EE123 Digital Signal Processing

Lecture 29

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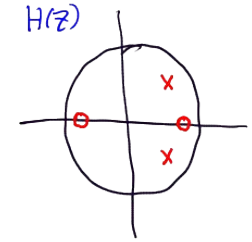
Minimum-Phase Systems

(7)

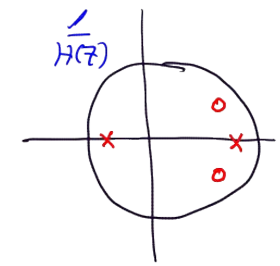
Definition: a stable and causal system $H(z)$
poles insid unit circle

who's inverse $\frac{1}{H(z)}$ is also stable & causal
zeros are insid unit circle.

$H(z)$



$\frac{1}{H(z)}$



AP-Min-Phase decomposition:

(8)

stable, causal system can be decomposed to:

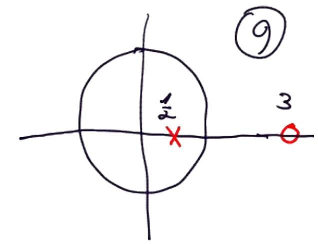
$$H(z) = \underbrace{H_{min}(z)}_{\text{min phase}} \cdot \underbrace{H_{ap}(z)}_{\text{all pass}}$$

Approach: ① first construct H_{ap} with all zeros outside unit circle
② compute $H_{min}(z) = \frac{H(z)}{H_{ap}(z)}$

Example

(9)

$$H(z) = \frac{1-3z^{-1}}{1-\frac{1}{2}z^{-1}}$$



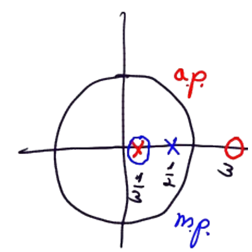
set:

$$H_{ap} = \frac{z^{-1}-\frac{1}{3}}{1-\frac{1}{3}z^{-1}}$$

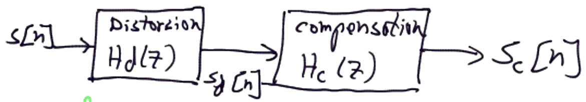
$$H_{min}(z) = \frac{1-3z^{-1}}{1-\frac{1}{2}z^{-1}} \cdot \frac{1-\frac{1}{3}z^{-1}}{z^{-1}-\frac{1}{3}} =$$

-3

$$= -3\,\frac{1-\frac{1}{3}z^{-1}}{1-\frac{1}{2}z^{-1}}$$



why m.p. property important? (10)



for ex. communication chan.

If $H_d(z)$ is minimum phase, design $H_c(z) = \frac{1}{H_d(z)}$ (stable!)

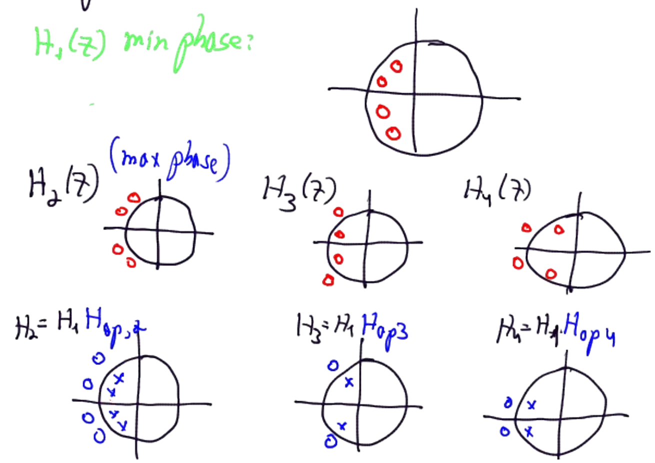
If not m.p., decompose: $H_d(z) = H_{d,mp}(z) \cdot H_{d,ap}(z)$

$$H_c(z) = \frac{1}{H_{d,min}(z)} \Rightarrow H_d H_c = H_{d,ap}(z)$$

only compensate for mag.

Why called "minimum phase"? (11)

Different systems can have same mag. response.



of all, $H_1(z)$ has minimum phase lag (12)

because:

$$\arg[H_i(e^{j\omega})] = \arg[H_1(e^{j\omega})] + \underbrace{\arg[H_{ap,i}]}_{\leq 0}$$

