EECS130
Integrated Circuit Devices

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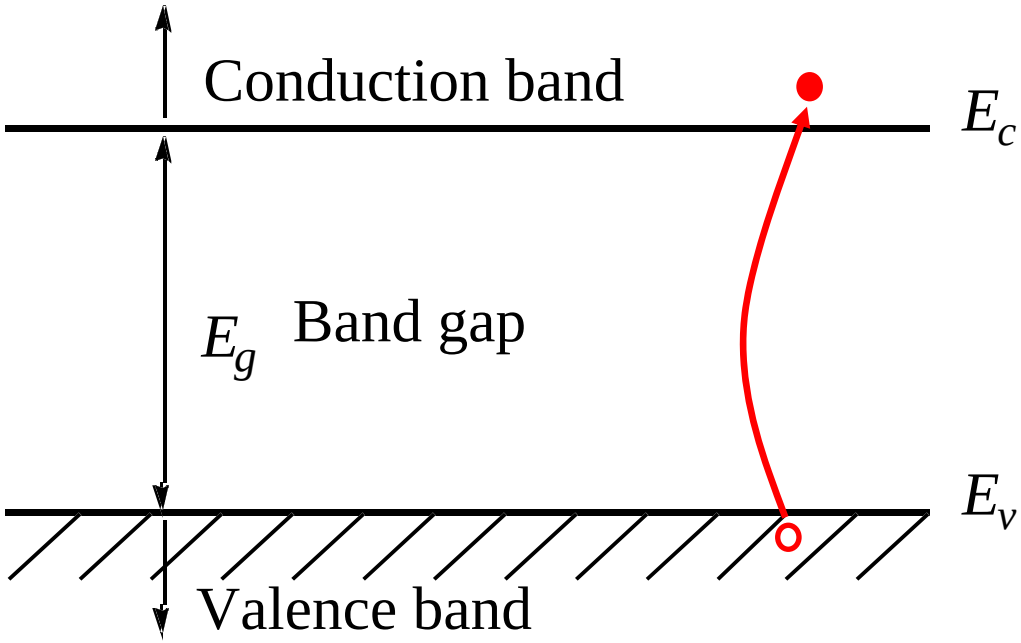
8/30/2007

<u>Semiconductor Fundamentals</u>

<u>Lecture 2</u>

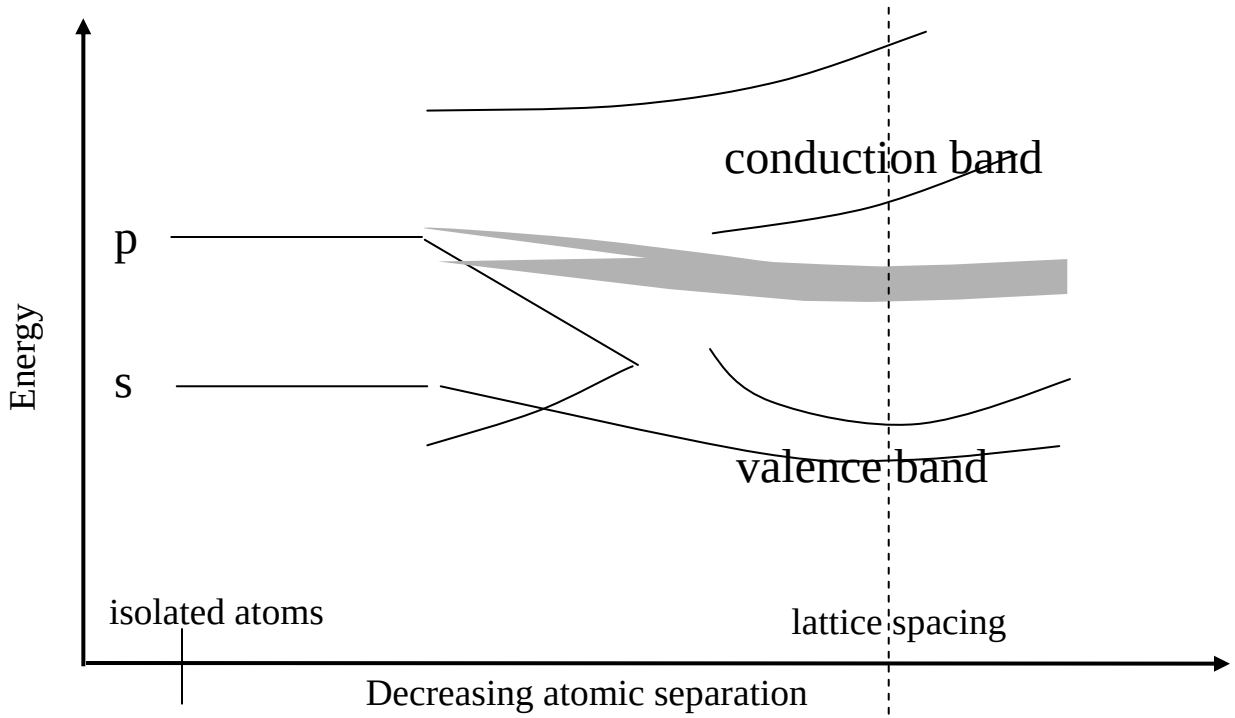
Read: Chapters 1 and 2

Last Lecture: Energy Band Diagram



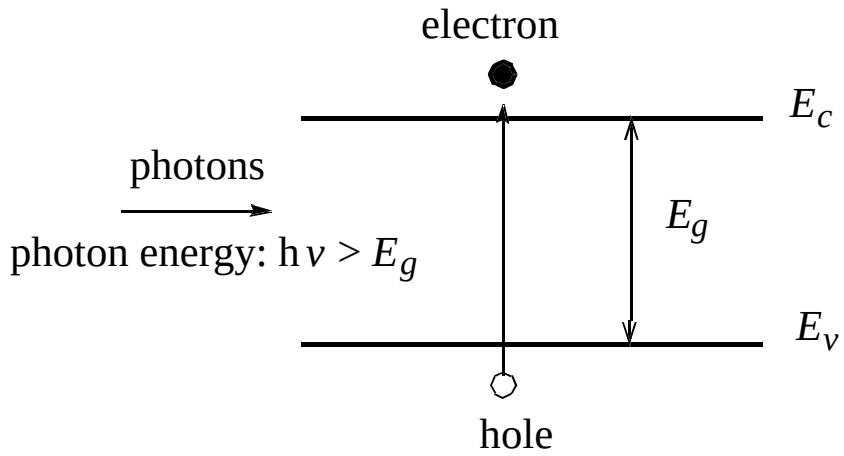
- ***Energy band diagram*** shows the bottom edge of conduction band, E_c , and top edge of valence band, E_v .
- E_c and E_v are separated by the ***band gap energy, E_g*** .

