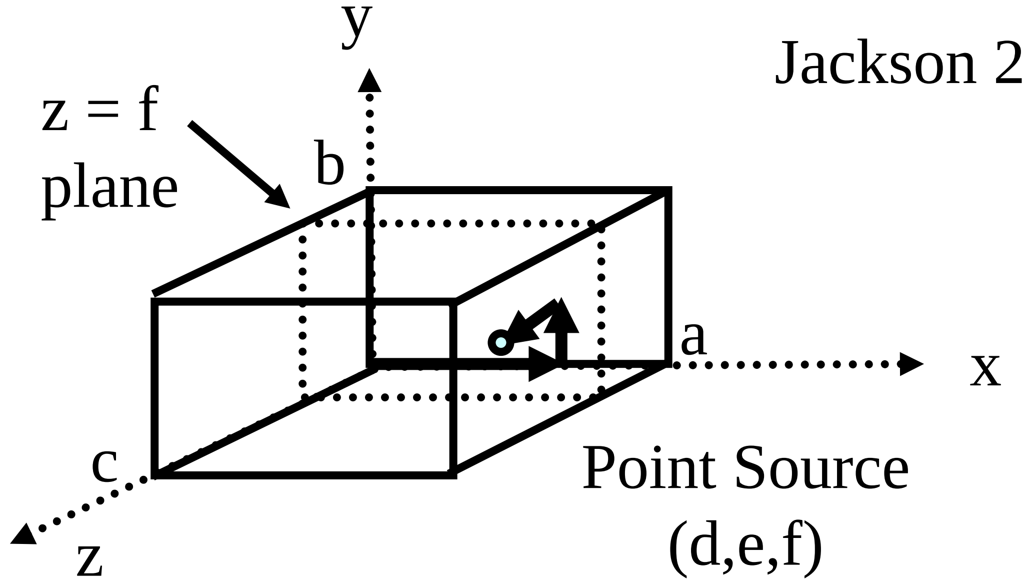
Plane Interface: Physical Effects





- Total Internal Reflection
  - Parallel part of $k_{med} > k_0$
- Brewster Angle
  - Polarization in plane of incidence reflection coefficient goes to zero giving 100% transmission
- Polarization dependent reflection phase change
  - Converts linear to part circular polarization
  - Beam energy penetration δ and spot shift D (Goos-Hanchen effect)

Separation of Variables: Source Strategy

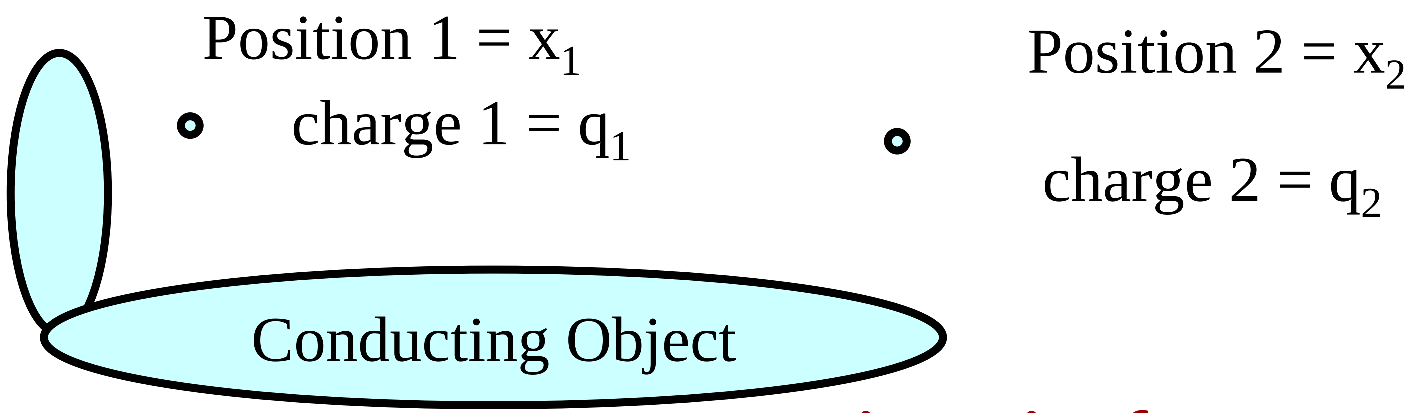


- View source as being on z = f plane.
- Require $\Phi_2 - \Phi_1 = D(x,y)/\varepsilon_0$ at z = f
- Also require at z =f $(\overline{E}_2 - \overline{E}_1)\cdot\hat{n} = \sigma_{SURFACE}(x,y)/\varepsilon_0$
- Multiply each of these equations by one of the composite eigenfunctions and integrate over x,y cross-section
- Gives two equations relating A_{nm} and B_{nm} for the same nm.