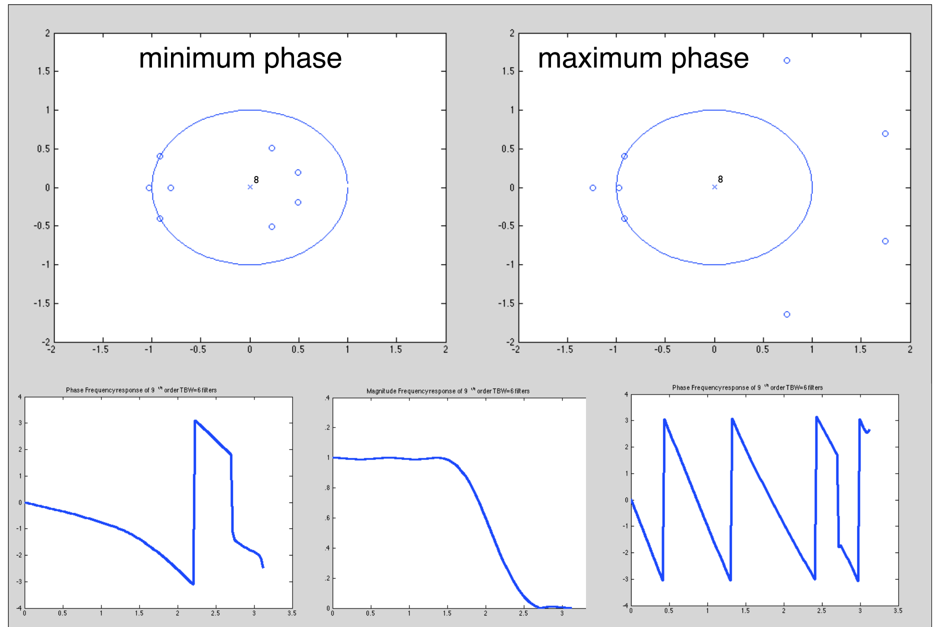
other properties:

minimum group delay:

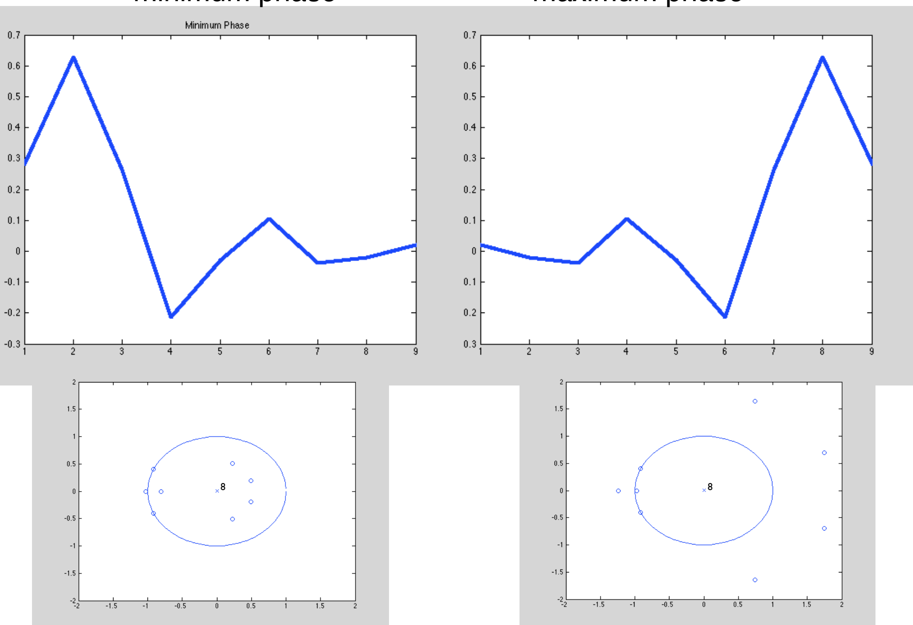
$$grd[H(e^{j\omega})] = grd[H_{min}] + \underbrace{grd[H_{ap}]}_{\geq 0}$$

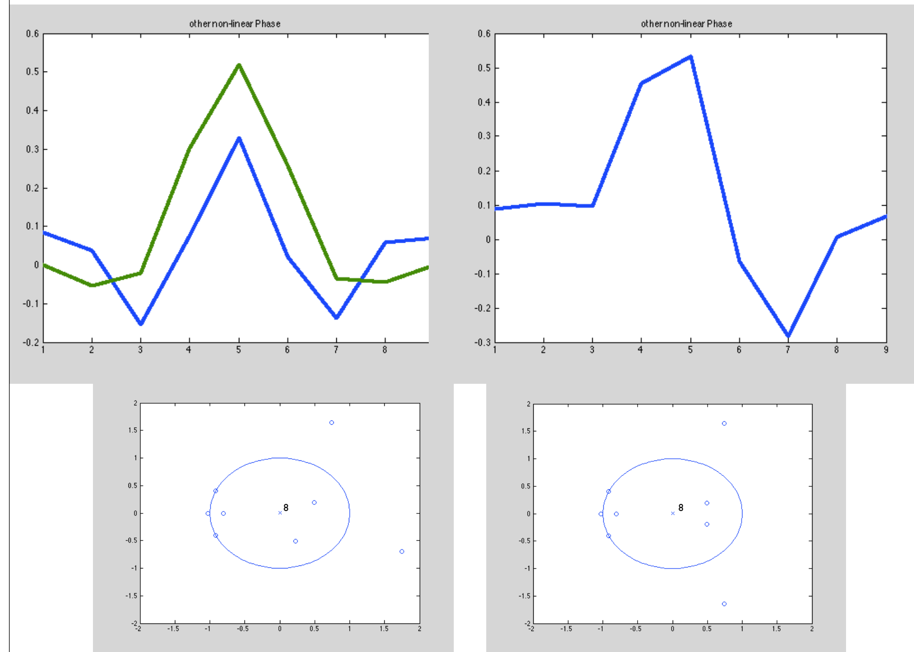
minimum energy delay:

Problem 5.72



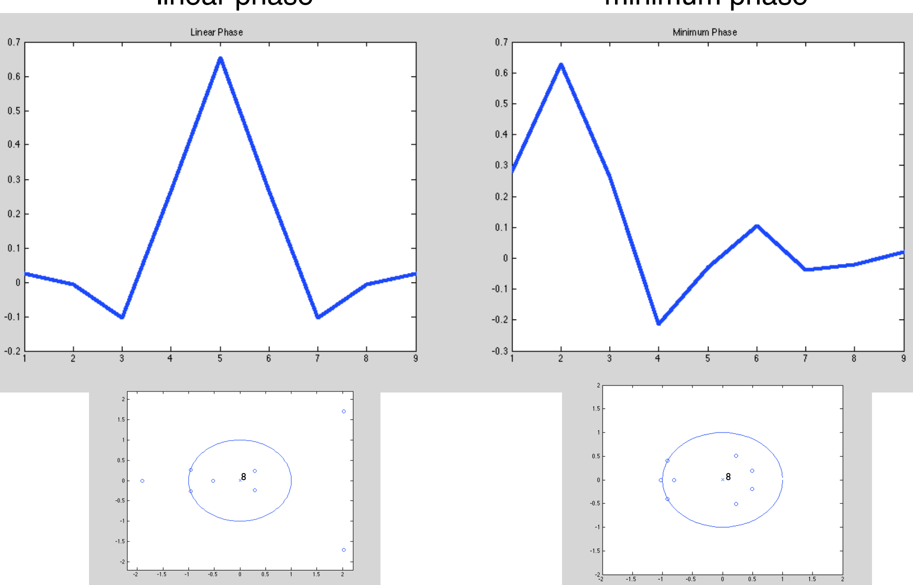
minimum phase   maximum phase

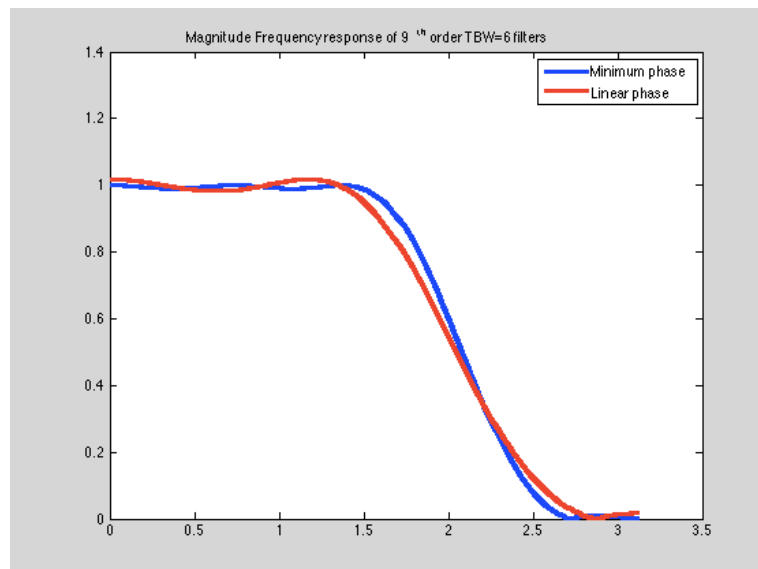




Minimum-phase VS Linear Phase

linear phase   minimum phase



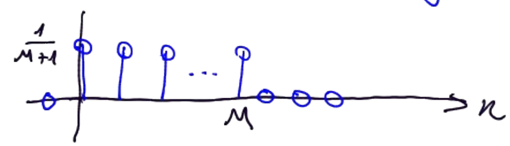


Generalized linear-phase systems

$$H(e^{j\omega}) = \underbrace{A(e^{j\omega})}_{\text{Real, allow sign change}} e^{-j\alpha\omega + j\beta}$$

