EECS130
Integrated Circuit Devices

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11/20/2007

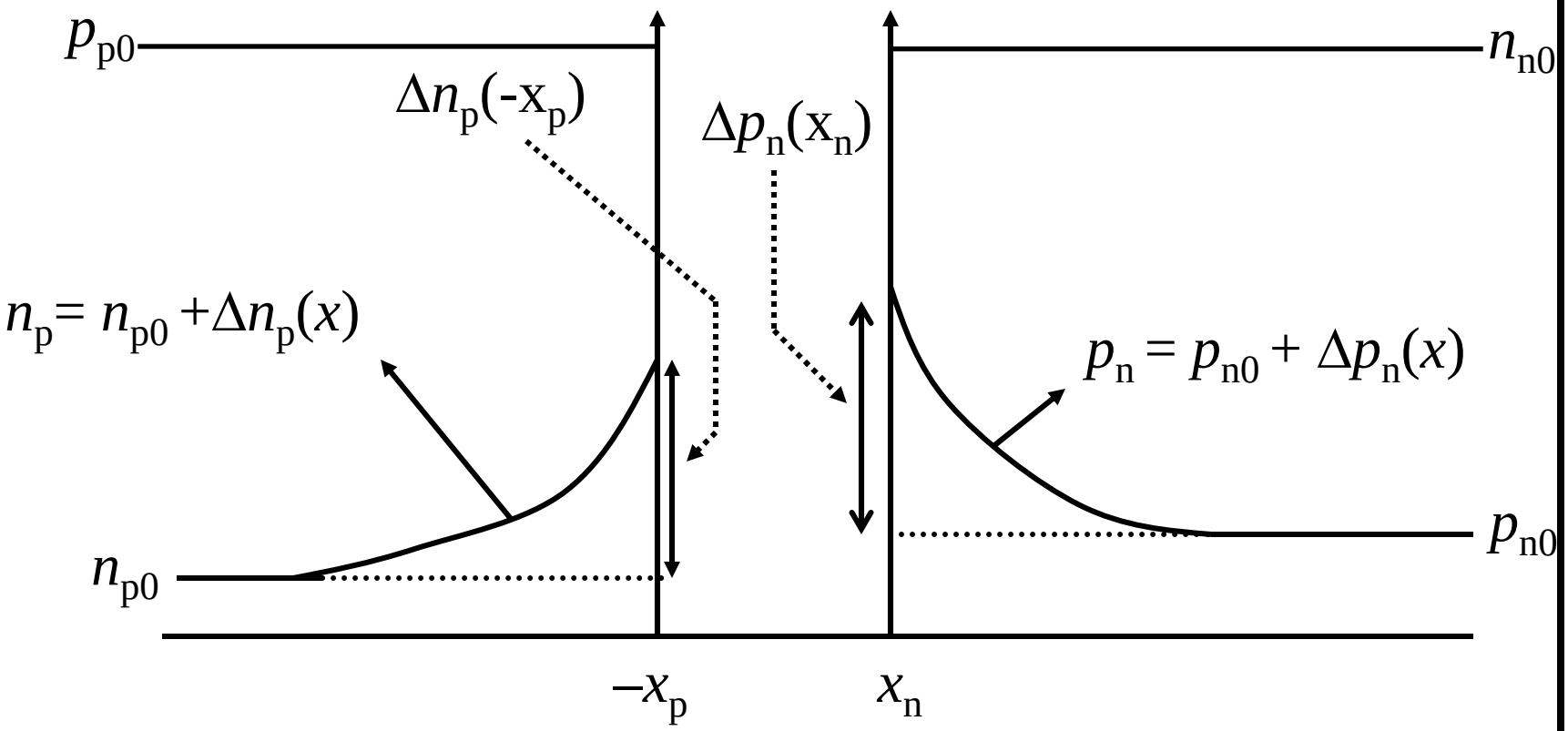
BJTs- Lecture 4

Reading Assignment: Chapter 11

Announcements

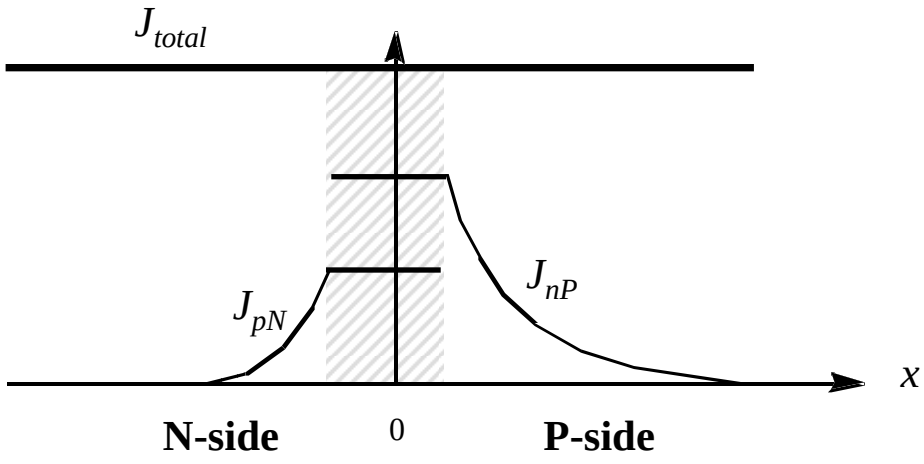
- HW9 is due now.