Temperature Effect on Band Gap



How does the band gap change with temperature?

Measuring the Band Gap Energy by Light Absorption

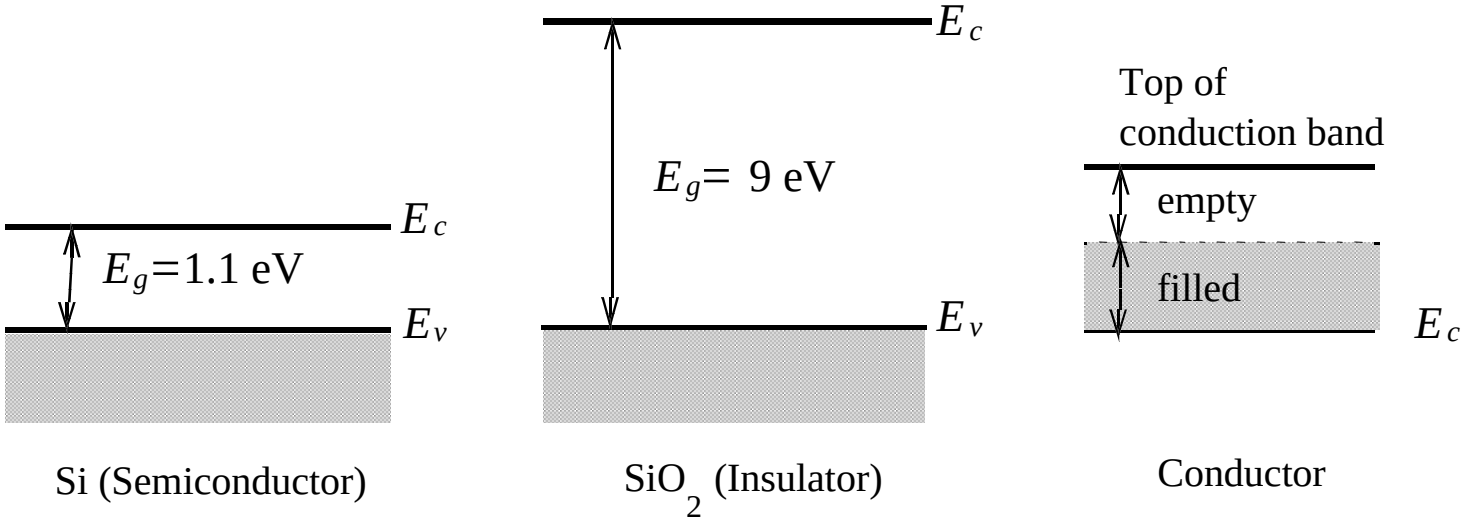


• E_g can be determined from the minimum energy ($h\nu$) of photons that are absorbed by the semiconductor.

Bandgap energies of selected semiconductors

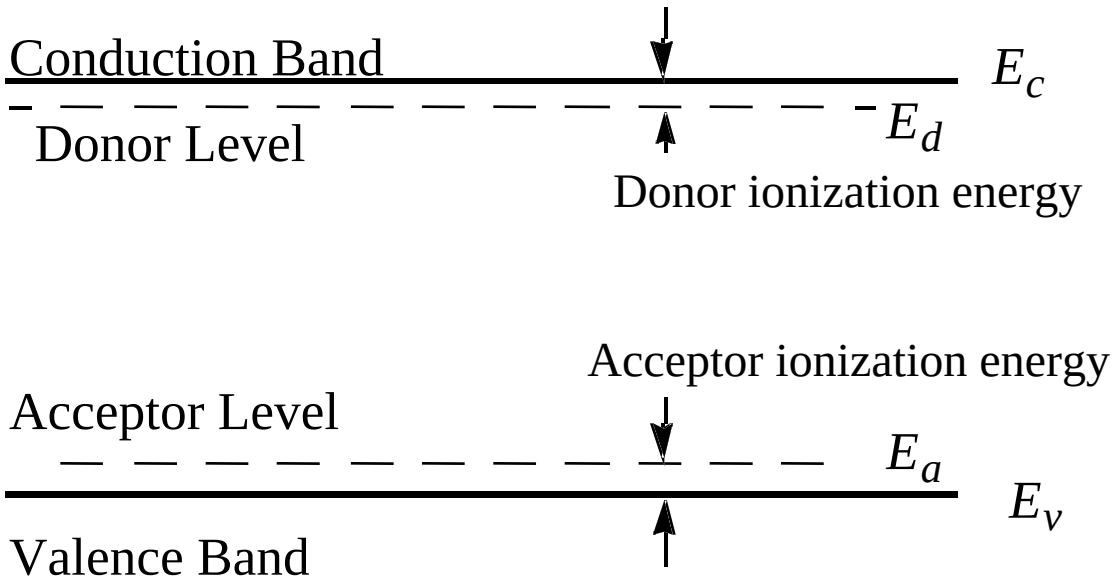
Material	PbTe	Ge	Si	GaAs	GaP	Diamond
E_g (eV)	0.31	0.67	1.12	1.42	2.25	6.0

Semiconductors, Insulators, and Conductors



- Totally filled bands and totally empty bands do not allow current flow. (Just as there is no motion of liquid in a totally filled or totally empty bottle.)
- Metal conduction band is half-filled.
- Semiconductors have lower E_g's than insulators and can be doped.

