Version 10/11/06

EECS 210 Applied Electromagnetic Theory

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Homework # 3: Solution

3.1) **Integral Equations by Hand and Cylindrical Geometry Practice**: Consider two small grounded conducting circular cylinders of radius **a = 0.1m** with centers at (x,y) = (**0,0**) for cylinder 1 and (x,y) = (1,0) for cylinder 2. A line charge of strength λ_{S1} C/m is brought into position (0, 0.5). Since **a** is much smaller that the separation between the charges and cylinders the charge induced on the cylinder can be modeled as a line charge λ_1 and λ_2.at the center of the cylinder

a) Write down the integral representation for the potential at a location (x,y) for λ_{S1} in the presence of a conducting cylinder at voltage V of arbitrary shape the Green's function for a line charge in free space.

$$\Phi(x,y) = -\frac{\lambda_{s1}}{2\pi\varepsilon}\ln|r_0 - r_s| + \oint_S \left(V\frac{\partial G}{\partial n} + G\sigma_s \right) da \quad \text{where } G = -\frac{1}{2\pi\varepsilon}\ln|r_0 - r_{surf}| \text{ and } r_{surf} \text{ is}$$

on the surface of the arbitrary cylinder.

b) Show how this integral representation specializes to multiple circular cylinder conductors that are grounded and for which the induced charge can be located at the center of the cylinder. Hint: For the self contribution to the potential use the uniform electric field on a circular cylinder of radius **a**.

When V = 0 the first surface term in the integral goes to zero. For the second term we lump the surface charge at the center giving

$$\Phi(x,y) = -\frac{\lambda_{S1}}{2\pi\varepsilon}\ln|r - r_{source}| - \frac{\lambda_1}{2\pi\varepsilon}\ln|r - r_1| - \frac{\lambda_2}{2\pi\varepsilon}\ln|r - r_2|$$

c) Generate a two-by-two matrix equation $\overline{\Phi} = \overline{0} = \overline{\overline{A}}\overline{q} + \overline{B}$ for the zero potential at observation points on the two circular cylinders as a function of the potential contributed by the induced charges on the two circular cylinders (that is the Aq term) and the contribution to the potential on each of the cylinders from the source q_{S1}(that is the B term).

$$0 = -\frac{\lambda_1}{2\pi\varepsilon}\ln|0.1| - \frac{\lambda_2}{2\pi\varepsilon}\ln|1.0| - \frac{\lambda_{S1}}{2\pi\varepsilon}\ln|0.5|$$

$$0 = -\frac{\lambda_1}{2\pi\varepsilon}\ln|1.0| - \frac{\lambda_2}{2\pi\varepsilon}\ln|0.1| - \frac{\lambda_{S1}}{2\pi\varepsilon}\ln|1.12|$$

d) Solve the matrix equation for the induced charges.

$\lambda_1 = -0.3\lambda_{s1}$, and $\lambda_2 = 0.05\lambda_{s1}$

e) Compute the potential at location (2,0).

$$\Phi(2,0) = -\frac{-0.3\lambda_{S1}}{2\pi\varepsilon}\ln|2.00| + \frac{+0.05\lambda_{S1}}{2\pi\varepsilon}\ln|1.00| - \frac{\lambda_{S1}}{2\pi\varepsilon}\ln|2.06| = -\frac{0.52\lambda_{S1}}{2\pi\varepsilon}$$

e) Repeat for a source of strength q_{S2} at location (2,0) and find the potential at (0,0.5). Show that reciprocity holds $q_{S1}\Phi(2,0) = q_{S2}\Phi(0,0.5)$ even though the induced charges on the cylinder appear to be unrelated.

$$0 = -\frac{\lambda_1}{2\pi\varepsilon}\ln|0.1| - \frac{\lambda_2}{2\pi\varepsilon}\ln|1.0| - \frac{\lambda_{S2}}{2\pi\varepsilon}\ln|2|$$

$$0 = -\frac{\lambda_1}{2\pi\varepsilon}\ln|1.0| - \frac{\lambda_2}{2\pi\varepsilon}\ln|0.1| - \frac{\lambda_{S1}}{2\pi\varepsilon}\ln|1|$$

Solution $\lambda_1 = +0.3\lambda_{s2}$, and $\lambda_2 = 0$.

$$\Phi(0,.5) = -\frac{+0.3\lambda_{S1}}{2\pi\varepsilon}\ln|.5| + \frac{0.0\lambda_{S1}}{2\pi\varepsilon}\ln|1.12| - \frac{\lambda_{S1}}{2\pi\varepsilon}\ln|2.06| = -\frac{0.52\lambda_{S1}}{2\pi\varepsilon}$$

$$\lambda_{S1}\Phi(0,0.5) = -\frac{0.52\lambda_{S1}\lambda_{S2}}{2\pi\varepsilon} = \lambda_{S2}\Phi(0,2) = -\frac{0.52\lambda_{S1}\lambda_{S2}}{2\pi\varepsilon}$$

3.2) **Creating Ortho-Normal Functions**: A power series expansion $y(x) = 1 + 0.5x$ on the interval [0,1] is to be converted to an expansion in orthonormal functions $y(x) = B_0\phi_0(x) + B_1\phi_1(x)$ where the first function is constant $\phi_0 = C_{00}$ and the second islinear $\phi_1 = C_{10} + C_{11}x$.

a) Find C by applying normalization to ϕ_0.

$C_{00} = 1$

b) Use orthogonality to find one condition on C_{10} and C_{11}.

$C_{10}+0.5C_{11} = 0$

c) Use normalization to find a second constraint on C_{10} and C_{11} and solve for ϕ_1.

$$C_{10}^2 + 2C_{10}C_{11}(0.5) + 1/3C_{11}^2 = 0$$

$$\phi_1 = \sqrt{3}(1-2x)$$

d) Compute the mean squared value of y(x) directly and check that it equals $MS = \sum_{n=0}^{1} B_n^2$.

MS = 19/12; B0 = 5/4; B1 = sqrt(3)/4; and summing squares gives same MS.

e) Suppose n were to go to 4 in defining orthonormal functions. Determine the number of unknown coefficients C that would have to be determined. Then show that the number of constraint equations from orthogonality plus the number from normalization give this number.

4 terms => 10 unknowns

Normalization => 4 constraints

Orthogonality = combination of 4 items taken 2 at a time = 6

3.3) **Constancy of the product of spatial width and Fourier Integral width**: For the definition of the Fourier transform in Jackson on pp. 69 take the product of the full-with half-maximum of $f(x)$ with the full-with half-maximum $A(k)$ for a) rectangular shape, b) triangular shape, and c) Gaussian. Are they approximately the same?

Rectangle = 7.6; Triangle 5.5; Gaussian 5.6

They are ballpark close. They depend a bit on how the 3π normalization is distributed between the transform and the inverse.

To check with equation 7.82 the rms deviations would need to be used. These rms deviations from the average are well less than ½ of the FWHM and the products are less than the numbers above divided by 4. (So the ½ in Jackson is really for rms and the FWHM is about 6.)