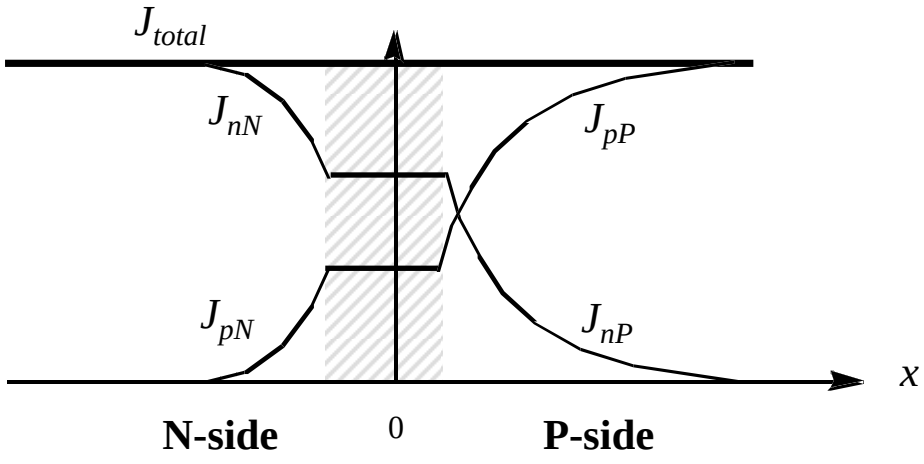
PN Junction Review: Carrier concentration profile under bias



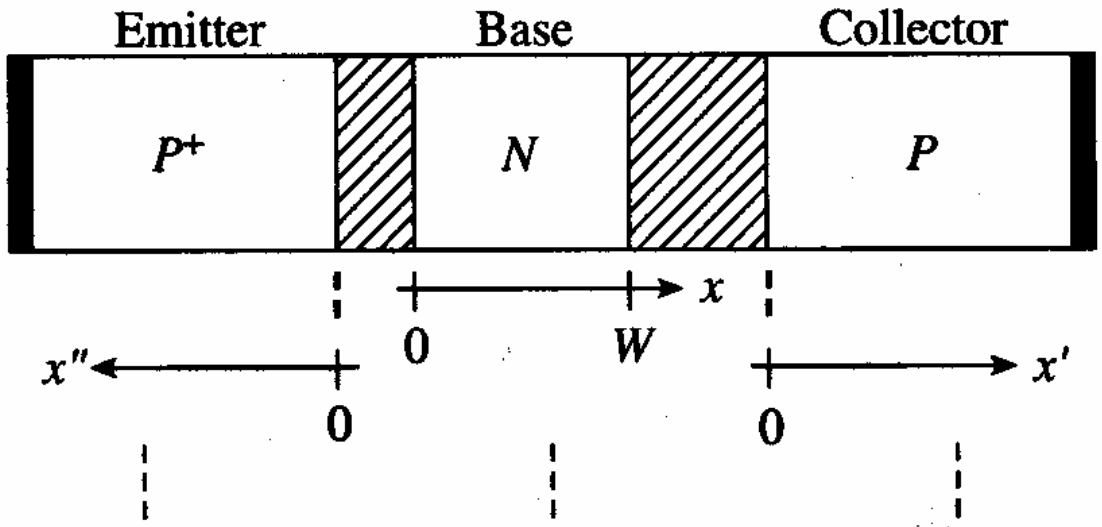
Question: how does this curve look for reverse bias?

PN Junction Review: Current Profile



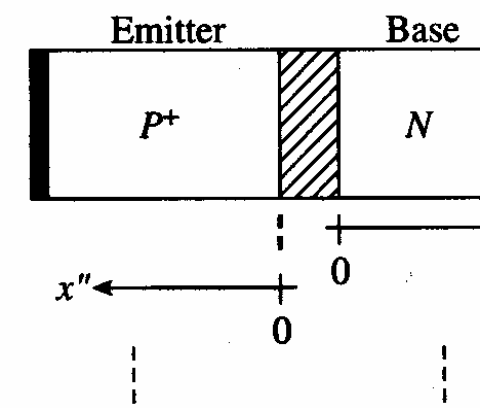


Qualitative Solution - Definitions



$N_E = N_{AE}$	$N_B = N_{DB}$	$N_C = N_{AC}$
$D_E = D_N$	$D_B = D_P$	$D_C = D_N$
$\tau_E = \tau_n$	$\tau_B = \tau_p$	$\tau_C = \tau_n$
$L_E = L_N$	$L_B = L_P$	$L_C = L_N$
$n_{E0} = n_{p0} = n_i^2/N_E$	$p_{B0} = p_{n0} = n_i^2/N_B$	$n_{C0} = n_{p0} = n_i^2/N_C$