Donor and Acceptor Levels in the Band Model



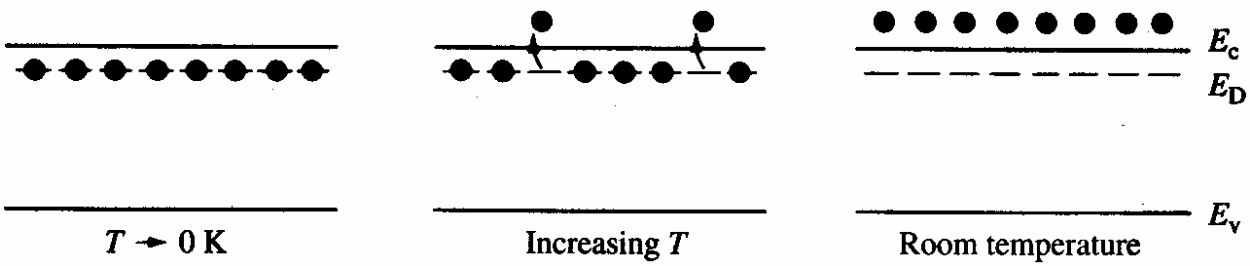
Ionization energy of selected donors and acceptors in silicon

	Donors			Acceptors		
Dopant	Sb	P	As	B	Al	In
Ionization energy, $E_c - E_d$ or $E_a - E_v$ (meV)	39	44	54	45	57	160

Hydrogen: $$E_{ion} = \frac{m_0 q^4}{8\varepsilon_0^2 h^2} = 13.6 \text{ eV}$$

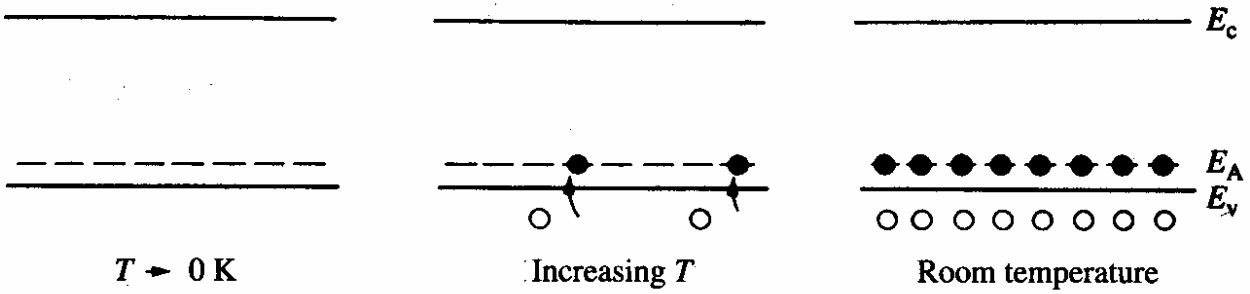
Dopants and Free Carriers

Donors
n-type



(a)

Acceptors
p-type



(b)

Dopant ionization
energy ~50meV (very low).

Effective Mass

In an electric field, $\mathcal{E}$, an electron or a hole accelerates.

Remember :
F=ma=-qE

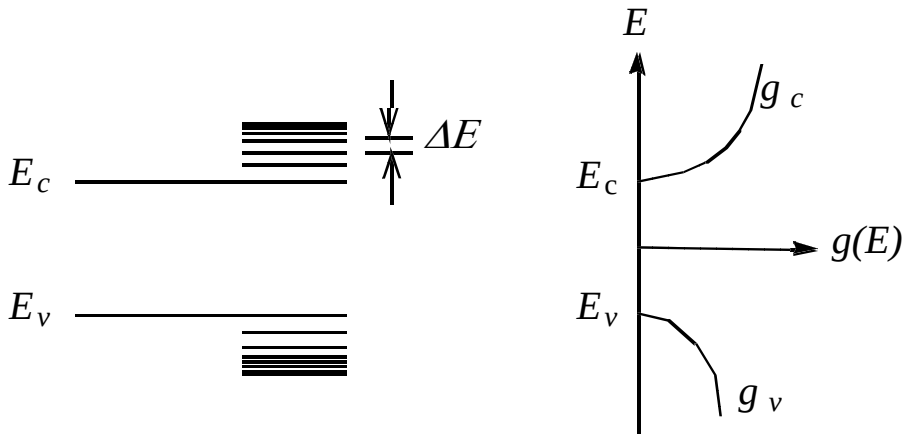
$$a = \frac{-q\mathcal{E}}{m_n} \qquad \text{electrons}$$

$$a = \frac{q\mathcal{E}}{m_p} \qquad \text{holes}$$

Electron and hole effective masses

	Si	**Ge**	**GaAs**	**GaP**
m_n/m_0	0.26	0.12	0.068	0.82
m_p/m_0	0.39	0.30	0.50	0.60

Density of States



$$g_c(E) \equiv \frac{\text{number of states in } \Delta E}{\Delta E \cdot \text{volume}} \quad \left(\frac{1}{\text{eV} \cdot \text{cm}^3}\right)$$

