Version 11/12/06

EECS 210 Applied Electromagnetic Theory

Fall 2006 | **Office Hours** | **Prof. A. R. Neureuther,**
Tu, Th 12:30-2 | **M, (W), 11AM** | **509 Cory Hall, 2-4590**
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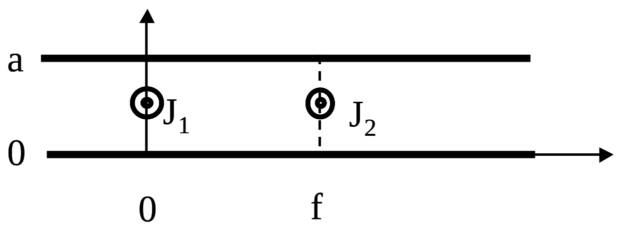
Homework # 7: Due Start of Class Thursday, Nov 9th
Project Specification is Posted



$$\bar{J}_1 = J_0 \sin\left(\frac{3\pi x}{a}\right)\delta(z)\hat{y}$$

$$\bar{J}_2 = J_0 \sin\left(\frac{3\pi x}{a}\right)\delta(z-f)\hat{y}$$

P.e.c Waveguide with height a in x Width b in y

7.1) **Source Matching Method:**

a) Explain why the source does not excite TM waves.

J_y will drive E_y. Uniformity in y requires n = 0. For TM E_y is proportional to n and hence TM cannot be driven.

b) Explain why the source only excites the $TE_{3,0}$ mode.

For the TE the drive by J_y of E_y will be zero for n not zero due to orthogonality of sin vs constant in y, and zero for m not 3 due to orthogonality between sin variations in x.

c) Analyze the contribution of source J_1 by using a modal representation and match boundary conditions at z = 0.

$$k_z = \sqrt{k_0^2 - \left(3\pi/a\right)^2}$$

Assume $E_y(z>0) = A_{30}\sin(3\pi x/a)e^{ik_z z}$ and $E_y(z<0) = B_{30}\sin(3\pi x/a)e^{-ik_z z}$

Want to find TE_{30} amplitudes A_{30} and B in +z and –z.. E_{tan} continuous implies $B_{30} = A_{30}$.

H_x must be discontinuous by J_0. Find H_x from curl. $H_x = (k_z/\omega u)E_y$. So using

$H_x(z=0^+) - H_x(z=0^-) = J_0$ gives $A_{30} = \left(\omega\mu J_0 / 2k_z\right)$

d) Shift the solution to c) to find the solution for source J_2.

Just change z to z-f.

e) Use superposition to find the solution when both J_1 and J_2 are present.

Add source contributions. $A_{30} = \left(\omega\mu J_0 / 2k_z\right)\left(1 + e^{ik_z f}\right)$

f) What values of f in terms of a and b will make the fields zero for z >f ?

For waves to cancel in forward direction $k_z f$ = odd multiple of π. This cancels the fields for all positions z > f. (It also happens to cancel the fields for all positions z < 0.

g) Is there a value of f for which the fields will be zero for 0 < z < f ?

Because there are waves to the left and right it is impossible to make the fields zero for all z in range 0 to f simultaneously.

7.2) **Reciprocity Analysis Method:**

Repeat problem 7.1 using the reciprocity method in of Jackson section 8.12C. Be sure to work out all of the normalization factors.

To find the TE_{03} wave to the right send in a TE_{30} test wave from right to left $E_{TEST} = A_{TEST} \sin\left(m\pi x / a\right) e^{-ik_z x}$. Note amplitude A_{TEST} with reference plane for 0 phase at z=0. Integral over sources gives $A_{TEST} J_0 \frac{a}{6}\left(1+e^{+ik_z f}\right)$. Integral over plane at z = 0- is zero as waves are in the same direction. Integral over cross-section at z= f+ is $2A_{TEST} A_{30} \frac{k_z}{\omega\mu}\frac{a}{6}$. Note that here the phase of the test wave and unknown wave cancel each other. Taking the ratio gives formula from above.

7.3) **Plasmons:**

Consider a silver surface and wavelengths of 350, 450 and 550 nm.

a) Obtain the refractive indices at these wavelength for silver from the RIT web site (see restricted pages for details).

350 nm 0.222 +j1.3586; 450 nm 0.1526 +j2.4696; 550 0.1251 +j3.3381

b) Evaluate k_z for a Plasmon at the given wavelengths.

This is a lot of complex algebra so only 550nm case is described here. Data from Chris Clifford on all cases is attached. Also to give an idea relative to free space the values are referenced to k_0 and λ_0.

$$n^2 = 11.16\angle 175.7\ ;\quad \frac{k_z}{\left(2\pi/\lambda_0\right)} = \sqrt{\frac{1}{1+1/n^2}} = 1.0408\angle 0.2113 = 1.041 + j0.00384$$

c) Find the surface wavelength of the plasmon at the given wavelengths.

The surface wavelength is 1/1.0408 =0.9543 that is 95% of free space.

d) Find the 1/e decay distance parallel to the surface for the electric field of the plasmon at the given wavelengths.

The number of wavelengths is $[2\pi\ 0.00386]^{-1}$ = 41 wavelengths

e) Find the 1/e decay distance away from the surface in air for the electric field of the plasmon at the given wavelengths.

$$\frac{v_1}{2\pi/\lambda_0} = \sqrt{-1/\left(1+n^2\right)} = 0.3137\angle 2.35 = 0.313 + i0.01286$$

The 1/e distance in wavelengths is $1/\mathrm{Re}(v_1) = \lambda_0 \cdot 1/(2\pi\ 0.313) = 0.51\lambda_0$

f) Evaluate the ratio of the 1/e decay distance away from the surface in air to the surface wavelength at the given wavelengths. (Is it 2π?)

$0.51\lambda_0 / 0.9543\lambda_0 = 0.53$ and this is about 12 times smaller than 2π.

Comments using Data from Chris Clifford:

- Even at 350 nm the surface wavelength is still 0.73 of that of air (30% effect and not 3X or 10X)
- By 350 nm the surface 1/e drops from 41 to 12 to less than a wavelength.
- Ratio 1/e in air to surface wavelength is 0.53, 0.40 and 0.23 so more or less scaling together.