$$\nabla^2\psi = -\frac{4\pi q}{\varepsilon_0}\delta(x-d)\delta(y-e)\delta(z-f)$$

Reciprocity

Lecture 4



Reciprocity for Green's Function in Jackson

$$G(\overline{x}, \overline{x}') = G(\overline{x}', \overline{x})$$

$\Phi(x_2,q_1) = \Phi(x_1,q_2)$

Proof:

- Green's Theorem
- Poisson's equation for $\Phi(x_2,q_1)$ and $\Phi(x_1,q_2)$ causes volume integral to give $\Phi(x_2,q_1) - \Phi(x_1,q_2)$
- In surface integral use homogeneous boundary condition to replace potential with derivative and integrand vanishes at every point on the boundary

Integral Equation to Find Surface Charge

$$\Phi(\bar{x}) = \frac{1}{4\pi\varepsilon_0}\int_V \rho(\bar{x}')G(\bar{x},\bar{x}')d^3x' + \frac{1}{4\pi}\oint_S \left[G(\bar{x},\bar{x}')\frac{\delta\Phi}{\delta n'} - \Phi(\bar{x}')\frac{\delta G(\bar{x},\bar{x}')}{\delta n'} \right] da'$$

- Example: Grounded Conducting Object and ρ(x)

Φ = 0 => all of the F surface term drop out

Lecture 5

$d\Phi/dn' = \sigma_{surface}$ remains

Since Φ is known at every point on object restrict x to be on the object

Gives and integral equation for the surface charge

$$0 = \left[\frac{1}{4\pi\varepsilon_0}\int_V \rho(\bar{x}')G(\bar{x},\bar{x}')d^3x' + \frac{1}{4\pi}\oint_S \left[G(\bar{x},\bar{x}')\frac{\delta\Phi}{\delta n'} \right] da' \right]_{\bar{x}\_on\_object}$$

Generally the Green's function for free space is used

$$0 = \left[\frac{1}{4\pi\varepsilon_0}\int_V \rho(\bar{x}')\frac{1}{|\bar{x}-\bar{x}'|}d^3x' + \frac{1}{4\pi}\oint_S \left[\frac{1}{|\bar{x}-\bar{x}'|}\frac{\delta\Phi}{\delta n'} \right] da' \right]_{\bar{x}\_on\_object}$$

Overview

- Source free guided wave solutions can exist on dielectric slabs, layers and fibers.
- The necessary conditions for their longitudinal propagation constant are found by representing the fields and matching boundary conditions on their transverse field to determine eigenvalues.
- The transverse behavior is exponential outside the dielectric and oscillatory inside the dielectric.
- The physical characteristics on the modes are quite similar to those in metal waveguides and include TE/TM classification, orthogonality, cut-off, etc.