$$g_c(E) \equiv \frac{m_n^* \sqrt{2m_n^*\left(E - E_c\right)}}{\pi^2 \hbar^3}$$

$$g_v(E) \equiv \frac{m_p^* \sqrt{2m_p^*\left(E_v - E\right)}}{\pi^2 \hbar^3}$$

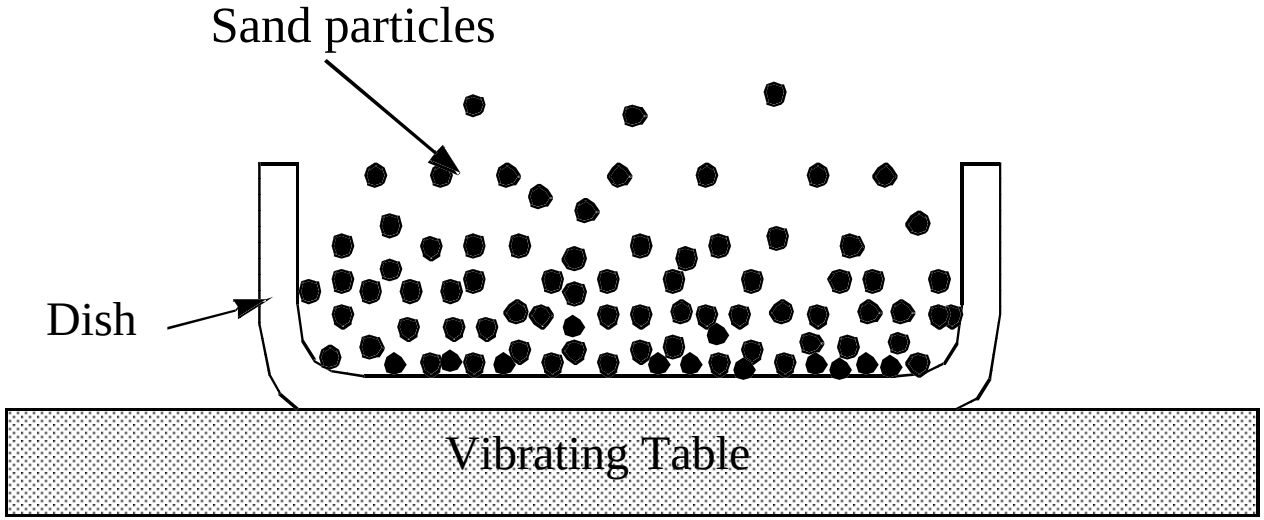
Thermal Equilibrium

- No external forces applied:
  - electric field = 0
  - magnetic field = 0
  - mechanical stress = 0
- Thermal agitation –> electrons and holes exchange energy with the crystal lattice and each other

  ⇒ Every energy state in the conduction and valence bands has a certain probability of being occupied by an electron

Thermal Equilibrium

An Analogy for Thermal Equilibrium



- There is a certain probability for the electrons in the conduction band to occupy high-energy states under the agitation of thermal energy (vibrating atoms, etc.)

Fermi Function

Probability that an available state at energy E is occupied:

$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$$

E_F is called the ***Fermi energy*** or the ***Fermi level***

There is only one Fermi level in a system at equilibrium.

At $E = E_F$, $f(E) = 1/2$

Boltzmann Approximation

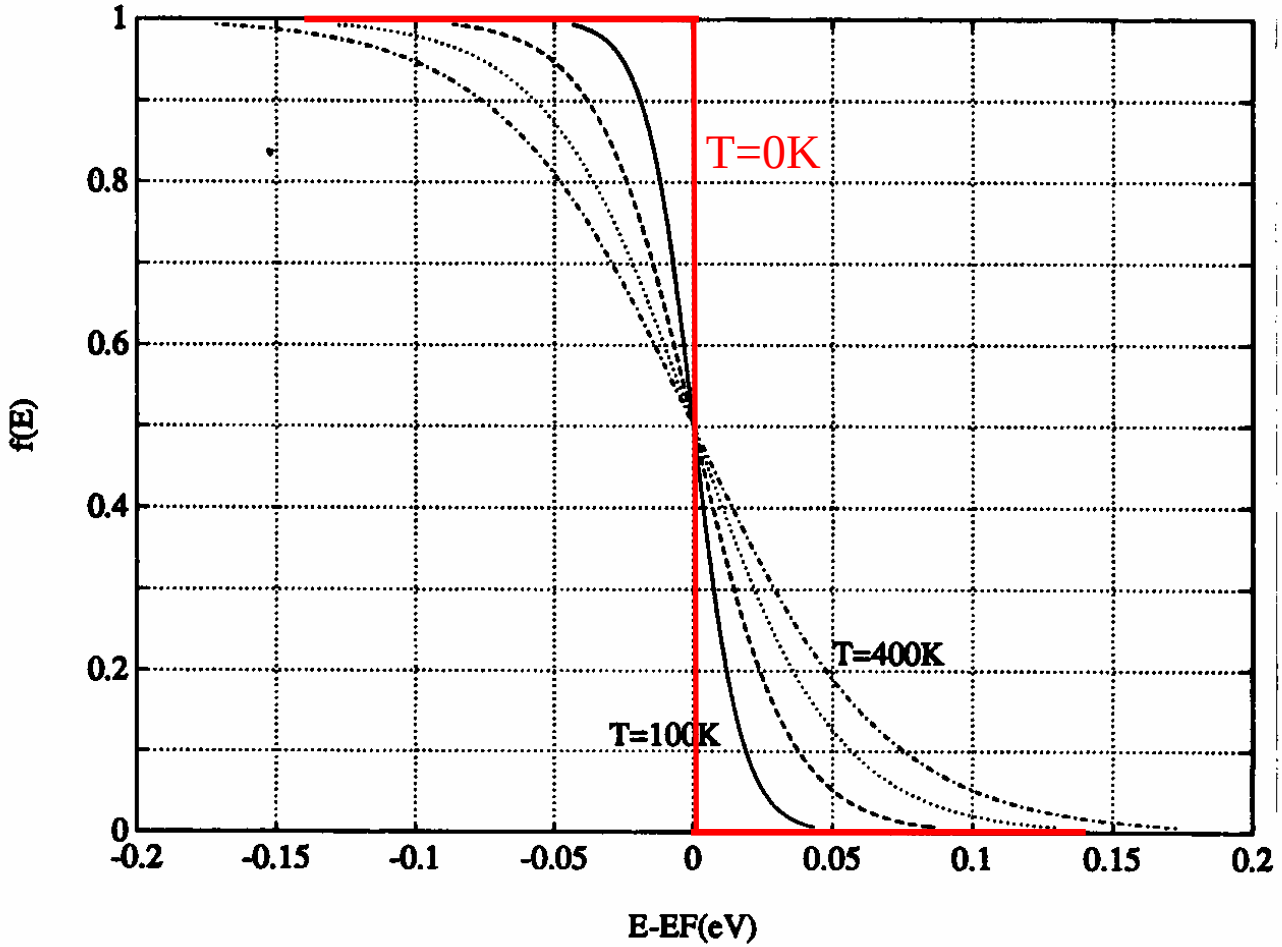
$$\text{If } E - E_F > 3kT, \;\; f(E) \approx e^{-(E-E_F)/kT}$$

$$\text{If } E_F - E > 3kT, \;\; f(E) \approx 1 - e^{E-E_F/kT}$$

<u>Assume the two extremes:</u>

High Energy (Large E): $E\text{-}E_f >> kT$, $f(E) \longrightarrow 0$
Low Energy (Small E): $E\text{-}Ef << kT$, $f(E) \longrightarrow 1$

Effect of T on f(E)



Question

- If $f(E)$ is the probability of a state being occupied by an electron, what is the probability of a state being occupied by a hole?

