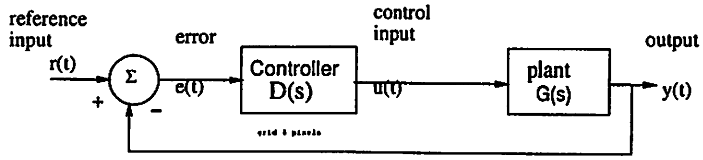
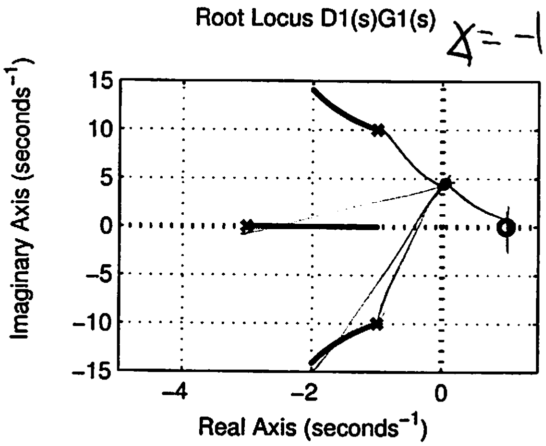
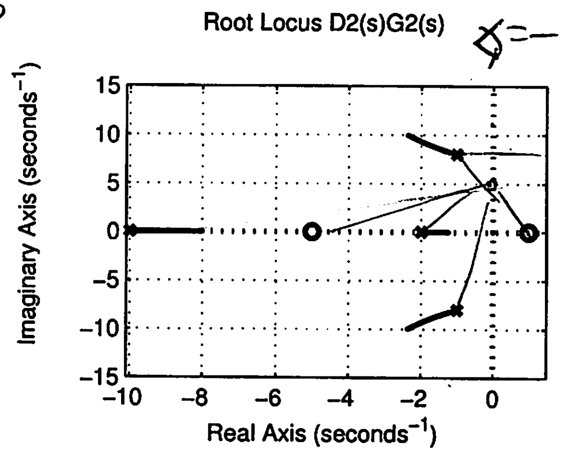
Problem 1 (16 pts) EE128 Fall 2011



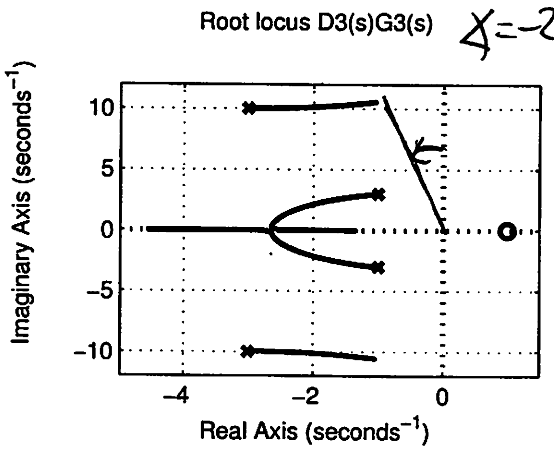
For the above system, the root locus is shown for 4 different controller/plant combinations, $D_1(s)G_1(s), ..., D_4(s)G_4(s)$. (Note: the root locus shows open-loop pole locations for $D(s)G(s)$, and closed-loop poles for $\frac{DG}{1+DG}$).



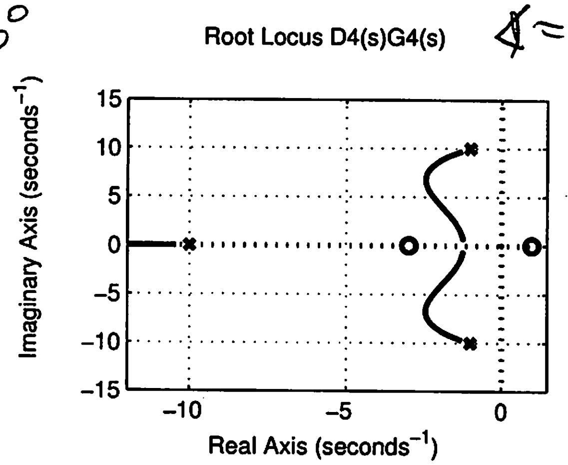
$\measuredangle = -180^\circ$



$\measuredangle = -180^\circ$



$\measuredangle = -270^\circ$



$\measuredangle = -90^\circ$

[4 pts] a) For each set of open-loop poles and zeros given above, choose the best corresponding open-loop Bode plot W,X,Y, or Z from the next page:

(i) $D_1(s)G_1(s)$: Bode Plot Y

(ii) $D_2(s)G_2(s)$: Bode plot X

(iii) $D_3(s)G_3(s)$: Bode plot Z since $\measuredangle D_3G_3 \to -270^\circ$

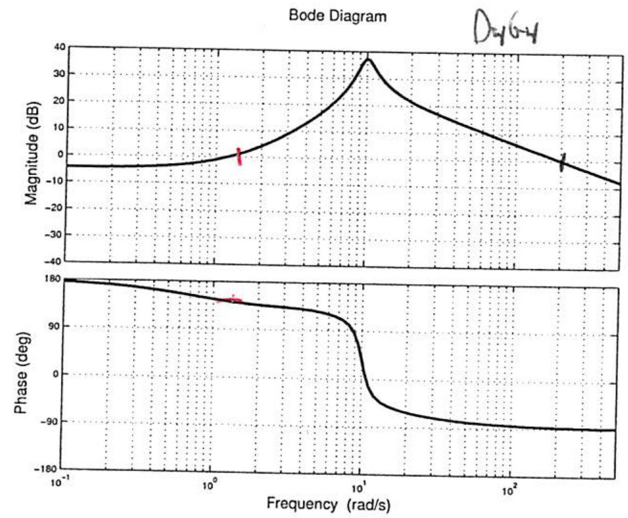
(iv) $D_4(s)G_4(s)$: Bode Plot W since $\measuredangle D_4G_4 \to -90^\circ$

