

**EECS 210**  
 Fall 2006  
 Tu, Th 12:30-2  
 400 Cory

**Applied Electromagnetic Theory**  
 Office Hours  
 M, (W), 11AM  
 Tu, Th, (F) 10AM

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## Homework # 4: Due start of class Th Sep 24th

4.1) **Boundary Value Problem N-1 Technique:** Consider the interior of a grounded box defined by the six planes,  $x = 0$ ,  $y = 0$ ,  $z = 0$ ,  $x = a$ ,  $y = b$ , and  $z = c$ . A surface charge distribution is on the plane  $z = f$  and has a constant value  $\sigma_{\text{SURFACE}}$  for  $a/2 - d < x < a/2 + d$ . Note that this source is not a function of  $y$ .

- Write down a general expression using eigenfunctions in  $x$  and  $y$  and exponentials in  $z$  that can represent the potential inside the box for  $z < f$  and for  $z > f$  for any source on the  $z = f$  plane in the grounded box (N-1 technique).
- Specify and apply the boundary condition on the potential at the source. Use the orthogonal nature of the expansion to show that this BC applies to each individual term in the expression.
- Specify and apply the boundary condition on the normal derivative of the potential at the source. Use the orthogonal nature of the expansion to in effect expand the source on the source plane and apply the BC to each term.
- Solve for the coefficients and write out the final N-1 infinite summation for the potential for  $z < f$ .
- Comment on the nature of the solution. (In which dimension is it constant? How did  $(\sin(u))/u$  appear?)

4.2) **Surface Boundary Value Problem N Technique:** Now reconsider the grounded box and source in Problem 4.1).

- Write down an expansion for the potential inside the box using eigenfunctions in all three directions (N technique). Use  $A_{ijk}$  as the unknown coefficient for the composite eigenfunction made up of the product of the eigenfunction in  $x$  times the eigenfunction in  $y$  times the eigenfunction in  $z$ .
- Substitute this expansion in to Poisson's Equation for the potential inside the box.
- Multiply both sides by the  $i,j,k$  composite eigenfunction and integrate over the volume of the box to determine a value for  $A_{ijk}$ .
- Write out the solution for the potential as the triple infinite summation over the eigenvalues.
- Comment on the nature of the solution. (In which dimension is it constant? How did  $(\sin(u))/u$  appear?)

4.3) **Dielectric Polarization:** Consider a charge  $q$  at a distance  $a$  along the  $z$  axis from a dielectric plane at  $z = 0$ . Use the image charge solution from Jackson.

- Evaluate the potential as a function of radius on  $z = 0$ . (Is constant or non constant?).
- Derive an expression for the polarization in the dielectric.
- Find the polarization charge on the surface of the dielectric as a function of radius from the divergence of  $P$ .