Equilibrium Distribution of Carriers

- Obtain $n(E)$ by multiplying $g_c(E)$ and $f(E)$
- Obtain $p(E)$ by multiplying $g_v(E)$ and $1-f(E)$

n = electron density : number of unbound electrons / cm^3
p = hole density : number of holes / cm^3

Equilibrium Carrier Concentrations

- Integrate $n(E)$ over all the energies in the conduction band to obtain n

$$n = \int_{E_c}^{\text{top of conduction band}} g_c(E) f(E) dE$$

- By using the Boltzmann approximation, and extending the integration limit to ∞, we obtain

$$n = N_c e^{-(E_c - E_F)/kT} \quad \text{where} \quad N_c = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}$$

N_c is called the ***effective density of states (of the conduction band)*** .

- Integrate $p(E)$ over all the energies in the valence band to obtain p

$$p = \int_{\text{bottom of valence band}}^{E_v} g_v(E)\left[1 - f(E)\right]dE$$

- By using the Boltzmann approximation, and extending the integration limit to -∞, we obtain

$$p = N_v e^{-(E_F - E_v)/kT} \quad \text{where} \quad N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$$

N_v is called the ***effective density of states of the valence band.***

Intrinsic Semiconductor

• Extremely pure semiconductor sample containing an insignificant amount of impurity atoms.

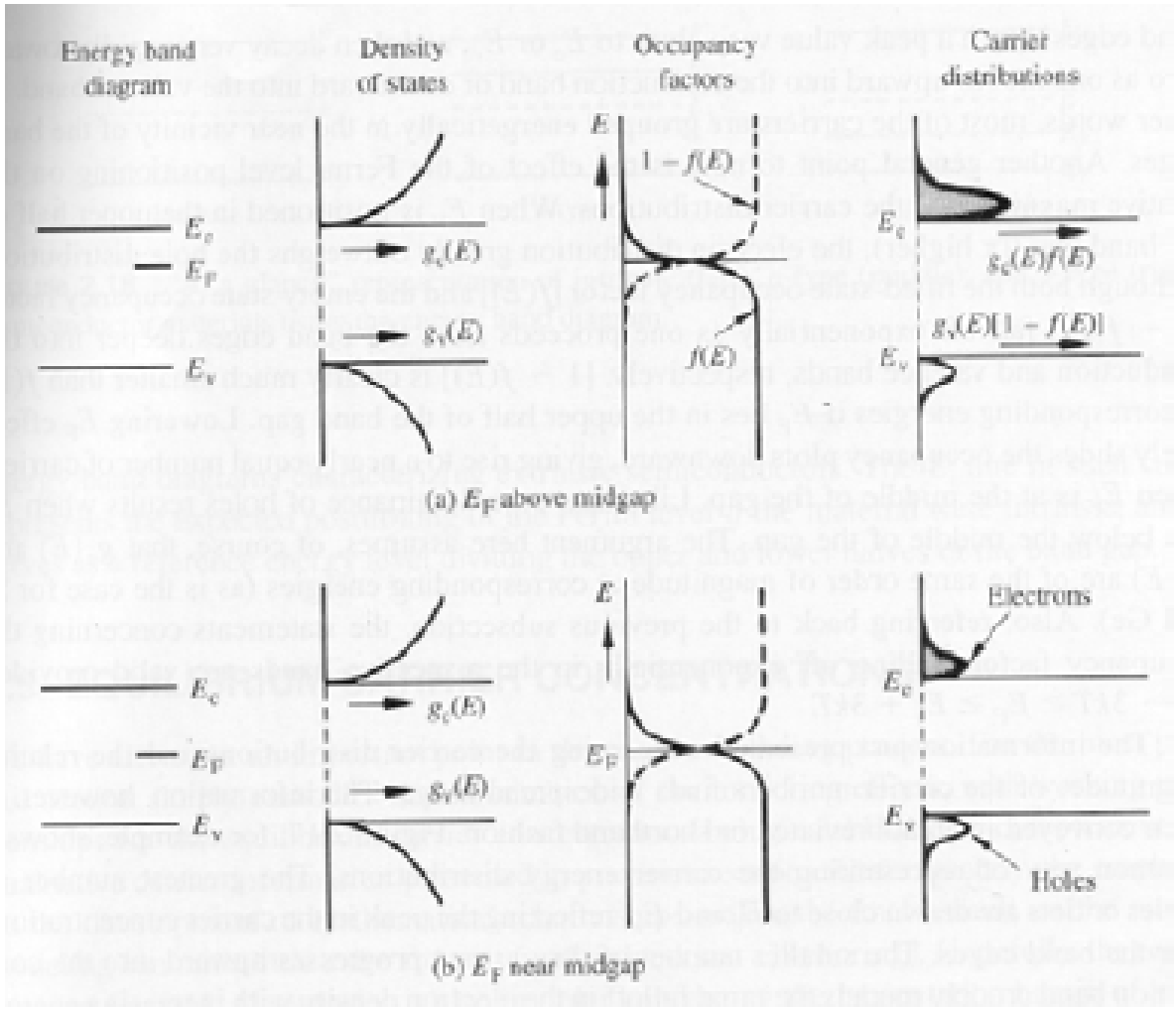
$$n = p = n_i$$

E_f lies in the middle of the band gap

Material	**Ge**	**Si**	**GaAs**
E_g **(eV)**	0.67	1.12	1.42
n_i **(1/cm^3)**	2×10^{13}	1×10^{10}	2×10^{6}

