Lab 5b: Luenberger Observer Design for Inverted Pendulum

I. Purpose

The objective of this lab is to design a full-state observer to estimate the state of an inverted pendulum system given just the position of the cart and the pendulum. We will utilize this estimate for full-state feedback control of the system.

II. Theory

Pole placement design is performed under the assumption that all states of the system are measurable. However, in physical systems not all states may be measurable and thus states need to be estimated based on the limited sensing available.

The state feedback now becomes $u = -K\hat{x}$, where $\hat{x}$ is the estimated state. We can't use $u = -Kx$ because the only measurements we have are y. The dynamics of the observer are:

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y}) \quad (1)$$

where $y = Cx$ and $\hat{y} = C\hat{x}$.

The first two terms of the above equation ($A\hat{x} + Bu$) can be called the predictor part and is a replica of the plant dynamics. However, because of uncertainties or errors in the plant model, estimate of the state using only the predictor will never match the actual state of the system. A corrective term $L(y - \hat{y})$ is thus needed and together these form the Luenberger observer. The corrector term corrects future estimates of the state based on the present error in estimation.

The gain L can be considered a parameter which weighs the relative importance between the predictor and the corrector in state estimation. Intuitively, it can be seen that a "low" value for L is chosen when our confidence in the model (i.e. the predictor) is high and/or confidence in measurement y is low (i.e. when the measurements are noisy) and vice-versa for a "high" L.

The objective of this lab is to design the observer gain matrix L and use the state estimator for feedback control of the inverted-pendulum system instead of our previous derivative-based approximation.

III. Pre-lab

(a) The model for inverted-pendulum system and the desired position of the closed-loop poles ($\mathbf{-1.7 \pm 9}\boldsymbol{j}$ and $\mathbf{-2 \pm 1.6}\boldsymbol{j}$) remain the same as in the previous lab.

The gain L is chosen such that the matrix $A - LC$ has eigenvalues in the left half-plane. Further, the exact eigenvalues of $A - LC$ govern the rate at which the state estimate ($\hat{x}$) converges to the actual state (x) of the system. It is normally desired that the observer estimate of the state converges to the actual state at least an order of magnitude faster than the performance desired of the system. This helps the controller in obtaining a "good" estimate of the actual state of the system in relatively short time and thus it can take appropriate control action.

Given that the size of $A - LC$ must be the same as A, **what are the dimensions of $\boldsymbol{L}$?**

For this lab, we want to place the eigenvalues of the observer at $-\mathbf{10} \pm \mathbf{15}\boldsymbol{j}$ and $-\mathbf{12} \pm \mathbf{17}\boldsymbol{j}$. Note that they have been chosen to be relatively "away" from the desired closed-loop poles. Using MATLAB, find the matrix L such that this is achieved. **Why does `acker` no longer work?**

(b) Implement the designed observer in MATLAB. **There should be NO derivative blocks used**. Remember that the observer is placed in feedback around the actual system. Utilize the estimate ($\hat{x}$) of the state for state feedback. You may use the feedback gain matrix K designed in the previous lab since the desired locations of the closed-loop poles have not changed.

Simulate the system for a *unit position* offset($\boldsymbol{x}(0) = [1\ 0\ 0\ 0]^T$) of the plant and zero reference input. You can do this by giving initial conditions to the state vector of the plant model in Simulink. Plot the output response (y) of the plant against time (i.e. position of both the cart and pendulum) and see how well the system performs.

Plot the observer estimate ($\hat{x}$) of the state and the actual state of the plant (x), which may be obtained from the plant model in Simulink (again, no derivative blocks), on 4 separate plots, one for each state variable. However, note that this cannot be done with the physical plant as we have no measurement of the actual state (x).

Plot the error in estimation ($e = \hat{x} - x$) and observe how it varies with time.

IV. Lab

(a) Implement the state feedback controller along with the Luenberger observer on the hardware.

Observe and record the output of the observer ($\hat{y}$) and the actual measurement (y). That is, plot both the estimated and actual signals on the same graph for the position of the cart and the pendulum. The difference between these two signals indicates how well the observer estimates the state of the system.

(b) We will now compare the controllers from lab 5a and 5b. Remember that the closed loop poles are the same with the both these systems. Run both systems and give them similar perturbations in time (this gets pretty subjective, but do your best). *Qualitatively* describe any noticeable differences in performance. Plot the performance of the cart position and the angular position of the rod for both controllers on top of each other and compare their tracking abilities.

(c) Now we will look at the differences in performances a little more closely. Compare the estimates of the cart and pendulum velocities from this lab with the measurements obtained by taking the derivatives of the position and angle signals from the previous lab. How do these two schemes differ when a noise is present in the actual measurement of the positions?

(d) Which scheme do you think gives the “better” performance, and more importantly, WHY? There is no definite answer here. Just form your own opinion and defend it.

V. Revision History

Semester and Revision	Author(s)	Comments
Fall 2010 Rev 1.3	Wenjie Chen and Jansen Sheng	Adjusted some minor wording.
Winter 2009 Rev. 1.2	Justin Hsia	Wording clarification following the Fall 2009 run-through.
Winter 2008 Rev. 1.1	Justin Hsia	Corrections and additions following Fall 2008 run-through and feedback.
Fall 2008 Rev. 1.0	Pranav Shah	Initial lab write-up