Emitter Region Formulation



- Diffusion equation:

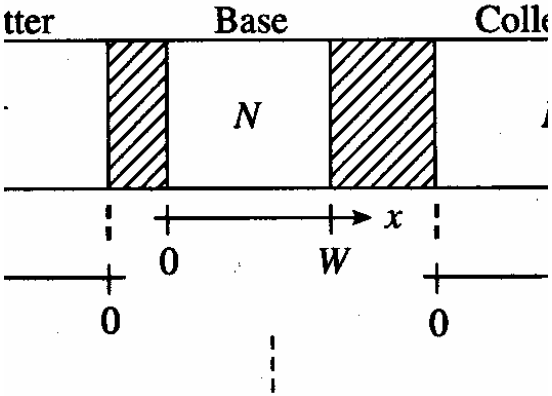
$$0 = D_E \frac{d^2 \Delta n_E}{dx''^2} - \frac{\Delta n_E}{\tau_E}$$

- Boundary Conditions

$$\Delta n_E (x'' \to \infty) = 0$$

$$\Delta n_E (x'' = 0) = n_{E0} (e^{qV_{EB}/kT} - 1)$$

Base Region Formulation



- Diffusion equation:

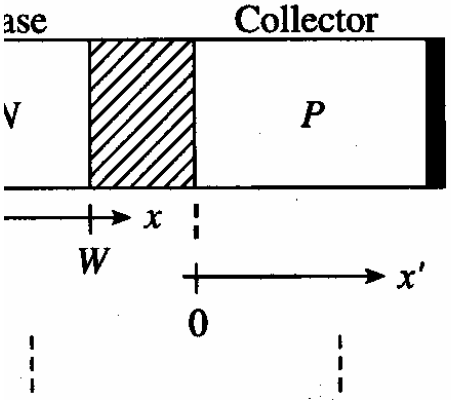
$$0 = D_B \frac{d^2 \Delta n_B}{dx^2} - \frac{\Delta p_B}{\tau_B}$$

- Boundary Conditions

$$\Delta p_B(0) = p_{B0}(e^{qV_{EB}/kT} - 1)$$

$$\Delta p_B(W) = p_{B0}(e^{qV_{CB}/kT} - 1)$$

Collector Region Formulation



- Diffusion equation:

$$0 = D_C \frac{d^2 \Delta n_C}{dx'^2} - \frac{\Delta n_C}{\tau_C}$$

- Boundary Conditions

$$\Delta n_C(x' \to \infty) = 0$$

$$\Delta n_C(x' = 0) = n_{C0}(e^{qV_{CB}/kT} - 1)$$

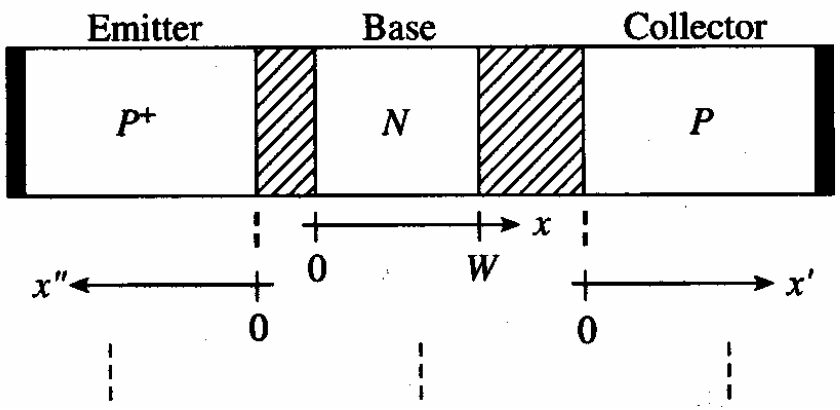
Current Formulation

$$I_{En} = -qAD_E \frac{d\Delta n_E}{dx''}\bigg|_{x''=0}$$

$$I_{Ep} = -qAD_B \frac{d\Delta p_B}{dx}\bigg|_{x=0}$$

$$I_{Cp} = -qAD_B \frac{d\Delta p_B}{dx}\bigg|_{x=W}$$

$$I_{Cn} = qAD_C \frac{d\Delta n_C}{dx'}\bigg|_{x'=0}$$



Emitter Region Solution

- The solution of: $0 = D_E \frac{d^2 \Delta n_E}{dx''^2} - \frac{\Delta n_E}{\tau_E}$

  is: $\Delta n_E(x'') = A_1 e^{-x''/L_E} + A_2 e^{x''/L_E}$

- From the boundary conditions:

$$\Delta n_E(x'' \to \infty) = 0$$

$$\Delta n_E(x'' = 0) = n_{E0}(e^{qV_{EB}/kT} - 1)$$

  we have:

$$\Delta n_E(x'') = n_{E0}(e^{qV_{EB}/kT} - 1)e^{-x''/L_E}$$

  and:

$$I_{En} = -qA\frac{D_E}{L_E} n_{E0}(e^{qV_{EB}/kT} - 1)$$

Collector Region Solution

- The solution of: $0 = D_C \frac{d^2 \Delta n_C}{dx'^2} - \frac{\Delta n_C}{\tau_C}$

  is:

$$\Delta n_C(x') = A_1 e^{-x'/L_C} + A_2 e^{x'/L_C}$$

- From the boundary conditions:

$$\Delta n_C(x' \to \infty) = 0$$

$$\Delta n_C(x' = 0) = n_{C0}(e^{qV_{CB}/kT} - 1)$$

  we have:

$$\Delta n_C(x') = n_{C0}(e^{qV_{CB}/kT} - 1)e^{-x'/L_C}$$

  and:

$$I_{Cn} = -qA\frac{D_C}{L_C} n_{C0}(e^{qV_{CB}/kT} - 1)$$

Base Region Solution

- The solution of: $0 = D_B \frac{d^2 \Delta n_B}{dx^2} - \frac{\Delta p_B}{\tau_B}$ is:

$$\Delta p_B(x) = A_1 e^{-x/L_B} + A_2 e^{x/L_B}$$

- From the boundary conditions:

$$\Delta p_B(0) = p_{B0}(e^{qV_{EB}/kT} - 1)$$

$$\Delta p_B(W) = p_{B0}(e^{qV_{CB}/kT} - 1)$$

we have:

$$\Delta p_B(x) = p_{B0}(e^{qV_{EB}/kT} - 1)\left(\frac{e^{(W-x)/L_B} - e^{-(w-x)/L_B}}{e^{W/L_B} - e^{-W/L_B}}\right)$$

$$+ p_{B0}(e^{qV_{CB}/kT} - 1)\left(\frac{e^{x/L_B} - e^{-x/L_B}}{e^{W/L_B} - e^{-W/L_B}}\right)$$

Base Region Solution (cont'd)

- Now, we know: $\sinh(\xi) = \frac{e^{\xi} - e^{-\xi}}{2}$
- Therefore, we can write:

$$\Delta p_B(x) = p_{B0}(e^{qV_{EB}/kT} - 1)\left(\frac{e^{(W-x)/L_B} - e^{-(w-x)/L_B}}{e^{W/L_B} - e^{-W/L_B}}\right)$$

$$+ p_{B0}(e^{qV_{CB}/kT} - 1)\left(\frac{e^{x/L_B} - e^{-x/L_B}}{e^{W/L_B} - e^{-W/L_B}}\right)$$

as:

$$\Delta p_B(x) = p_{B0}(e^{qV_{EB}/kT} - 1)\frac{\sinh\left[(W-x)/L_B\right]}{\sinh\left(W/L_B\right)}$$

$$+ p_{B0}(e^{qV_{CB}/kT} - 1)\frac{\sinh\left[x/L_B\right]}{\sinh\left(W/L_B\right)}$$

Base Region Solution (cont'd)

- Now, we know: $\cosh(\xi) = \frac{e^{\xi} + e^{-\xi}}{2}$
- Therefore, we have:

$$I_{Ep} = qA\frac{D_B}{L_B}\, p_{B0}\left[\frac{\cosh(W/L_B)}{\sinh(W/L_B)}\left(e^{qV_{EB}/kT} - 1\right) - \frac{1}{\sinh(W/L_B)}\left(e^{qV_{CB}/kT} - 1\right)\right]$$

and:

$$I_{Cp} = qA\frac{D_B}{L_B}\, p_{B0}\left[\frac{1}{\sinh(W/L_B)}\left(e^{qV_{EB}/kT} - 1\right) - \frac{\cosh(W/L_B)}{\sinh(W/L_B)}\left(e^{qV_{CB}/kT} - 1\right)\right]$$

Terminal Currents

- We know:

$$I_{En} = -qA\frac{D_E}{L_E}n_{E0}(e^{qV_{EB}/kT}-1)$$

$$I_{Ep} = qA\frac{D_B}{L_B}p_{B0}\left[\frac{\cosh(W/L_B)}{\sinh(W/L_B)}(e^{qV_{EB}/kT}-1)-\frac{1}{\sinh(W/L_B)}\left(e^{qV_{CB}/kT}-1\right)\right]$$

$$I_{Cp} = qA\frac{D_B}{L_B}p_{B0}\left[\frac{1}{\sinh(W/L_B)}(e^{qV_{EB}/kT}-1)-\frac{\cosh(W/L_B)}{\sinh(W/L_B)}\left(e^{qV_{CB}/kT}-1\right)\right]$$