$\measuredangle D_1G_1$: -180°/decade for $\omega > 0.3$, $\omega_d = 10$

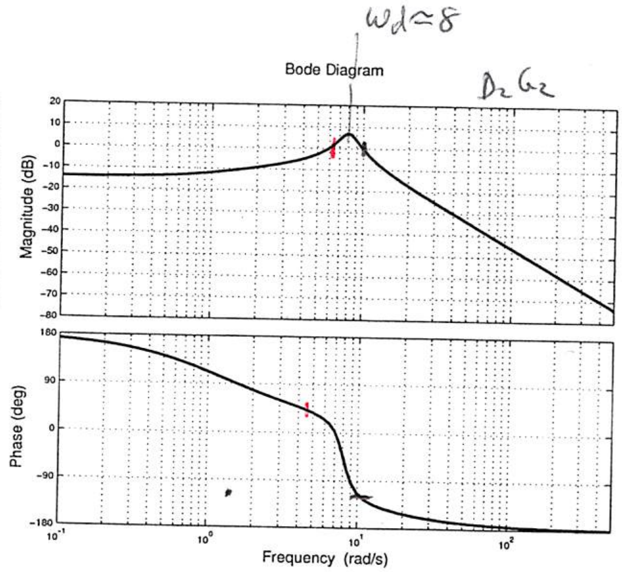
$\measuredangle D_2G_2$: -135°/decade for $\omega > 0.5$ $\omega_d = 8$

Problem 1, cont. KEY

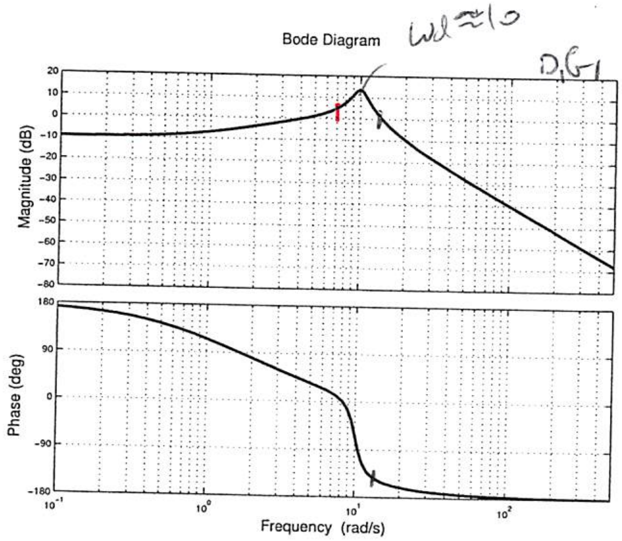
The open-loop Bode plots for 4 different controller/plant combinations, $D_1(s)G_1(s), ..., D_4(s)G_4(s)$ are shown below.



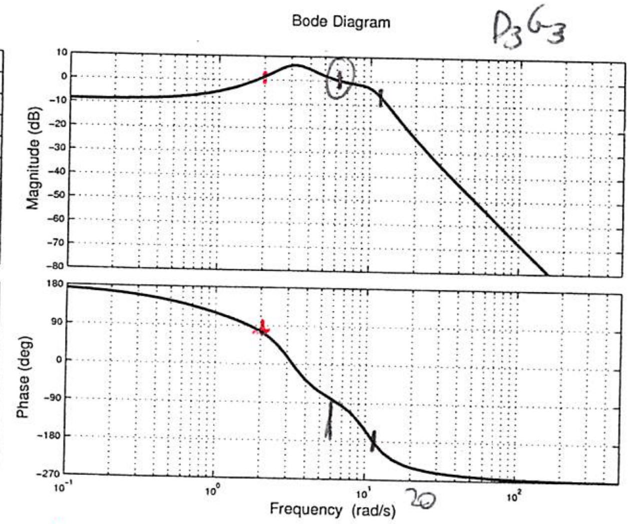
W



X



Y



Z

+45 at $\omega = 1.3$

[8 pts] b) For each Bode plot, estimate the phase and gain margin:

(i) Bode plot W: phase margin ~~70~~ 90° (degrees) at $\omega =$ ~~200~~
Bode plot W: gain margin 5 dB at $\omega =$ 0

(ii) Bode plot X: phase margin 60 (degrees) at $\omega =$ 10
Bode plot X: gain margin 15 dB at $\omega =$ 0

(iii) Bode plot Y: phase margin 30 (degrees) at $\omega =$ ~~20~~ 12
Bode plot Y: gain margin 10 dB at $\omega =$ 0
or 110

(iv) Bode plot Z: phase margin 10 90 (degrees) at $\omega =$ 6 =2
Bode plot Z: gain margin ~~10~~ dB at $\omega =$ ~~0~~
5dB 11

Problem 1, cont.

s

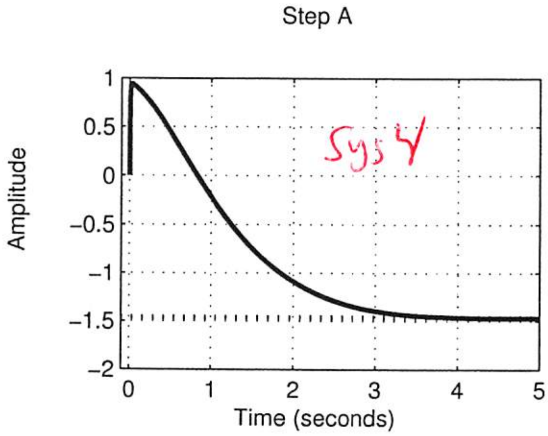
c) For each closed loop controller/plant with root locus as given in part a), choose the best corresponding closed-loop step response (A-D)

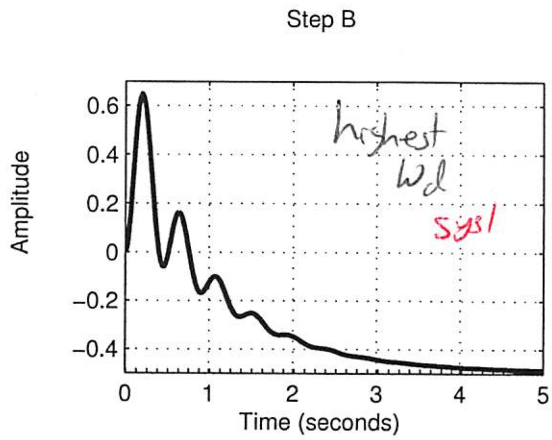
(i) $D_1(s)G_1(s)$: step response B $w_d = 14$ $(s+2 \pm 14j)$ all have other CLP at $s=-4$