Dielectric Waveguides

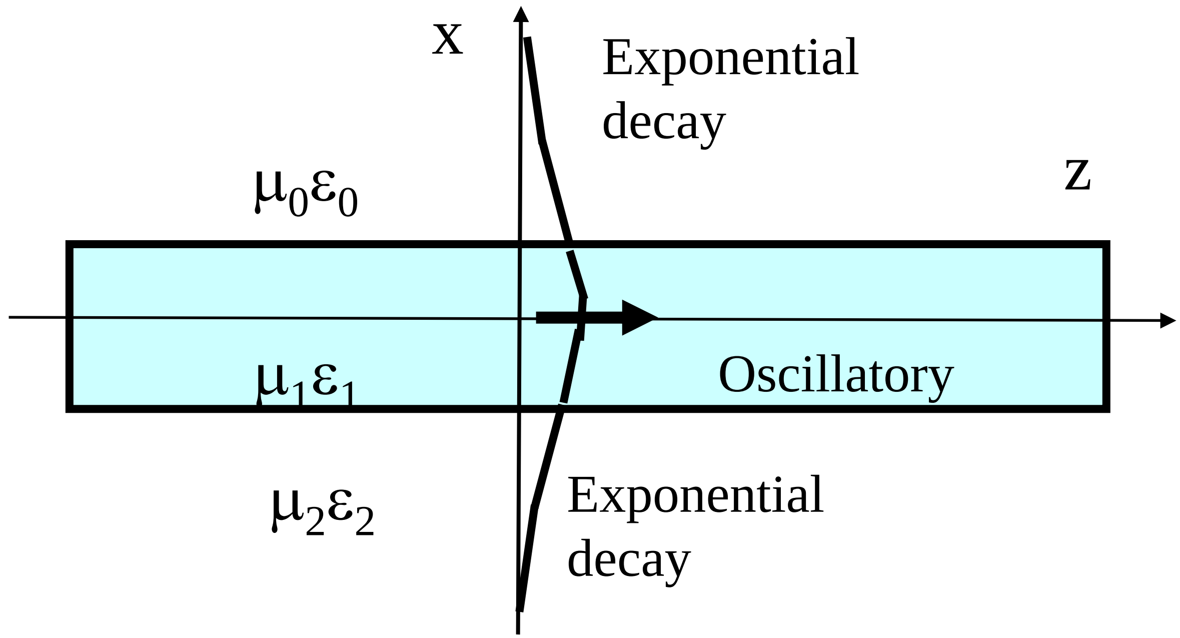
$$e^{j\omega t}$$

$$e^{-jk_z z}$$

$$v_0 = \sqrt{k_z^2 - \omega^2 \mu_0 \varepsilon_0}$$

$$k_x = \sqrt{\omega^2 \mu_0 \varepsilon 1 - k_z^2}$$

$$v_2 = \sqrt{k_z^2 - \omega^2 \mu_2 \varepsilon_2}$$



- Three regions
- Choose TM (or TE)
- Will have Hy, Ez and Ex (Ey, Hx, and Hz)

Dielectric Waveguides

$$H_{0y}^{+}(\overline{x}) = H_0^{+}\hat{y}e^{-v_0 x}e^{-jk_z z}$$

$$H_{1y}(\overline{x}) = H_1^{+}\hat{y}e^{-jk_x x}e^{-jk_z z} + H_1^{-}\hat{y}e^{+jk_x x}e^{-jk_z z}$$

$$H_{2y}^{-}(\overline{x}) = H_2^{-}\hat{y}e^{+v_0 x}e^{-jk_z z}$$

- Consider TM w/r z case
- Write expression for Hy in each of three regions (above, in and below dielectric).
- Note: Include Kinetic boundary condition in expressions

Dielectric Waveguides

$$\nabla \times \overline{H} = j\omega\varepsilon\overline{E}$$

$$E_z(\overline{x}) = \frac{-1}{j\omega\varepsilon}\frac{\partial H_y(\overline{x})}{\partial x}$$

$$E_{0z}^{+}(\overline{x}) = \frac{+v_0}{j\omega\varepsilon} H_{0y}^{+}(\overline{x})$$

$$E_{1z}^{+}(\overline{x}) = \frac{-jk_x}{j\omega\varepsilon} H_{1y}^{+}(\overline{x}) + \frac{jk_x}{j\omega\varepsilon} H_{1y}^{-}(\overline{x})$$

$$E_{2z}^{-}(\overline{x}) = \frac{-v_2}{j\omega\varepsilon} H_{2y}^{+}(\overline{x})$$

- Find Ez in each of three regions
- Apply dynamic boundary conditions (four)
  - – Hy continuous at top and bottom of dielectric
  - – Ez continuous at top and bottom of dielectric

Dielectric Waveguide: Dispersion Eq.