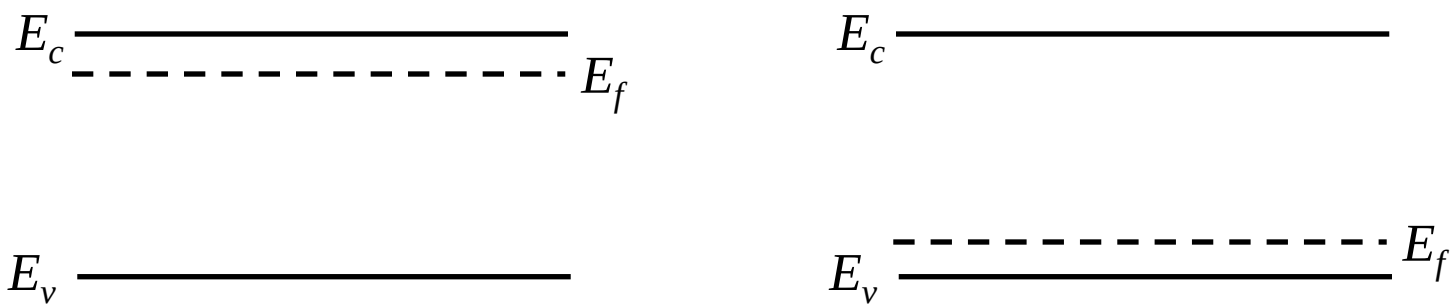
n-type

intrinsic



Remember: the closer E_f moves up to E_c, the larger n is; the closer E_f moves down to E_v , the larger p is.

For Si, $N_c = 2.8\times10^{19}\,cm^{-3}$ and $N_v = 1.04\times10^{19}\,cm^{-3}$.

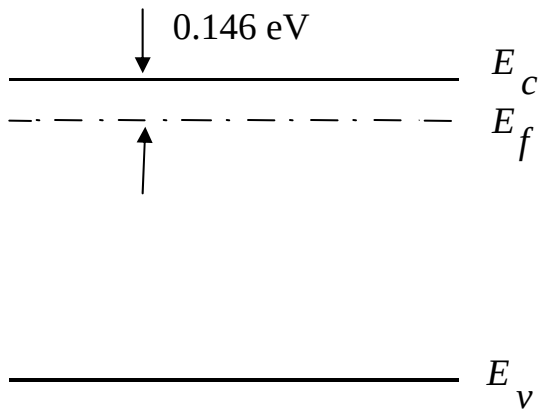


Example: The Fermi Level and Carrier Concentrations

Where is E_f for $n = 10^{17}\ cm^{-3}$? ***Solution:***

$$n = N_c e^{-(E_c - E_f)/kT}$$

$$E_c - E_f = kT \ln\left(N_c / n\right) = 0.026 \ln\left(2.8 \times 10^{19} / 10^{17}\right) = 0.146\ \text{eV}$$



The *np* Product and the Intrinsic Carrier Concentration

Multiply $n = N_c e^{-(E_c - E_f)/kT}$ and $p = N_v e^{-(E_f - E_v)/kT}$

$$np = N_c N_v e^{-(E_c - E_v)/kT} = N_c N_v e^{-E_g/kT}$$

$$\boxed{np = n_i^2}$$

$$\boxed{n_i = \sqrt{N_c N_v}\, e^{-E_g/2kT}}$$

- In an intrinsic (undoped) semiconductor, $n = p = n_i$.

EXAMPLE: Carrier Concentrations

Question: *What is the hole concentration in an N-type semiconductor with 10^{15} cm^{-3} of donors?*

Solution: *$n = 10^{15}$ cm^{-3}.*

$$p = \frac{n_i^{\,2}}{n} \approx \frac{10^{20}\,\text{cm}^{-3}}{10^{15}\,\text{cm}^{-3}} = 10^{5}\,\text{cm}^{-3}$$

After increasing T by 60 ℃, n remains the same at 10^{15} cm^{-3} while p increases by about a factor of 2300 because $n_i^{\,2} \propto e^{-E_g/kT}$.

Question: *What is n if $p = 10^{17}$cm^{-3} in a P-type silicon wafer?*

Solution:

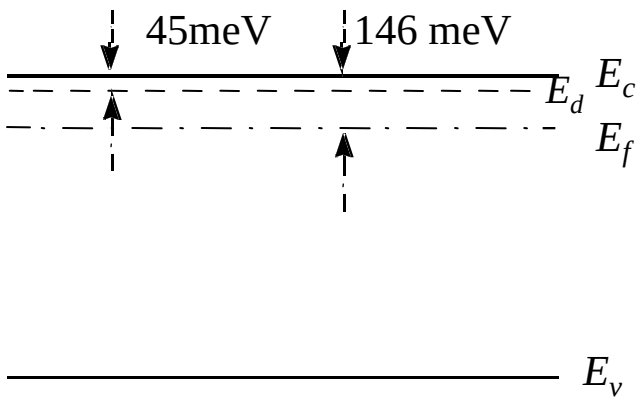
$$n = \frac{n_i^{\,2}}{p} \approx \frac{10^{20}\,\text{cm}^{-3}}{10^{17}\,\text{cm}^{-3}} = 10^{3}\,\text{cm}^{-3}$$

EXAMPLE: Complete ionization of the dopant atoms

$N_d = 10^{17}$ cm^{-3} and E_c-E_d=45 meV. What fraction of the donors are not ionized?

Solution: *First assume that all the donors* ***are*** *ionized.*

$$n = N_d = 10^{17}\,\text{cm}^{-3} \Rightarrow E_f = E_c - 146\text{meV}$$



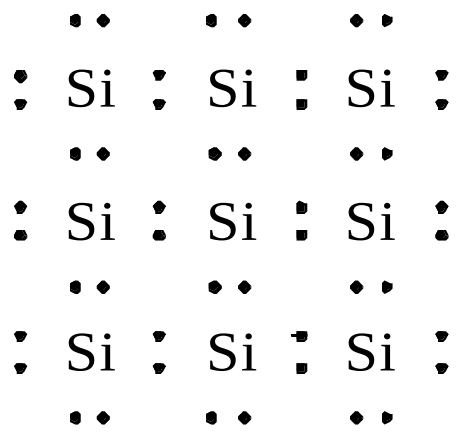
$$\text{Probability of non-ionization} \approx \frac{1}{1+e^{(E_d - E_f)/kT}} = \frac{1}{1+e^{((146-45)\text{meV})/26\text{meV}}} = 0.02$$

Therefore, it is reasonable to assume complete ionization, i.e., $n = N_d$.

