Hints for Inverted Pendulum Dynamics

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- One easy way to analyze the motion of the pendulum is by describing the motion of the pendulum as observed from the frame of the cart.

- In order to describe the motion of the pendulum appropriately in the cart frame, we need to apply a "fictitious" force on the pendulum.

  This force is given as:

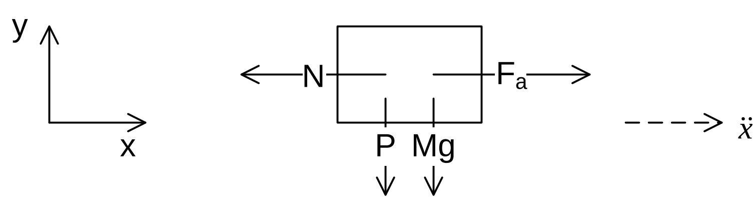
$$F_{\text{fictitious}} = \begin{pmatrix}\text{mass of the object}\\ \text{whose motion is}\\ \text{being described}\end{pmatrix} \times \begin{pmatrix}\text{acceleration}\\ \text{of the frame}\end{pmatrix}$$

  Thus, $F_{\text{fictitious}} = m\ddot{x},$ where $m =$ mass of pendulum and $x =$ accel. of cart

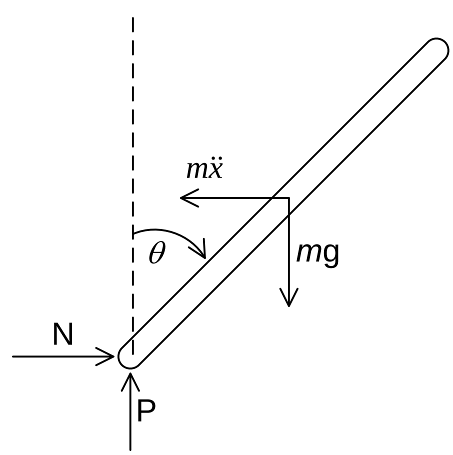
- The direction of the fictitious force is opposite to the direction of acceleration of the frame. Thus, if the cart is accelerating to the right, the fictitious force on the pendulum acts to the left.

- Now we have the following free body diagrams of the cart and the pendulum:

  - Cart: Motion of the cart is described from the ground reference frame.



  - Pendulum: Motion of pendulum described in the frame of the cart.



  The motion of the pendulum in this frame is that of a hinged rod falling under gravity.

- Now, apply Newton's laws and get the required equations.