Harrington 4.7 Special case of air on top and bottom, thickness a

	TM	TE
odd	$\frac{k_x a}{2}\tan\frac{k_{x0}a}{2}=\frac{\varepsilon_1}{\varepsilon_0}\frac{v_0 a}{2}$	$\frac{k_x a}{2}\tan\frac{k_{x0}a}{2}=\frac{\mu_1}{\mu_0}\frac{v_0 a}{2}$
even	$-\frac{k_x a}{2}\cot\frac{k_{x0}a}{2}=\frac{\varepsilon_1}{\varepsilon_0}\frac{v_0 a}{2}$	$-\frac{k_x a}{2}\cot\frac{k_{x0}a}{2}=\frac{\mu_1}{\mu_0}\frac{v_0 a}{2}$

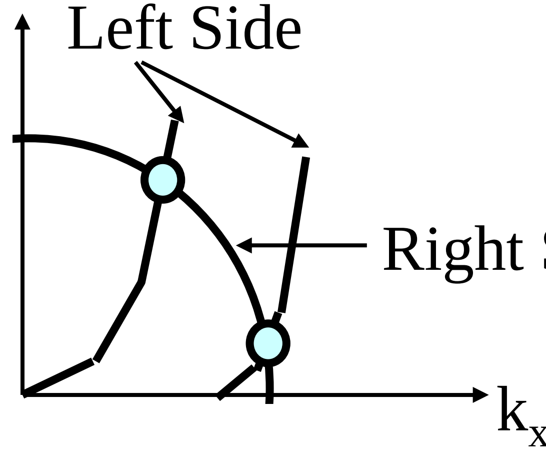
- Convenient within TM and TE to distinguish between even ($\cos k_x x$) and odd $\sin(k_y x)$ variations
- Results in four dispersion relationships
  - Two for TM
  - Two for TE

Dielectric Waveguide: Physical Nature

Harrington 4.7 Special case of air on top and bottom, thickness a

TM

odd $$\frac{k_x a}{2}\tan\frac{k_{x0} a}{2} = \frac{\varepsilon_1}{\varepsilon_0}\frac{v_0 a}{2}$$



- Right hand side is a circle; Left hand side is spikes in tan (See H Fig 4-11)
- Odd sin(k_yx) variations have no cut-off (always exist) in both TM and TE
- Mutiple solutions (intersections) give multiple modes
- Additional new mode about every half wavelength of oscillatory variation.
- Weighted by material contrast sqrt ($\mu_1\varepsilon_1$-$\mu_0\varepsilon_0$)