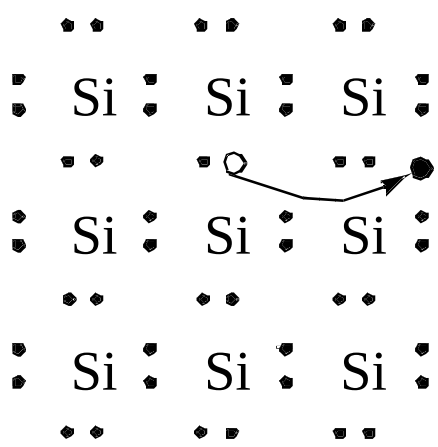
Doped Si and Charge

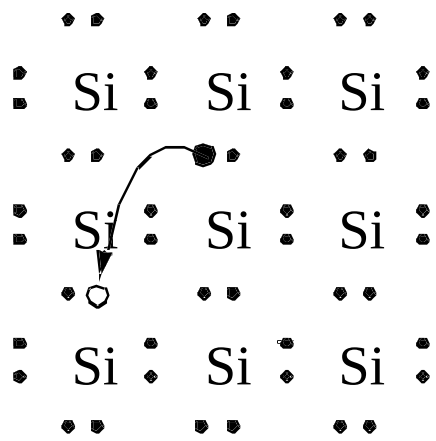
- What is the net charge of your Si when it is electron and hole doped?

Bond Model of Electrons and Holes (Intrinsic Si)



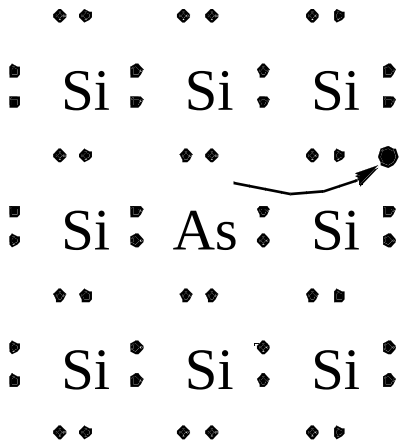
- Silicon crystal in a two-dimensional representation.



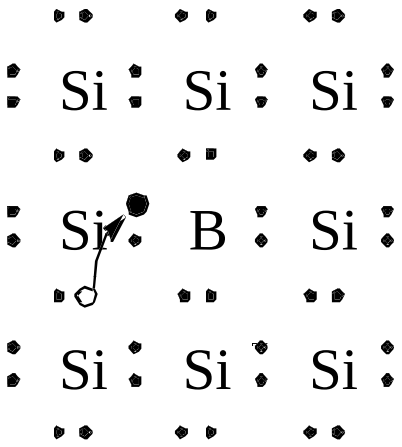


- When an electron breaks loose and becomes a ***conduction electron***, a ***hole*** is also created.

Dopants in Silicon



N-type Si



P-type Si

- As (Arsenic), a Group V element, introduces conduction electrons and creates ***N-type silicon,*** and is called a ***donor.***
- B (Boron), a Group III element, introduces holes and creates ***P-type silicon***, and is called an ***acceptor***.
- Donors and acceptors are known as dopants.

General Effects of Doping on n and p

Charge neutrality: $n + N_a^- - p - N_d^+ = 0$

N_a^- : number of ionized acceptors /cm3

N_d^+ : number of ionized donors /cm3

Assuming total ionization of acceptors and donors:

$$n + N_a - p - N_d = 0$$

N_a : number of ionized acceptors /cm3

N_d^+ : number of ionized donors /cm3

General Effects of Doping on n and p

I. $N_d - N_a >> n_i$ (i.e., N-type)

$$\boxed{\begin{aligned} n &= N_d - N_a \\ p &= n_i^2 / n \end{aligned}}$$

If $N_d >> N_a$, $n = N_d$ and $p = n_i^2 / N_d$

II. $N_a - N_d >> n_i$ (i.e., P-type)

$$\boxed{\begin{aligned} p &= N_a - N_d \\ n &= n_i^2 / p \end{aligned}}$$

If $N_a >> N_d$, $p = N_a$ and $n = n_i^2 / N_a$

EXAMPLE: Dopant Compensation

What are n and p in Si with (a) $N_d = 6\times10^{16}$ cm^{-3} and $N_a = 2\times10^{16}$ cm^{-3} and (b) additional 6×10^{16} cm^{-3} of N_a?

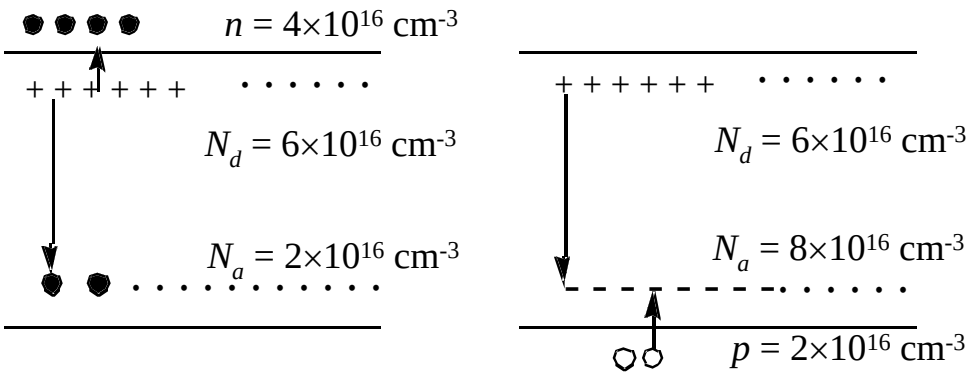
(a) $n = N_d - N_a = 4\times10^{16}\,\text{cm}^{-3}$