$$I_{Cn} = -qA\frac{D_C}{L_C}n_{C0}(e^{qV_{CB}/kT}-1)$$

- Therefore:

$$I_E = qA\left[\left(\frac{D_E}{L_E}n_{E0}+\frac{D_B}{L_B}p_{B0}\frac{\cosh(W/L_B)}{\sinh(W/L_B)}\right)(e^{qV_{EB}/kT}-1)-\left(\frac{D_B}{L_B}p_{B0}\frac{1}{\sinh(W/L_B)}\right)\left(e^{qV_{CB}/kT}-1\right)\right]$$

$$I_C = qA\left[\left(\frac{D_B}{L_B}p_{B0}\frac{1}{\sinh(W/L_B)}\right)(e^{qV_{EB}/kT}-1)-\left(\frac{D_C}{L_C}n_{C0}+\frac{D_B}{L_B}p_{B0}\frac{\cosh(W/L_B)}{\sinh(W/L_B)}\right)\left(e^{qV_{CB}/kT}-1\right)\right]$$

Question: What is I_B?

Simplification

- In real BJTs, we make W << L_B so that we have a high gain. Then, we note:

$$\sinh(\xi) \rightarrow \xi \ldots \xi << 1$$

$$\cosh(\xi) \rightarrow 1 + \tfrac{\xi^2}{2} \ldots \xi << 1$$

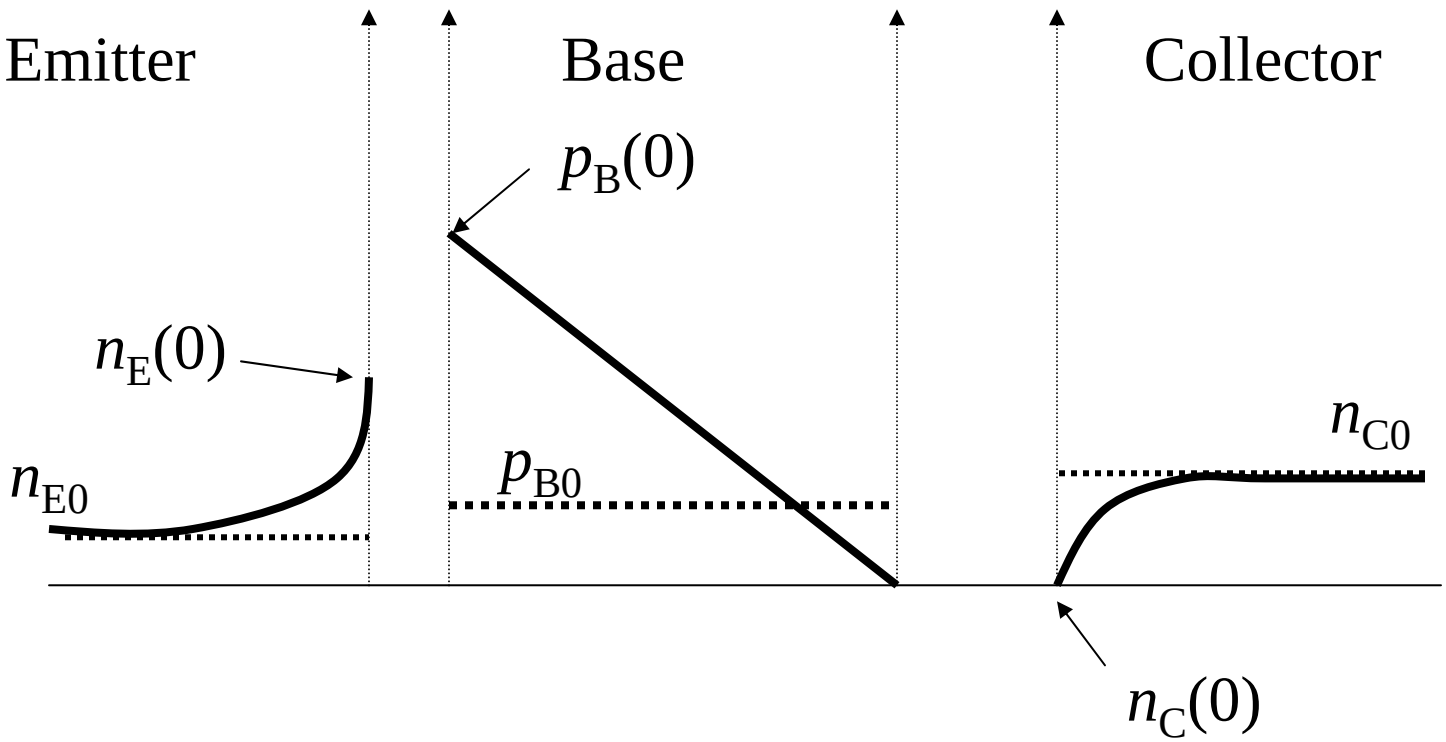
- So, we have:

$$\Delta p_B(x) \cong p_{B0}(e^{qV_{EB}/kT} - 1)\left(1 - \tfrac{x}{W}\right)$$

$$+ p_{B0}(e^{qV_{CB}/kT} - 1)\left(\tfrac{x}{W}\right)$$

Simplified Analysis

Consider the carrier distribution in a forward active pnp transistor



Question: why does the carrier concentration show linear dependence in the base?