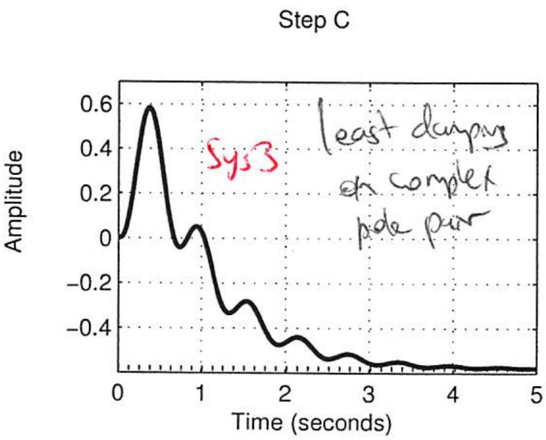
(ii) $D_2(s)G_2(s)$: step response D $w_d = 10$ $(s+2 \pm 10j)$ most damping.

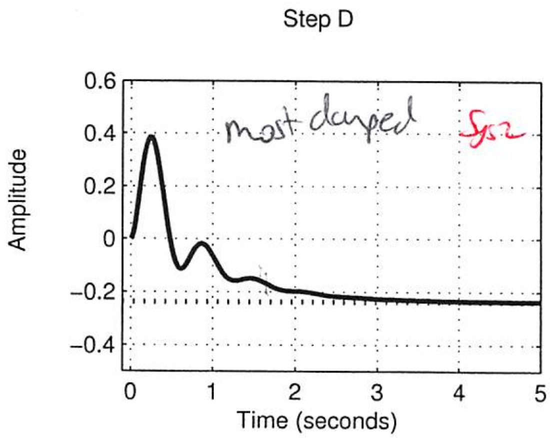
(iii) $D_3(s)G_3(s)$: step response C $w_d = 11$, $s \mp 1 \pm 11j$ least damping.

(iv) $D_4(s)G_4(s)$: step response A



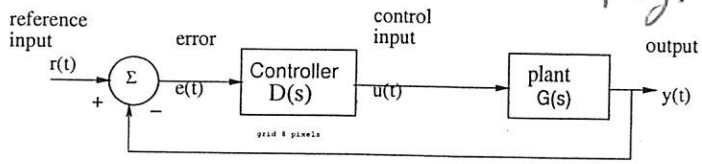






Problem 2 (12 pts)

Key.

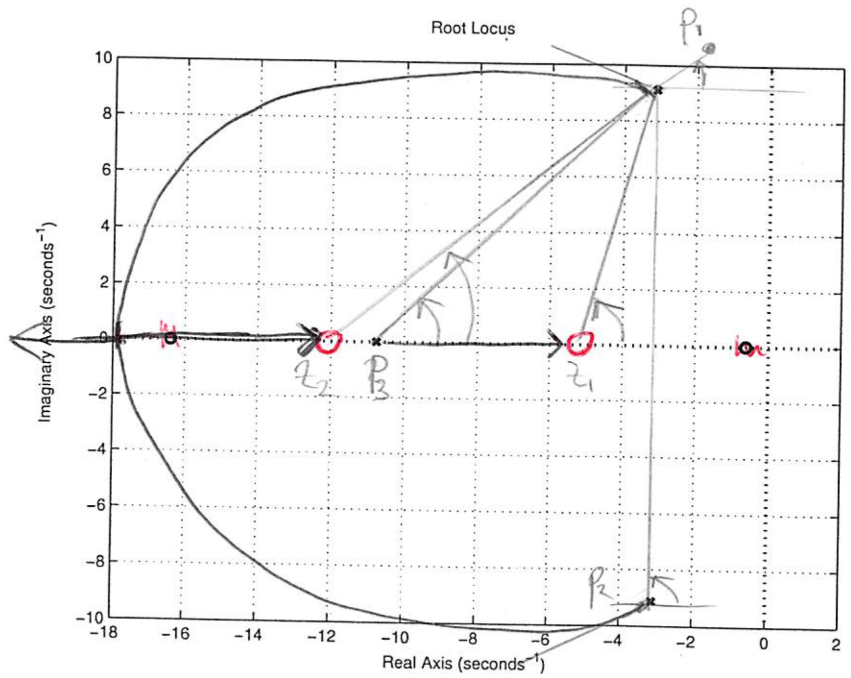


You are given the open loop plant $G(s) = \frac{100}{s^2+17s+60}$. The system is to be controlled using a lag controller. with $D(s) = \frac{s+10}{s+\alpha}$.

Given: the roots of $s^3 + 17s^2 + 160s + 1000 \approx (s + 10.7)(s + 3.11 + 9.1j)(s + 3.11 - 9.1j)$

[8 pts] a) Sketch the positive root locus as α varies, noting asymptote intersection point and angle of departure.

[4 pts] b)
(i) approximate asymptote intersection point $s =$ 0
(ii) approximate angle of departure for the poles: $0°$ $164°$ $-164°$



Angle of departure

$-\angle P_1 - 90° - \angle P_3 + \angle Z_1 + \angle Z_2 = -180°$

$-\angle P_3 + \angle Z_2 \approx 0$ (less than 10°) but > 0.

