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# EECS 210 Applied Electromagnetic Theory

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## Solution Homework # 9

9.1) **Scattering at Long Wavelengths:** Consider both a dielectric with $\varepsilon_r = 2$ and a perfectly conducting sphere of radius a. A plane wave is incident in the +z direction with electric field in the x direction.

a) Evaluate **the bracket [ ]** in equation 10.2 of Jackson for the following four combinations of orientation of the electric field and position (orientation, position). (x, -z), (x, +z), (x, +y)and (y, +**x**) when **p** is in the x direction and **m** is in the y direction.

$$[(\hat{n}\times\overline{p})\times\hat{n}-\hat{n}\times\overline{m}/c]=[(-\hat{z}\times\hat{x})\times(-\hat{z})p-(-\hat{z})\times m\hat{y}/c]=p\hat{x}-m\hat{x}$$
$$[(\hat{n}\times\overline{p})\times\hat{n}-\hat{n}\times\overline{m}/c]=[(+\hat{z}\times\hat{x})\times(+\hat{z})p-(+\hat{z})\times m\hat{y}/c]=p\hat{x}+m\hat{x}$$
$$[(\hat{n}\times\overline{p})\times\hat{n}-\hat{n}\times\overline{m}/c]=[(+\hat{y}\times\hat{x})\times(+\hat{y})p-(+\hat{y})\times m\hat{y}/c]=p\hat{x}-0$$
$$[(\hat{n}\times\overline{p})\times\hat{n}-\hat{n}\times\overline{m}/c]=[(+\hat{x}\times\hat{x})\times(+\hat{x})p-(+\hat{x})\times m\hat{y}/c]=0\hat{x}-m\hat{z}$$

b) For the spheres evaluate $\mathbf{p}_{Dielectric}$, $\mathbf{p}_{PEC}$ and $\mathbf{m}_{PEC}$.

$$p_{Diel}=1\pi\varepsilon_0 a^3 E_{x,Inc}\hat{x}\ \&\ p_{PECl}=4\pi\varepsilon_0 a^3 E_{x,Inc}\ \&\ {m_{PEC}}\big/{c}=-\frac{2\pi a^3}{c}H_{y,Inc}=-\frac{2\pi a^3}{cZ_0}E_{x,Inc}=2\pi\varepsilon_0 a^3 E_{x,Inc}$$

c) Evaluate **the bracket [ ]** in equation 10.2 of Jackson for the dielectric sphere and the metallic sphere for the orientations and positions in part a).

$$[(-\hat{z}\times\hat{x})\times(-\hat{z})p-(-\hat{z})\times m\hat{y}/c]=p\hat{x}-m\hat{x}=\pi\varepsilon_0 a^3 E_{x,Inc}\ \&\ (4+2)\pi\varepsilon_0 a^3 E_{x,Inc}=6\pi\varepsilon_0 a^3 E_{x,Inc}$$
$$[(+\hat{z}\times\hat{x})\times(+\hat{z})p-(+\hat{z})\times m\hat{y}/c]=p\hat{x}+m\hat{x}=1\pi\varepsilon_0 a^3 E_{x,Inc}\ \&\ (4-2)\pi\varepsilon_0 a^3 E_{x,Inc}=2\pi\varepsilon_0 a^3 E_{x,Inc}$$
$$[(+\hat{y}\times\hat{x})\times(+\hat{y})p-(+\hat{y})\times m\hat{y}/c]=p\hat{x}-0=1\pi\varepsilon_0 a^3 E_{x,Inc}\ \&\ 4\pi\varepsilon_0 a^3 E_{x,Inc}$$
$$[(+\hat{x}\times\hat{x})\times(+\hat{x})p-(+\hat{x})\times m\hat{y}/c]=0\hat{x}-m\hat{z}=0\ \&\ 2\pi\varepsilon_0 a^3 E_{x,Inc}$$

d) Make a table of the values from part c) and comment on 1) the coherence of the interaction between $\mathbf{p}_{PEC}$ and $\mathbf{m}_{PEC}$ , 2) the front to back asymmetry in the metallic case, and 3) the non-zero scattering in the y direction for the metallic sphere.

| Case | Backscatter | Forwardscatter | Sidescatter | Topscatter (z^) |
|---|---|---|---|---|
| Dielectric | 1 | 1 | 1 | 0 |
| PEC | 6 | 2 | 4 | 2 |

The formulation in Jackson clearly adds electric fields and thus assume the contributions from the dielectric and magnetic dipole moments add coherently. There is a 3 times stronger backscatter than forward scatter for a PEC whereas they are equal for a dielectric sphere. The magnetic dipole adds radiation in the plane of the incident polarization in the direction of the polarization which does not exist for the dielectric sphere.