Question:

- Plot carrier distribution profile for a BJT under cut-off.

Performance Parameters

$$\gamma = \frac{1}{1 + \frac{D_E}{D_B}\frac{N_B}{N_E}\frac{W}{L_E}}$$

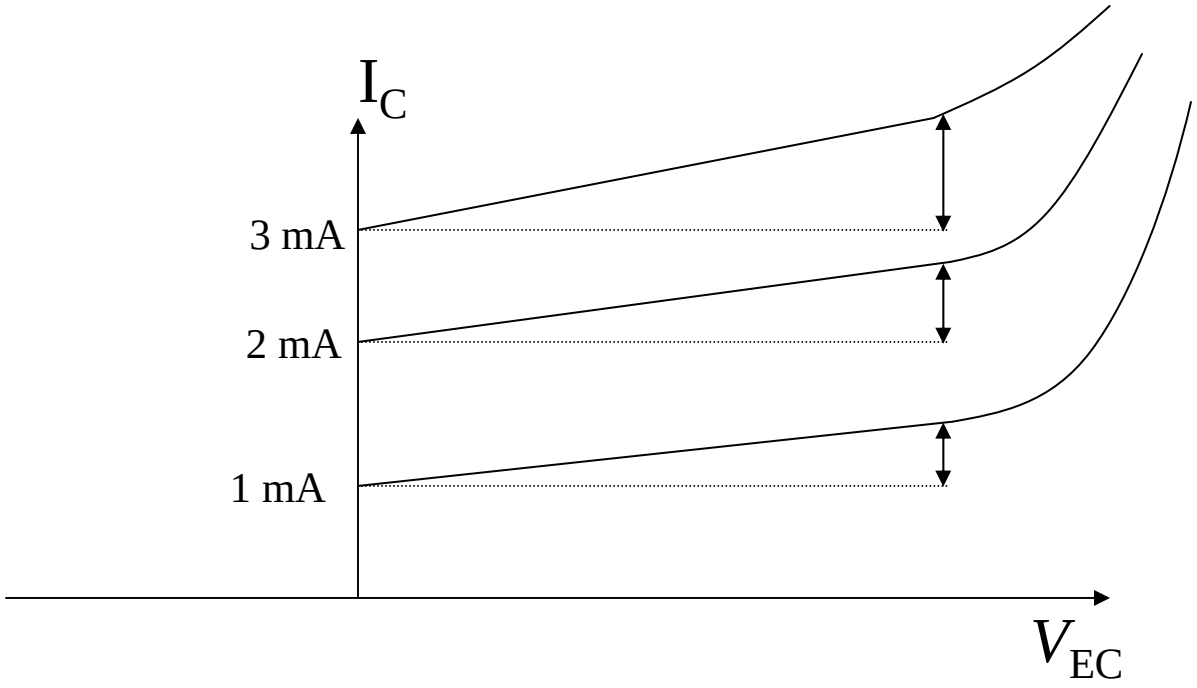
$$\alpha_T = \frac{1}{1 + \frac{1}{2}\left(\frac{W}{L_B}\right)^2}$$

$$\alpha_{dc} = \frac{1}{1 + \frac{D_E}{D_B}\frac{N_B}{N_E}\frac{W}{L_E} + \frac{1}{2}\left(\frac{W}{L_B}\right)^2}$$

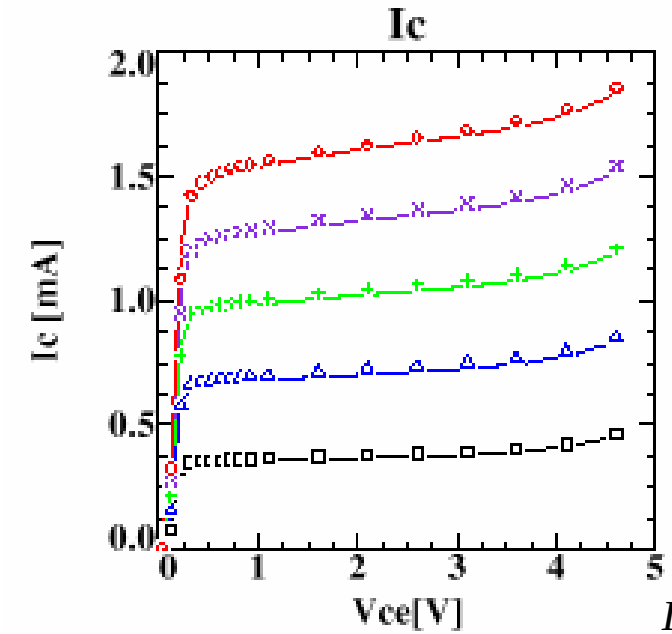
$$\beta_{dc} = \frac{1}{\frac{D_E}{D_B}\frac{N_B}{N_E}\frac{W}{L_E} + \frac{1}{2}\left(\frac{W}{L_B}\right)^2}$$

Base width modulation

When the reverse bias applied to the C-B junction increases, the C-B depletion width increases and W decreases. Thus, the collector current, I_C increases. This is also known as "Early Effect". More prominent in narrow-base transistors.