$\angle Z_1$: $\approx 14°$, $\approx 76°$, 9, 2, $\approx$ 4, 1

$-\angle P_1 - 90° + 76° = -180°$

$-\angle P_1 = -166°$

$1 + D(s)G(s) = 1 + \frac{100}{s^2+17s+60} \cdot \frac{s+10}{s+\alpha} = 0$

$D(s)G(s) \approx \frac{1}{(s+\alpha)}$,

$(s+\alpha)(s^2+17s+60) + 100s + 1000 = 0$

$\alpha = \frac{a_1 - b_1}{n-m} = \frac{0}{1}$

$s^3 + 17s^2 + 60s$
$+ \alpha s^2 + 17\alpha s + 60\alpha$ $+ s^3 + 17s^2 + 160s + 1000 = 0$

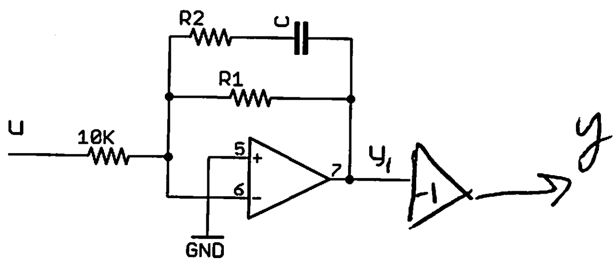
$\alpha(s^2 + 17s + 60) + s^3 + 17s^2 + 160s + 1000$

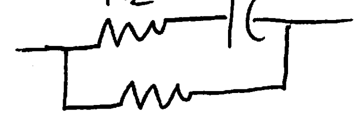
+2 if this equation and bad root locus.

$1 + \alpha \cdot \frac{s^2+17s+60}{s^3+17s^2+160s+1000} \Longrightarrow$ "zeros" at $-12, -5$

KEY

Problem 3 (8 pts)
Consider the following circuit for a *lag* compensator:





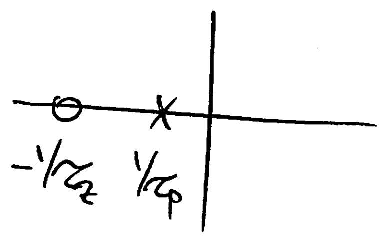
$R_1 \parallel \left(R_2 + \frac{1}{sC}\right) = R_1 \parallel \frac{1+sR_2C}{sC}$

$Z = \frac{R_1R_2}{R_1+R_2} \cdot \frac{s+\frac{1}{R_2C}}{s+\frac{1}{C(R_1+R_2)}}$

[4 pts] a) Find the transfer function $\frac{Y(s)}{U(s)}$

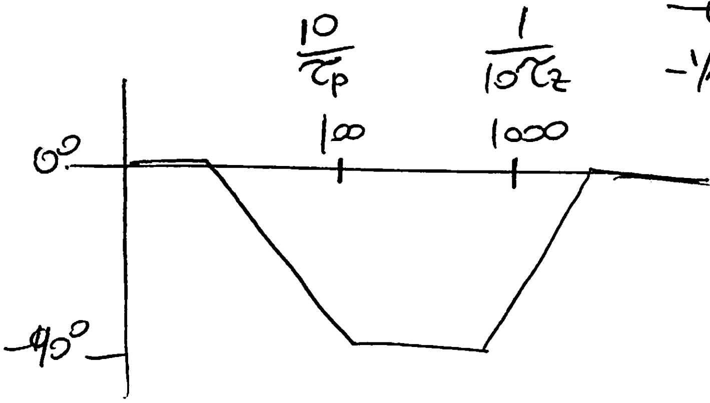
$$\frac{Y}{U} = \frac{Z}{10K} = \frac{R_1R_2}{10K(R_1+R_2)} \cdot \frac{s+\frac{1}{R_2C}}{s+\frac{1}{C(R_1+R_2)}}$$

[4 pts]b) Suppose the desired behaviour of this circuit is that the (asymptotic) phase response is $-90°$ between $100 rad/s$ and $1000 rad/s$. At every other frequency the phase response should be greater than $-90°$. If $C = 1\mu F$, what are the resistor values, R_1 and R_2?



$R_2C = \tau_{zero}$

$(R_1+R_2)C = \tau_{pole}$



$10^{-4} = R_2 10^{-6} F$

$R_2 = 100\Omega$

$\frac{10}{\tau_p} = 100, \quad \tau_p = 0.1$

$(R_1+R_2)10^{-6} = 0.1$

$= 10^5$

$\frac{1}{10\tau_z} = 1000, \quad \tau_z = 10^{-4}$

$R_2 = 99,900$

Problem 4 (16 pts)

You are given the following plant

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\mathbf{u}(t), \quad y = [4\ 1]\ \mathbf{x} \qquad \mathbf{x}(t=0) = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

[2 pts] a) Determine if the system is controllable and observable.

$\mathcal{C} = [B \ AB] = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$, rank 2 $\Rightarrow$ Controllable

$\mathcal{O} = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 0 & 0 \end{bmatrix}$, rank 1 $\Rightarrow$ not observable.

[4 pts] b) Find feedback gains $K = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix}$ such that with control $\mathbf{u} = K(\mathbf{r} - \mathbf{x})$, the controller has closed loop poles at -2 and -4.

$k_1 = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$ or $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$