9.2) **Imaging**: An optical projection printer has a wavelength of 193 nm and numerical aperture of **sinθ = 0. 5**. The mask is 4 times larger (16 times the area) of the image at the wafer and has lines with a 50% duty cycle so that the opaque linewidth equals the open linewidth. Start by neglecting vector effects. **(In a resist with n = 1.7 the angle becomes Sinθ =0.85/1.7 = 0.5). Answers assume NA in air is 0.85 and changes to 0.5 in resist.**

a) Draw a k-vector diagram with radius $NAk_0$ and find the minimum pitch on the wafer for which an on-axis ray will produce 3-wave imaging.

$$\frac{2\pi}{P} = NAk_0 = NA\frac{2\pi}{\lambda} \Rightarrow P = \frac{\lambda}{NA} = 1.176$$

b) For the 3-wave imaging in a) determine the contrast C = (Imax – Imin)/(Imax + Imin). (Hint: Find the E field from plane waves and use I = EE*.)

$$E(x) = 0.5 + (2/\pi)\cos(k_x x) \Rightarrow \min = 0 \Rightarrow C = 1$$

c) Find the maximum off-axis angle, pitch and contrast for two-wave off-axis imaging. (Hint: The two waves E1 and E0 are unequal.)

$$E(x) = 0.5e^{ik_x x} + (1/\pi)e^{-ik_x x} \Rightarrow \min = 0.5 - 1/\pi = 0.182 \;\&\; \max = 0.5 + 1/\pi = 0.818 \Rightarrow C = \frac{0.670 - 0.033}{0.670 + 0.033} = 0.0.906$$

d) A 0 and 180 degree phase shifting mask is introduced and illuminated on-axis. Show that this doubles the resolution of problem a) and also produces a contrast of 1.

$$E(x) = (2/\pi)e^{ik_x x} + (2/\pi)e^{-ik_x x} \Rightarrow \min = 0 \Rightarrow C = 1 \;\&\; \frac{2\pi}{P} = 2NAk_0 \Rightarrow P = \frac{\lambda}{2NA} = 0.588\lambda$$

e) Find the contrast C when vector effects are added on the wafer side. (Hint: Due to lack of co-linearity of vectors the maximum decreases and the minimum increases.)

Assume case d) of the phase shifting mask.

Use $NA_{RESIST} = NA_{AIR}/1.7 = 0.5$ and $I = E_x E_x^* + E_y E_y^* + E_z E_z^*$ where

$$\overline{E} = (2/\pi)(\cos\theta_{RESIST}\hat{x} + \sin\theta_{RESIST}\hat{z})e^{ik_x x} + (2/\pi)(\cos\theta_{RESIST}\hat{x} - \sin\theta_{RESIST}\hat{z})e^{-ik_x x}$$

$$\overline{E} = (4/\pi)(\cos k_x x \cos\theta_{RESIST}\hat{x} + j\sin k_x x \sin\theta_{RESIST}\hat{z})$$

$$E_{MAX} = 4/\pi\cos\theta_{RESIST} \Rightarrow I_{MAX} = (4/\pi\cos\theta_{RESIST})^2$$

$$E_{MIN} = j\sin\theta_{RESIST} \Rightarrow I_{MIN} = (4/\pi\sin\theta_{RESIST})^2$$

$$C = \frac{(4/\pi\cos\theta_{RESIST})^2 - (4/\pi\sin\theta_{RESIST})^2}{(4/\pi\cos\theta_{RESIST})^2 - (4/\pi\sin\theta_{RESIST})^2} = \frac{\cos^2\theta_{RESIST} - \sin^2\theta_{RESIST}}{\cos^2\theta_{RESIST} - \sin^2\theta_{RESIST}} = \frac{0.75 - 0.25}{0.75 + 0.25} = 0.5$$

9.3) **Planewave Standing Waves:** Consider a film of thickness d with refractive index 2 on a substrate with refractive index 4. Use the equation in slide 4 of lecture 25 to evaluate the field inside the film at a depth z.

a) Derive this formula by considering the multiple scattering waves. (You must consider both the downward and upward waves and correct them for their phases at depth z. First consider the dominant downward and then the dominant upward waves. Then add the 2nd most dominant downward and upward and notice that they differ from the dominant waves by a common round trip factor z. Then use 1 + z + $z^2$+ … = 1/(1-z). Finally use the fact that $\rho_{12} = -\rho_{21}$.

b) Make a spot check that the formula is correct by making the film thickness λ/4 to produce quarter wave matching which makes E2 = E1 at z = 0.

Evaluation gives 8/9 over 8/8 =1

c) Evaluate the accuracy for just the dominant downward and upward waves (numerator) compared to including the infinite reflections (full expression).

One round trip gives 8/9 of the final value.

d) Show that $E_{2ma}x/E_{2min}$ is independent of d and given by $(1+\rho_{23})/(1-\rho_{23})$.

e) First knock out the d dependence in the denominator by assuming a given d. Find two z values that give the pure constructive addition and pure destructive subtraction of the two terms in the numerator. Then take the ratio and formula results.