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EECS 210 Applied Electromagnetic Theory

Fall 2006 | **Office Hours** | **Prof. A. R. Neureuther,**
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Solution Homework # 8:

These problems are based on the solution of the coupled modes pp.. 71-73 of the Kogelnik Chapter in Tamir. Assume a wavelength of 500nm for estimating distances.

8.1) **Couple Modes: Physical Properties:** Assume the co-directional case.

a) For the phase matched case when $\delta = 0$, determine the κ value required to couple 100% in a distance of 20 μm. From this value suggest a rule of thumb for the value of $\kappa L/\lambda$.

$\kappa L = \pi/2 \Rightarrow \kappa = \pi/2L = 0.785/\mu$m. Rule of Thumb is $kL/\lambda = \pi/2\lambda$.

b) For optical image Lord Rayleigh pointed out that a phase error of 90 degrees from center to edge of the lens can be considered as a good measure of the defocus limit. Consider the coupler when the phase error in each wave is equal and opposite at $\delta L = \pi/4$. Assume the 100% coupling condition from a) holds, namely that $\kappa L = \pi/2$. Evaluate the complex quantities S and R. Hint: S comes out very similar to the Strehl ratio for the point spread function with one Rayleigh Unit defocus which is 0.78. Hint: Check your answer by conservation of energy as SS* + RR* = 1.

$\delta = \kappa/2 \Rightarrow \mathrm{Sqrt}(\kappa^2 + \delta^2) = 1.118\kappa$; S(L) = -jsin[($\pi$/2)1.118]/1.118= 0.879; SS*= 0.773

R(L) = cos[(π/2)1.118]+j0.5sin[(π/2)1.118]/1.118= -0.1843+j0.4395; RR* =0.226

7.2) **Coupled Modes: Phase-Matching for Periodic Case:** Consider the Contra-directional coupler. The device structure consists of a film with refractive index 2.0 on a substrate with refractive index 1.5. Assume that a mode on this film has a k-vector 1.9 larger than $k_0 = 2\pi/\lambda$. The coupling of a plane wave into a mode in the film is produced a periodic height modulation with period P. The structure is uniform in the y-direction and the fields have no y-variation.

a) Determine the period P that would be suitable for coupling a planewave making a 30^o angle with the x-axis with its k-vector in the y = 0 plane in the backward direction. (The wave is propagating in the (-x,-z) direction and the mode is propagating in the +z direction.)

Sin(30) = 0.5 so $2\pi/P = (2\pi/\lambda)(0.5+1.9) = (2\pi/\lambda)2.4 = P = \lambda/2.4 = 0.417\lambda$

b) Determine the largest period that could be used in the coupler to not produce substrate radiation by the -2 spatial harmonic. Is the incident angle to couple via the -1 harmonic still backward? What is the minimum angle?

$2(2\pi/P) = (2\pi/\lambda)(1.5+1.9) \Rightarrow 2\pi/P = 1.7 \Rightarrow n = 1$ is at $k = 0.02 \Rightarrow \theta = 1.15^o$.

7.3) **Coupled Mode Eigenvalues and Eigenvectors:** Use the equations 2.6.28 and 2.6.29 and boundary conditions given just below with the eigenvalue-eigenfunction method to derive equations 2.6.30 and 2.6.31.

a) Find the eigenvalues.

$$M - lI = \begin{matrix} j(\delta+\lambda) & -j\kappa \\ -j\kappa & -j(\delta-\lambda) \end{matrix} \Rightarrow \lambda = \pm\sqrt{\kappa^2+\delta^2}$$

b) Find the eigenvectors and normalize to unit energy.

Put one value in first row of matrix and solve for ratio of elements. Normalization is by squaring each component and adding to get unit energy. But is is skipped here due to the algebra.

c) Match boundary conditions to find the amplitudes that multiply the eigenfunctions.

$$\begin{bmatrix}1\\0\end{bmatrix} = c_1\begin{bmatrix}1\\ \delta+\sqrt{\kappa^2+\delta^2}\end{bmatrix}e^{-j\sqrt{\kappa^2+\delta^2}z} + c_2\begin{bmatrix}1\\ \delta-\sqrt{\kappa^2+\delta^2}\end{bmatrix}e^{-j\sqrt{\kappa^2+\delta^2}z}$$

Find c_1 =c$\begin{bmatrix}1\\ \delta+\sqrt{\kappa^2+\delta^2}\end{bmatrix}$ =0.5

d) Rearrange your results to obtain equations 2.6.30 and 2.6.31.

Break up two exponentials into sin where differ and cos where same.

e) Evaluate the two eigenfunctions at the three phase values of δ = -2κ, 0, +2κ. Show that the relative weights of the first and second components of the eigenfunctions for λ_1 and λ_2 tend to switch identities as the phase mismatch goes from negative to positive. (Example: Define eigenvalue 1 as coming from the +sign in sqrt. This eigen vector for this eigenvalue will have its top component smaller than its lower component for negative values of δ. At δ =0 they will be equal and for δ positive the top component will be larger than the bottom component.)

When d is 2xκ the sqrt = κsqrt(5). When signs agree lower term is bigger. When signs differ lower term is smaller. Then when δ sweeps from negative to positive this same bigger to small or opposite change takes place.