$$p = n_i^2 / n = 10^{20} / 4\times10^{16} = 2.5\times10^{3}\,\text{cm}^{-3}$$

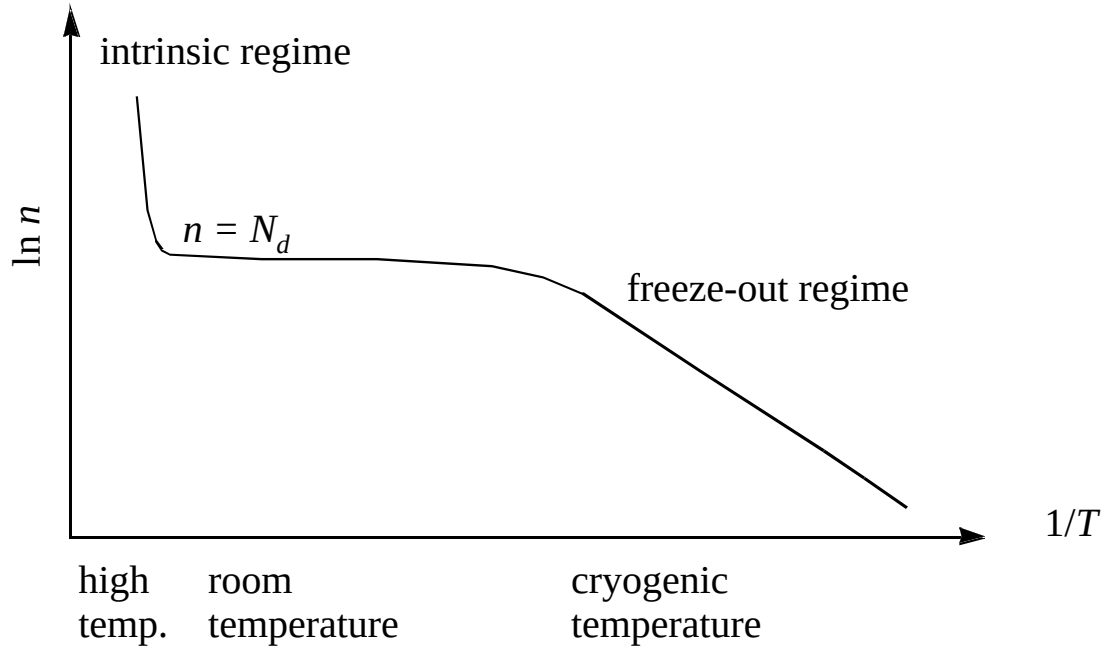
(b) $N_a = 2\times10^{16} + 6\times10^{16} = 8\times10^{16}$ cm^{-3} > N_d!

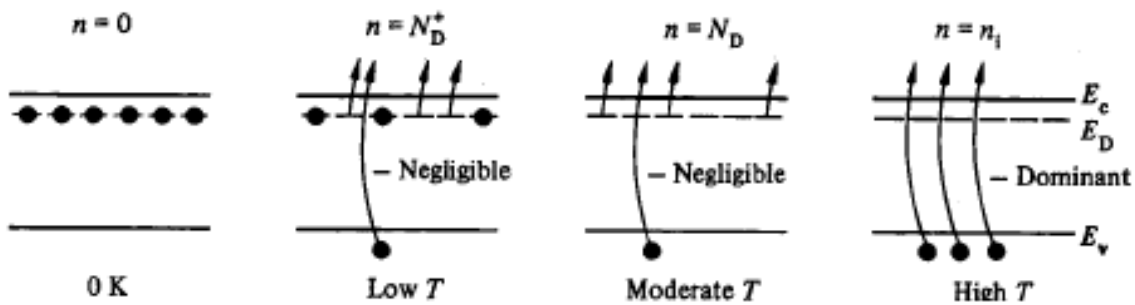
$$p = N_a - N_d = 8\times10^{16} - 6\times10^{16} = 2\times10^{16}\,\text{cm}^{-3}$$

$$n = n_i^2 / p = 10^{20} / 2\times10^{16} = 5\times10^{3}\,\text{cm}^{-3}$$



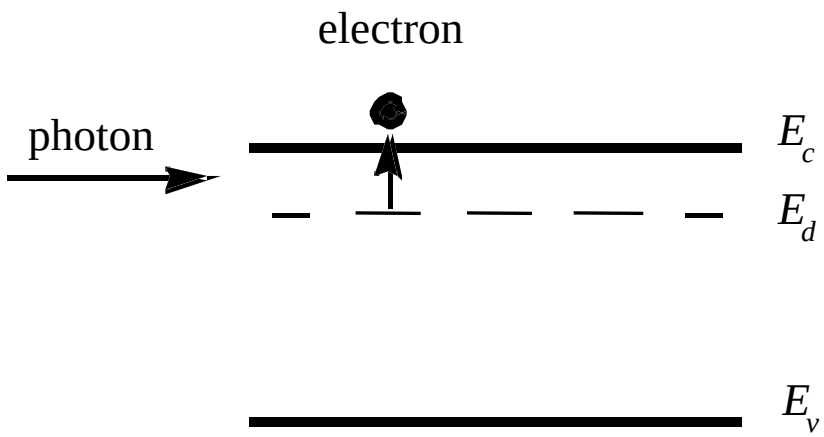
Carrier Concentrations at Extremely High and Low Temperatures





Infrared Detector Based on Freeze-out

To image the black-body radiation emitted by tumors requires a photodetector that responds to $h\nu$'s around 0.1 eV. In doped Si operating in the freeze-out mode, conduction electrons are created when the infrared photons provide the energy to ionized the donor atoms.



Chapter Summary

Energy band diagram. Acceptor. Donor. m_n, m_p. Fermi function. E_f.

$$n = N_c e^{-(E_c - E_f)/kT}$$

$$p = N_v e^{-(E_f - E_v)/kT}$$

$$n = N_d - N_a$$

$$p = N_a - N_d$$

$$np = n_i^2$$