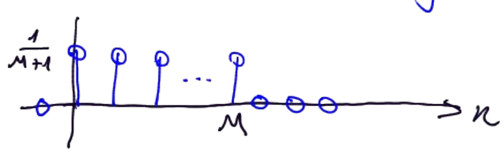
$$\text{grd}\left[H(e^{j\omega})\right] = \alpha \quad \left(\text{except when } A(e^{j\omega}) \text{ changes sign}\right)$$

Example (M+1)-point moving average ②



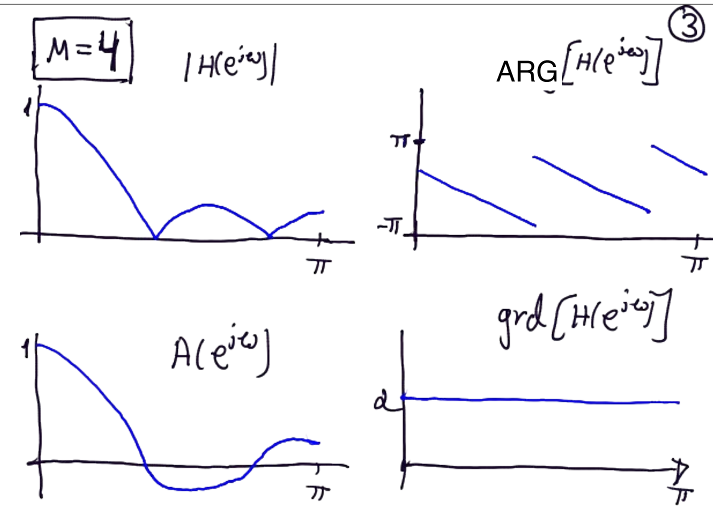
$$H(e^{j\omega}) = \underbrace{\boxed{}}_{A(e^{j\omega})} \boxed{}$$

Example (M+1)-point moving average ②



$$H(e^{j\omega}) = \underbrace{\boxed{\frac{1}{M+1}\frac{\sin[\omega(M+1)/2]}{\sin(\omega/2)}}}_{A(e^{j\omega})} \boxed{e^{-j\omega\underbrace{\frac{M}{2}}_{\alpha}}}$$

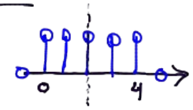
③



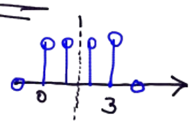
GLP for FIR → MUST have symmetry ④

$h[n] = h[M-n]$:

Type I (M even)



Type II (M odd)



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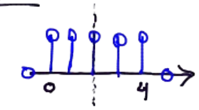
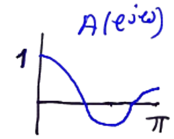
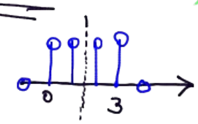
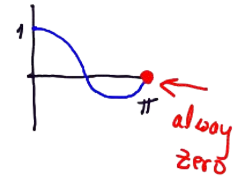




Type II (M odd)

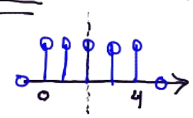


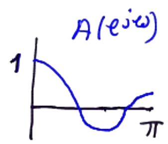


GLP for FIR → MUST have symmetry ④

$h[n] = h[M-n]$:

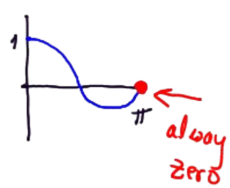
Type I (M even)





$$A(e^{j\omega}) = h[\tfrac{M}{2}] + 2\sum_{k=1}^{\frac{M}{2}} h[\tfrac{M}{2}-k]\cos(\omega k)$$

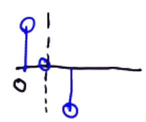
Type II (M odd)

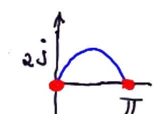


$A(e^{j\omega}) =$ In the text

$h[n] = -h[M-n]$ ⑤

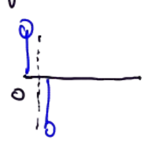
Type III (M even)

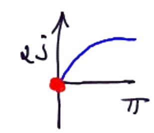




$$A(e^{j\omega}) = j\,2\sum_{k=1}^{\frac{M}{2}} h[\tfrac{M}{2}-k]\sin(\omega k)$$

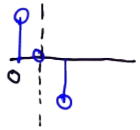
Type IV (M odd)





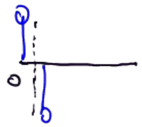
$h[n] = -h[M-n]$

Type III (M even)



$$A(e^{j\omega}) = j\,2\sum_{k=1}^{M/2} h\left[\tfrac{M}{2}-k\right]\sin(\omega k)$$

Type IV (M odd)



$A(e^{j\omega}) =$ see text