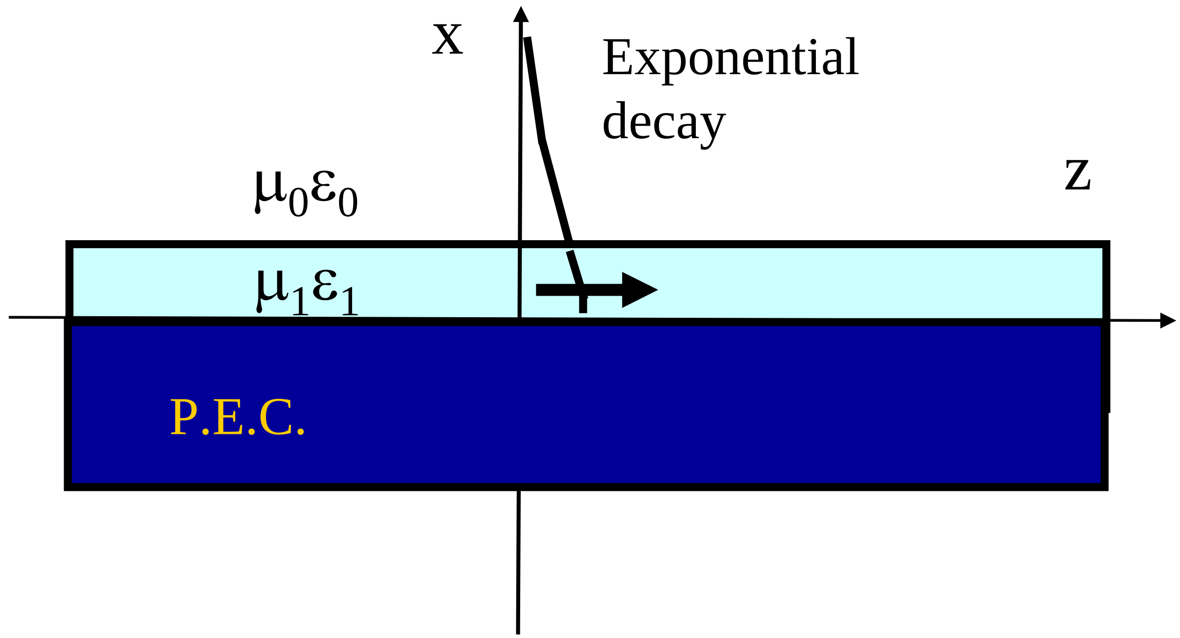
Surface-Guided Waves

$$e^{j\omega t}$$

$$e^{-jk_z z}$$

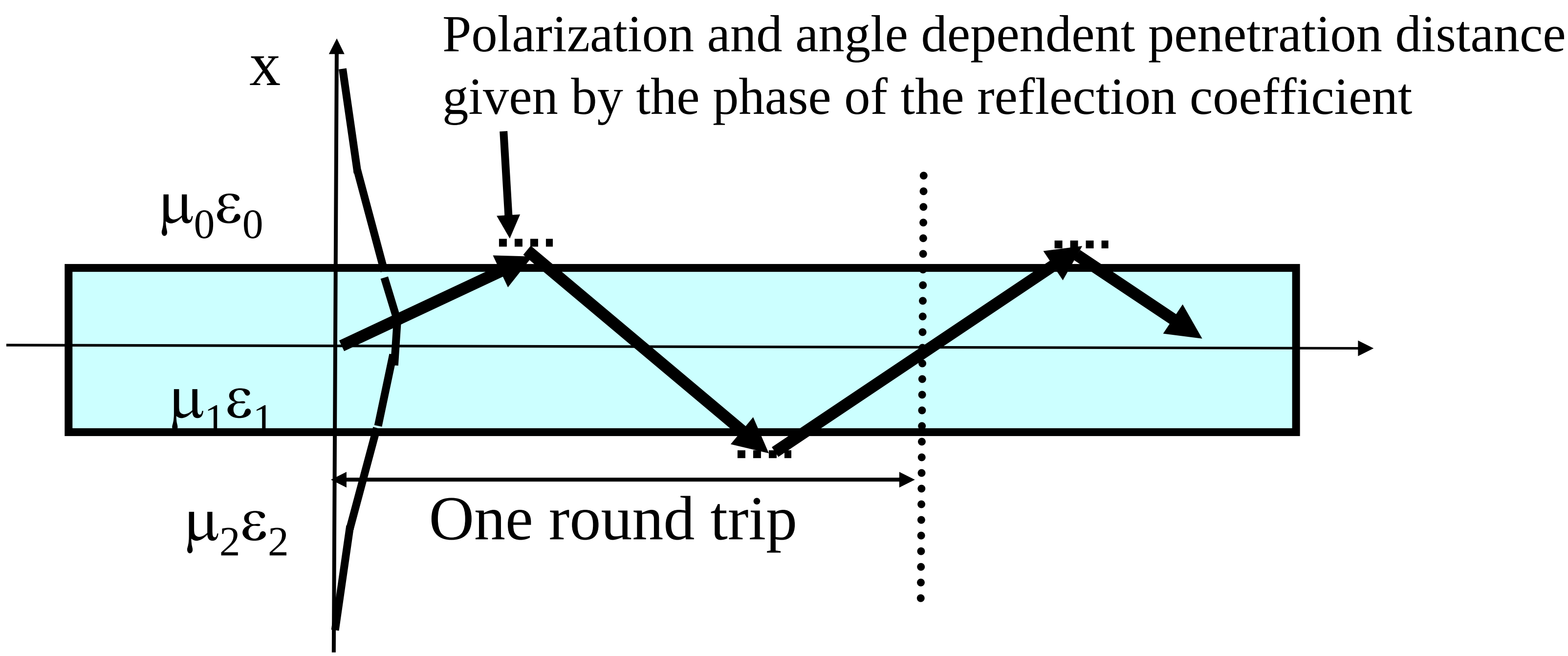
$$v_0 = \sqrt{k_z^2 - \omega^2 \mu_0 \varepsilon_0}$$

$$k_x = \sqrt{\omega^2 \mu_0 \varepsilon 1 - k_z^2}$$



- Two regions
- Choose TM (or TE)
- Will have half of the solutions from the symmetric dielectric slab: TM odd and TE even of the slab

Dielectric Waveguides: Resonance View



- Add up phase of transverse round trip = $n2\pi$
- Use the phase of the reflection coefficient to account for penetration of fields outside dielectric
- This phase will depend on polarization and angle and is thus must be found iteratively