$k_2 =$ ______

$\dot{x} = Ax + BK(r - x)$

$= (A - BK)x + BKr$

$= \begin{bmatrix} -2k_1 & 0 \\ 0 & -k_2 \end{bmatrix}$

$|sI - A| = \begin{vmatrix} s+2k_1 & 0 \\ 0 & s+k_2 \end{vmatrix}$

$= s^2 + (2k_1 + k_2)s + 2k_1k_2$

by coeff matching

$(s+2)(s+4) = s^2 + 6s + 8 = 0$

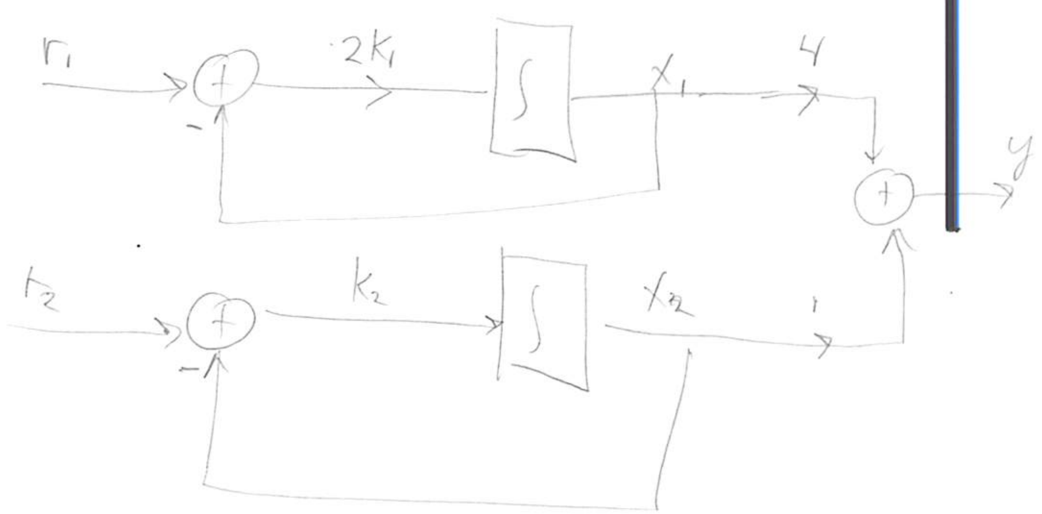
$s^2 + (2k_1 + k_2)s + 2k_1k_2 = 0$

$2k_1 + k_2 = 6, \quad 2k_1k_2 = 8,$

[2 pts] c) Draw a block diagram of the controlled system using integrators, summing junctions, and scaling functions. (Every signal should be a scalar, no vectors.)

$\dot{x}_1 = -2k_1x_1 + 2k_1r_1$

$\dot{x}_2 = -k_2x_2 + k_2r_2$



Problem 4, cont Key.

You are given the following plant

$$\dot{\mathbf{x}} = A_1\mathbf{x} + B_1\mathbf{u} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\mathbf{u}(t), \quad y = [4\ 1]\,\mathbf{x} \qquad \mathbf{x}(t=0) = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

[2 pts] d) Determine if the system $\{A_1, B_1, C_1\}$ is controllable and observable.

$\mathcal{C} = [B\ AB] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, rk = 2 $\Rightarrow$ Controllable.

$\mathcal{O} = \begin{bmatrix} C \\ \cancel{A}CA \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 0 & 4 \end{bmatrix}$, rk = 2 $\Rightarrow$ observable.

[4 pts] e) Find feedback gains $K = [k_1 k_2]$ such that with control $u = K(\mathbf{r} - \mathbf{x})$, the controller has closed loop poles at -2 and -4.

$k_1 =$ 8

$k_2 =$ 6

$\dot{x} = (A - BK)x + BKr$, $BK = \begin{bmatrix} 0 & 0 \\ k_1 & k_2 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1 \\ -k_1 & -k_2 \end{bmatrix} x + BKr$

$|sI - A| = \begin{vmatrix} s & -1 \\ k_1 & s+k_2 \end{vmatrix} = s^2 + k_2 s + k_1$

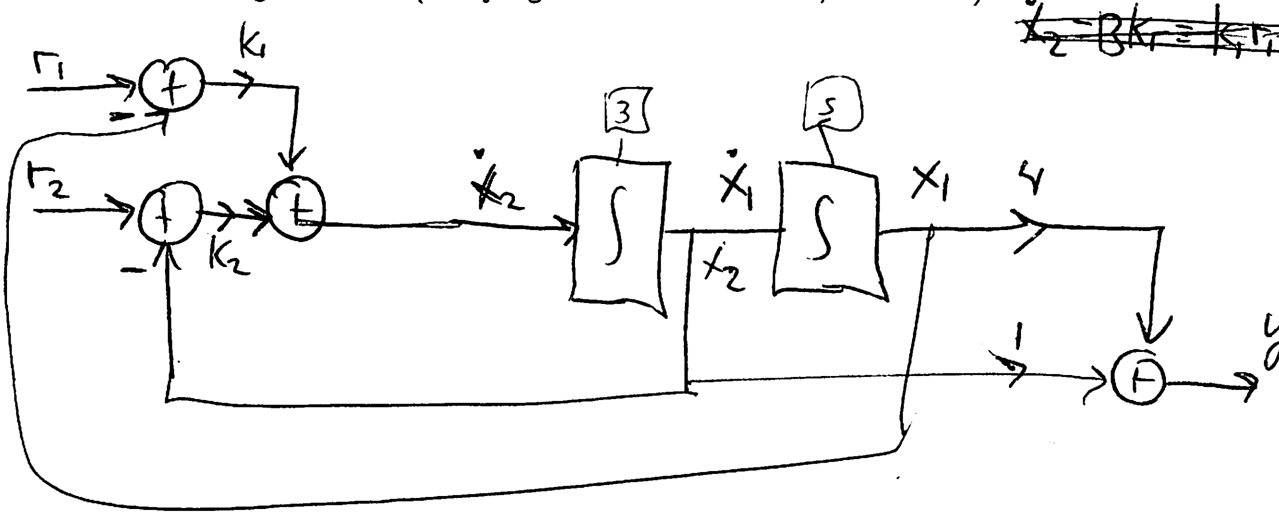
match coeff:

$s^2 + 6s + 8 = 0$

$\Rightarrow k_2 = 6,\ k_1 = 8$

[2 pts] f) Draw a block diagram of the controlled system using integrators, summing junctions, and scaling functions. (Every signal should be a scalar, no vectors.)

~~$\dot{x}_2 = BKr = k_1 r_1 + k_2 r_2$~~



Problem 5 (11 pts)

Key.

[3 pts] a) Given the following system:

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu \qquad y = C\mathbf{x}$$

The state is transformed by a non-singular P such that $\bar{\mathbf{x}} = P\mathbf{x}$. Thus $\dot{\bar{\mathbf{x}}} = \bar{A}\bar{\mathbf{x}} + \bar{B}u$ and $y = \bar{C}\bar{\mathbf{x}}$.