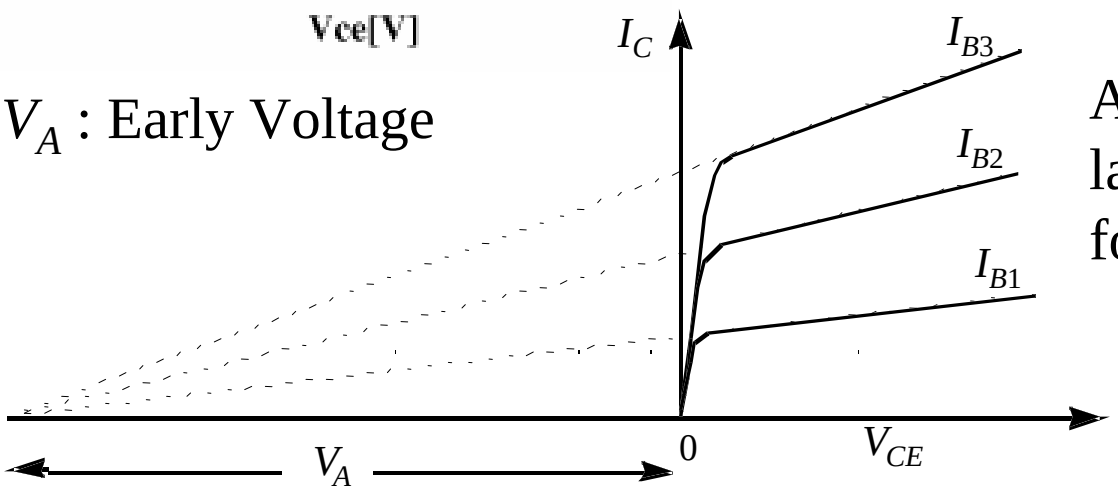
Base-Width Modulation



Output resistance :

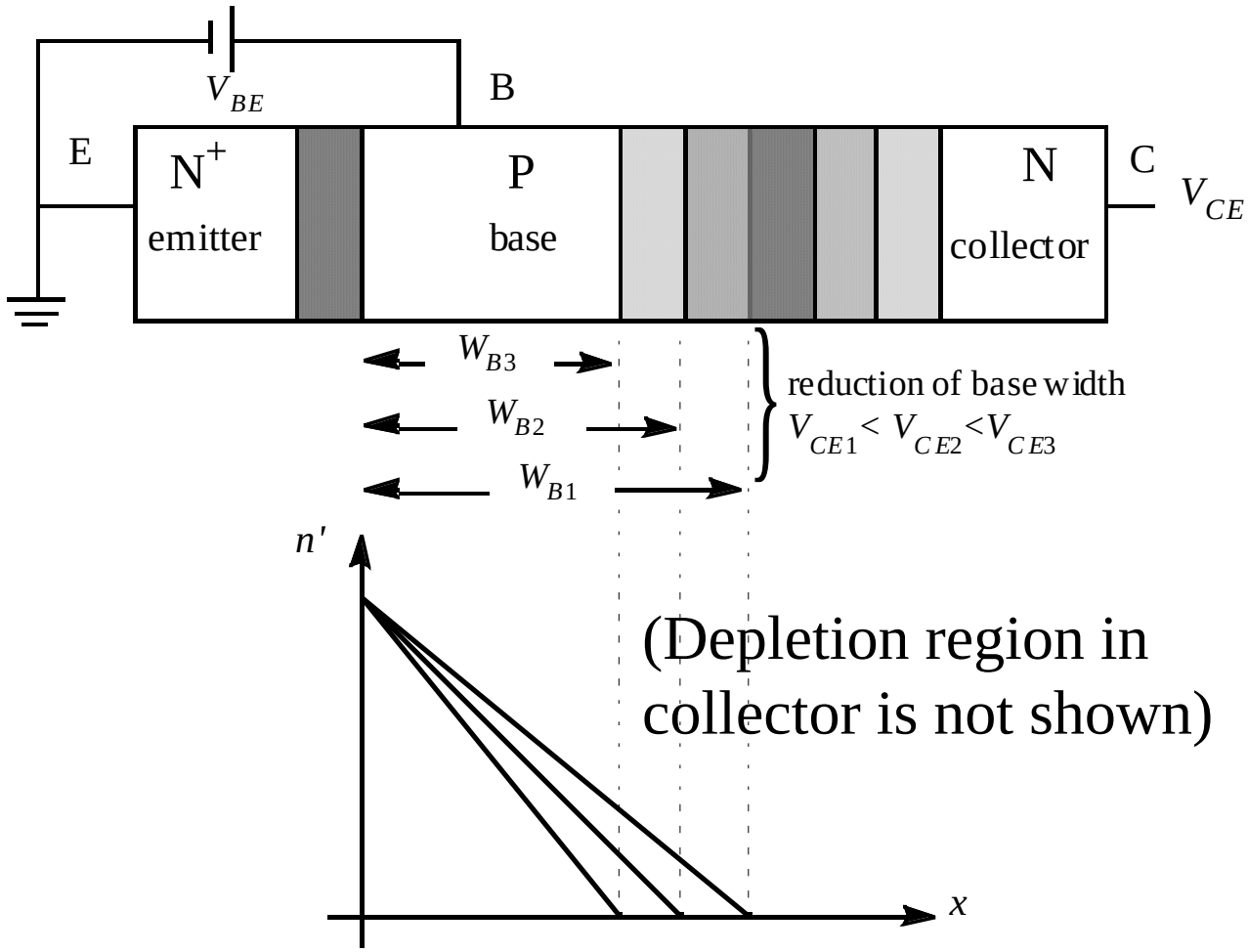
$$r_0 \equiv \left(\frac{\partial I_C}{\partial V_{CE}} \right)^{-1} = \frac{V_A}{I_C}$$