Find $\bar{A}$ $\bar{B}$ $\bar{C}$ in terms of A, B, C, P:

$\bar{A} =: PAP^{-1}$

$\bar{B} =: PB$

$\bar{C} =: CP^{-1}$

$x = P^{-1}\bar{x} = P^{-1}Px$

$\dot{x} = AP^{-1}\bar{x} + Bu$

$P\dot{x} = \dot{\bar{x}} = \underbrace{PAP^{-1}}_{\bar{A}}\bar{x} + \underbrace{PB}_{\bar{B}}u$

$y = \underbrace{CP^{-1}}_{\bar{C}}\bar{x}$

[4 pts] b) You are given the following system:

$$\dot{x} = \begin{bmatrix} -2 & -1 \\ -9 & 6 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t), \quad y = [3 \;\; -1]\,\mathbf{x}$$

Find the transformation P and $\bar{A}$ such that $\bar{A} = P^{-1}AP$ is in **modal canonical** (diagonal) form.

$$P = \begin{bmatrix} 1 & 1 \\ -9 & 1 \end{bmatrix} \qquad \bar{A} = \begin{bmatrix} 7 & 0 \\ 0 & -3 \end{bmatrix}$$

$P = [e_1 \mid e_2]$ eigenvectors $\bar{x} = P^{-1}x$, $x = P\bar{x}$

$Ae_i = \lambda e_i$; $Ae_1 = \begin{bmatrix} 7 \\ -63 \end{bmatrix}$

$|\lambda I - A| = \begin{vmatrix} \lambda+2 & +1 \\ +9 & \lambda-6 \end{vmatrix} = \lambda^2 - 4\lambda - \cancel{12} - 9$

$= \lambda^2 - 4\lambda - 21$

$\cancel{(\lambda+7)(\lambda-3)}$

$(\lambda-7)(\lambda+3) = 0$

$\lambda_i = 7, -3$

$$\begin{bmatrix} -2 & -1 \\ -9 & 6 \end{bmatrix}\begin{bmatrix} e_{11} \\ e_{12} \end{bmatrix} = \begin{bmatrix} 7e_{11} \\ 7e_{12} \end{bmatrix}, \quad e_1 = \begin{bmatrix} 1 \\ -9 \end{bmatrix}, \quad e_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$-2e_{11} - e_{12} = 7e_{11}$ $\quad -e_{12} = 9e_{11}$

$-9e_{11} + 6e_{12} = 7e_{12}$, $\quad -9e_{11} = e_{12}$

$-2e_{21} - e_{22} = -3e_{21}$

$-9e_{21} + 6e_{22} = -3e_{22}$

$-e_{22} = -e_{21}$

[4 pts] d) Find $e^{\bar{A}t}$ and e^{At}:

$$e^{\bar{A}t} = \begin{bmatrix} e^{7t} & 0 \\ 0 & e^{-3t} \end{bmatrix} \qquad e^{At} = \begin{bmatrix} e^{7t} + 9e^{-3t} & -e^{7t} + e^{-3t} \\ -9e^{7t} + 9e^{-3t} & 9e^{7t} + e^{-3t} \end{bmatrix} \cdot \frac{1}{10}$$

$$P^{-1} = \begin{bmatrix} 1 & 1 \\ -9 & 1 \end{bmatrix}^{-1} = \frac{\begin{bmatrix} 1 & -1 \\ 9 & 1 \end{bmatrix}}{10}$$

$$P^{-1}AP = \frac{\begin{bmatrix} 1 & -1 \\ 9 & 1 \end{bmatrix}}{10} \cdot \begin{bmatrix} 7 & -3 \\ -63 & -3 \end{bmatrix} = \frac{\begin{bmatrix} 70 & 0 \\ 0 & -30 \end{bmatrix}}{10}$$

$$e^{At} = \begin{bmatrix} e^{7t} & e^{-3t} \\ -9e^{7t} & e^{-3t} \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 9 & 1 \end{bmatrix}\frac{1}{10} = \frac{1}{10}\Big[\quad \Big]$$

$\dot{\bar{x}} = e^{\bar{A}t}\bar{x}_0$, $\cancel{P\dot{x} = Pe^{\bar{A}t}x_0}$

$\dot{x} = e^{At}x_0$

$P^{-1}\dot{x} = e^{\bar{A}t}P^{-1}x_0$

$\dot{x} = \underbrace{Pe^{\bar{A}t}P^{-1}}_{e^{At}}x_0$

Problem 6 (16 pts)

The simplified dynamics of a magnetically suspended steel ball are given by:

$$m\ddot{y} = mg - c\frac{u^2}{y^2}$$

$\frac{\text{amps}^2}{m^2}$

y is the position of the ball; u is the current through the coil (in amps); c is a constant that describes the magnetic force between the coil and the ball. The system is linearized at equilibrium position y_0 with equilibrium input u_e:

$$y = y_0 + \delta y \qquad u = u_e + \delta u$$

$$u_e = y_0\sqrt{\frac{mg}{c}}$$

The linearized state space equations are:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ \frac{2g}{y_0} & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ \frac{-2}{y_0}\sqrt{\frac{cg}{m}}\end{bmatrix}\delta u$$

$$= \begin{bmatrix}0 & 1\\ 200 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ -20\end{bmatrix}\delta u$$

$$\delta y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

$\ddot{y} = g - m^{-1}c\frac{u^2}{y^2}$

$u = k_1 y + k_2\dot{y}$

$k_1^2 y^2 + k_2^2\dot{y}^2$

$\cancel{+} 2k_1 k_2 y_1 \dot{y}$

[1 pts] (a) What are the units of c? Assume that all other quantities are SI standard (kilograms, meters, amps, etc).

newtons $\cdot \frac{m^2}{\text{amp}^2}$

[4 pts] (b) We want to build a regulator to keep the ball at y_0. We will design a state feedback scheme, $\delta u = -Kx$, so that the poles of the linearized system are at $s = -20, -12$. Find K.