V_A : Early Voltage



A large V_A (i.e. a larger r_o) is desirable for voltage gain

Base-Width Modulation by Collector Voltage

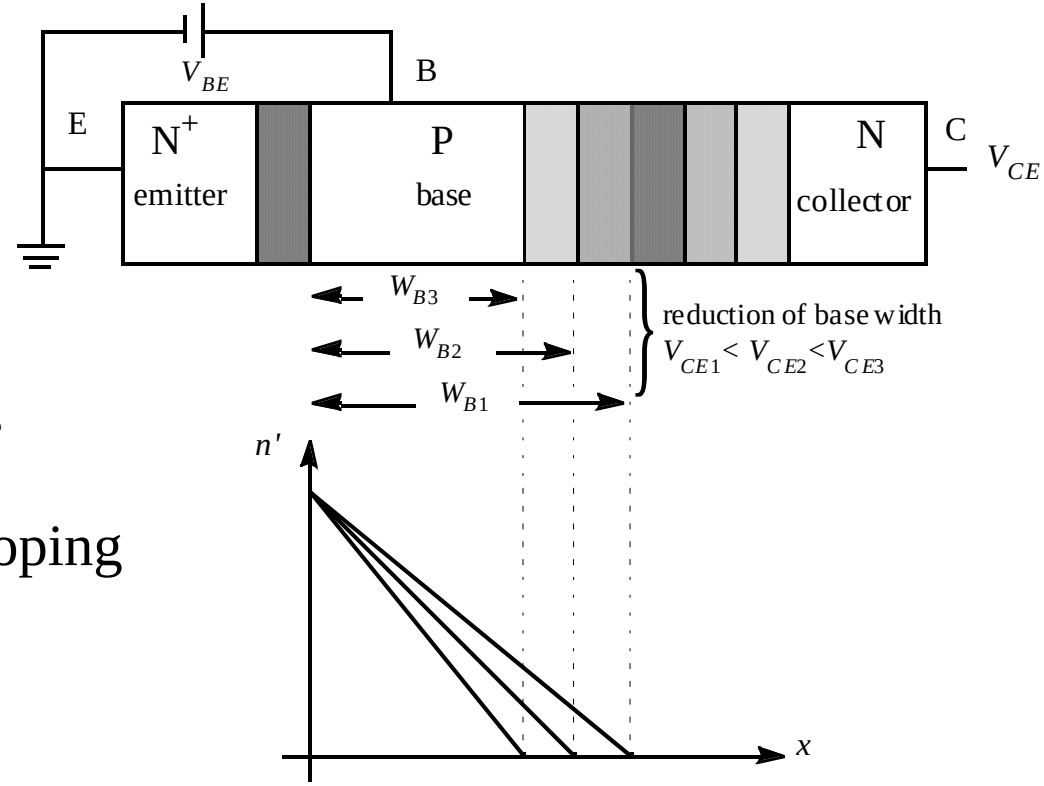


How can we reduce the base-width modulation effect?

Base-Width Modulation by Collector Voltage

The base-width modulation effect is reduced if we

(A) Increase the base width,
(B) Increase the base doping concentration, N_B, or
(C) Decrease the collector doping concentration, N_C.



Which of the above is the most acceptable action?