$K = [\qquad\qquad]$

$A - BK = \begin{bmatrix}0 & 1\\ 200 & 0\end{bmatrix} - \begin{bmatrix}0 & 0\\ -20k_1 & -20k_2\end{bmatrix}$

$k_1 = -22$

$k_2 = -1.6$

[3 pts] (c) Assume that you can directly access x_1 and x_2. You build your regulator as described above, and it successfully levitates the ball. You decide to try levitating four steel balls at the same time. Now m is four times bigger; everything else stays the same. Is your linearized control system still stable? Will the steel balls be stable at y_0? Why or why not?

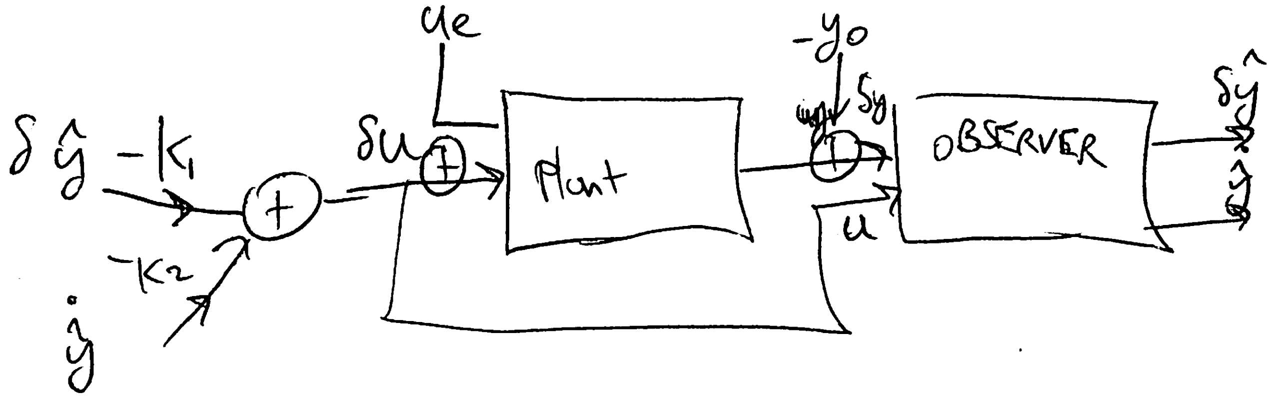
~~$s^2 + 16s + 20$ Still stable~~

u_e did not change, linearization not valid

y_0 will change

~~new $B = \begin{bmatrix}0\\ -10\end{bmatrix}$~~

[4 pts] (d) Return to the one-ball problem. Assuming that the only accessible output of the plant is y, you will need an observer in order to implement state feedback. Draw a block diagram of your regulator system. Use one block labelled "Plant", with input u and output y; one block labelled "Observer", with output $\hat{x}_1$ and $\hat{x}_2$ (you decide what the input should be); static gains; and addition junctions. Every signal should be scalar (no vectors). Label as many signals as you can. (Note that you're not being asked to design the observer gain).



[2 pts] (e) What are some sensible values for the poles of the observer?

-200, ~~-100~~ -200

[2 pts] (f) Does using an observer introduce any new problems if you try to levitate four balls, as in (c)?

GIGO

need accurate A, B, C, D.

Problem 7. (13 pts) Key.

You are given a continuous time plant described by the following state equation.

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu$$ $u[0] =$

The system is driven with a D/A converter such that $u(t) = u[n]$ for $nT < t < nT + T$. (That is, the input is held constant, by a zero-order hold equivalent.) Every T seconds the state of the system is measured with an A/D converter, that is $\mathbf{x}[n] = \mathbf{x}(nT)$.

Recall that the solution for the continuous time system is given by:

$$\mathbf{x}(t) = e^{A(t-t_o)}\mathbf{x}(t_o) + \int_{t_o}^{t} e^{A(t-\tau)}Bu(\tau)d\tau. \qquad (1)$$

[3 pts] a) For the zero input response, $(\mathbf{x}(t=0) = \mathbf{x}_o, u(t) = 0)$
Find:(in terms of A and $\mathbf{x_o}$)

$\mathbf{x}[0] =$: x_o
$\mathbf{x}[1] =$: $e^{AT} x_o$
$\mathbf{x}[n] =$: $e^{AnT} x_o$

$e^{A(T-0)} = G(T)$

$x[2] = Gx[1] + Hu[1]$
$= GHu[0] + Hu[1]$
$x[3] = Gx[2] + Hu[2]$
$= G^2Hu[0] + GHu[1] + Hu[2]$

[3 pts] b) For the zero state response, $(\mathbf{x}(t=0) = \mathbf{0}$)
Find:(in terms of A, B, u)

$\mathbf{x}[0] =$: 0
$\mathbf{x}[1] =$: $H(T)Bu(0)$
$\mathbf{x}[n] =$: $= \sum_{j=0}^{n-1} G^{n-j-1} Hu[j]$

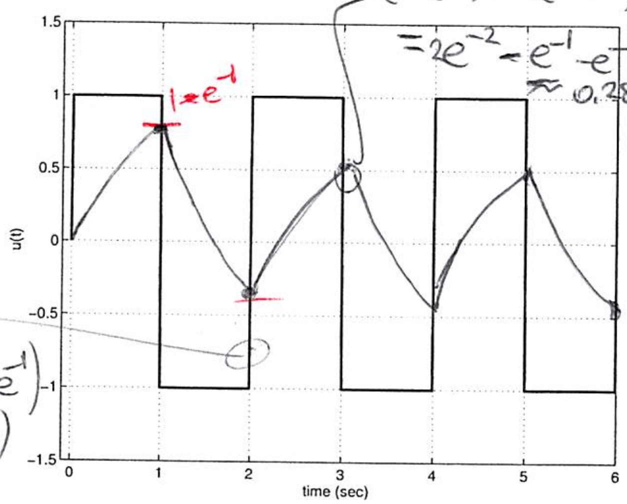
$x(t=T) = \int_0^T e^{A(T-\tau)} Bu(\tau)d\tau = \underbrace{e^{AT}\int_0^T e^{-A\tau} d\tau B}_{=H(T)} u(0)$

[3 pts] c) Consider the CT system $\dot{x} = -x + u$. With $u(t)$ as shown, sketch $x(t)$ for $0 < t < 6$sec with initial condition $x(0) = 0$.

$(1-e^{-3}) - 2(1-e^{-2}) + 2(1-e^{-1})$
$= 2e^{-2} - e^{-1} - e^{-3} + 1$
$\approx 0.28 - 0.37$
$- 0.05$

$s X(s) = -X(s) + U(s)$

$\frac{X(s)}{U(s)} = \frac{1}{s+1}$



1.26
$-.86$
$.4$

$(1-e^{-2}) - 2(1-e^{-1})$
$0.86 - 2(.63)$
$\approx$

x step response $u(t) = $ step.

$X(s) = \frac{1}{s\cdot(s+1)} = \frac{1}{s} + \frac{-1}{s+1}$

$y(t) = (1-e^{-t})u(t)$

$y(t) - 2y(t-1)$
$+ 2y(t-2)$
$- 2y(t-3) + \cdots$

$\tau = 1 sec$

s

d) Let $T = 1sec$. For the CT system $\dot{x} = -x + u$, $x_o = 0$, with zero-order hold on input, determine the value of x at following steps (Answers may be left in terms of e.) Consider $u[0] = 1, u[1] = -1$ etc.

$\mathbf{x}[0] =$: 0
$\mathbf{x}[1] =$: $1-e^{-1}$
$\mathbf{x}[2] =$: $1-e^{-2} - 2(1-e^{-1})$
$\mathbf{x}[3] =$: $1-e^{-3} - 2(1-e^{-2}) + 2(1-e^{-1})$

Problem 8 (8 pts)

You are given the following plant

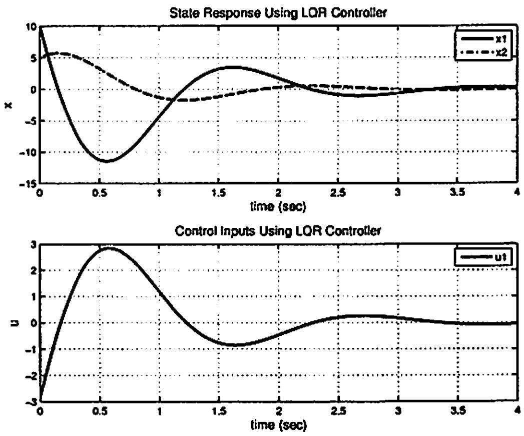
$$\dot{\mathbf{x}} = A\mathbf{x} + Bu = \begin{bmatrix} -2 & -10 \\ 1 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}u(t), \quad y = [1\ 4]\ \mathbf{x} \qquad \text{and } \mathbf{x}(t=0) = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

The LQR method is used to find the linear control $u = -Kx$ which minimizes the cost $J = \int_0^\infty (x^T Q x + u^T R u)dt$, where Q and R are positive semi-definite. Four responses of the closed-loop system A, B, C, D are shown below for different choices of Q, R. Match the plots with Q R weights below.

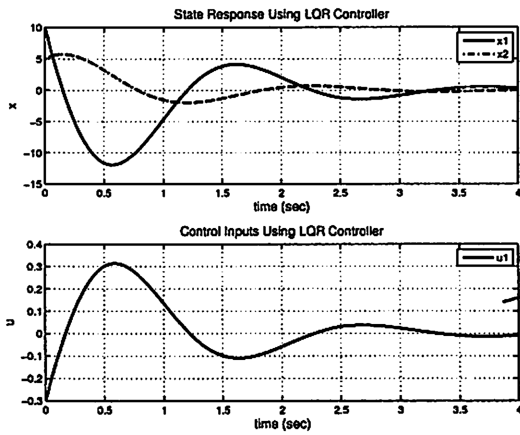
max $|x_1| = 12$

$q_1 = 1$

max $u = 3$



A



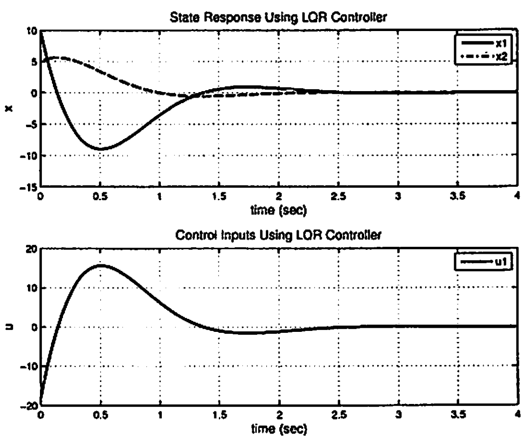
— min $u^T R u$

B

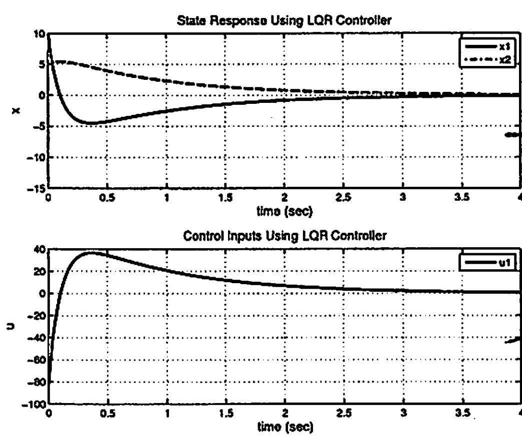
max $|x_1| = 9$

$q_1 = 10$

max $u = 10$



C



— min $x^T Q x$

— biggest $u^T R u$

D

[4pts] 8 For each plot of $\mathbf{x}(t)$ and $u(t)$ choose the appropriate Q, R pair, writing in the appropriate letter.

$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $R = [1]$ Plot: A	$Q = \begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix}$ $R = [1]$ Plot: C
$Q = \begin{bmatrix} 100 & 0 \\ 0 & 1 \end{bmatrix}$ $R = [1]$ Plot: D	$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $R = [10]$ Plot: B

$x^T Q x$

$= [x_1\ x_2] \begin{bmatrix} q_1 x_1 \\ q_2 x_2 \end{bmatrix}$

$= q_1 x_1^2 + q_2 x_2^2$

min $x^T Q x$