EE123 Digital Signal Processing

Lecture 28

Based on lecture notes by Prof. Murat Arcak

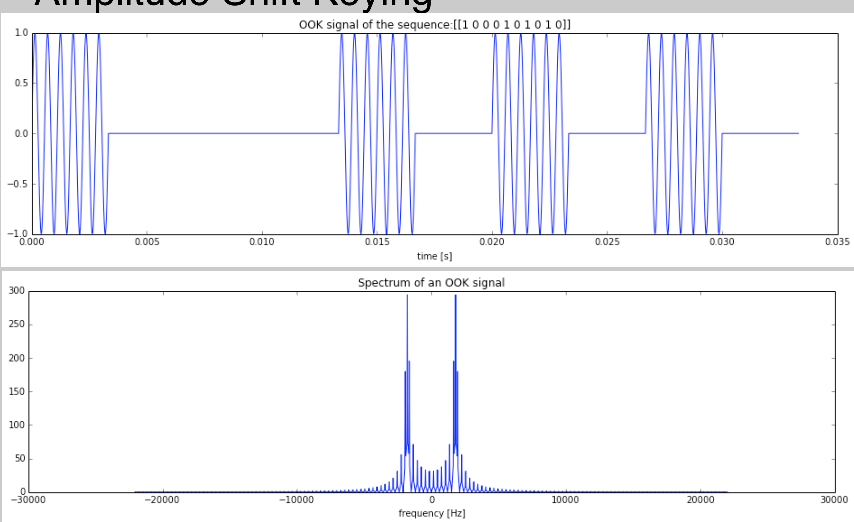
Lab 3

- Part I Due Friday
- Part II Due next Friday
- Part III Due Next Friday

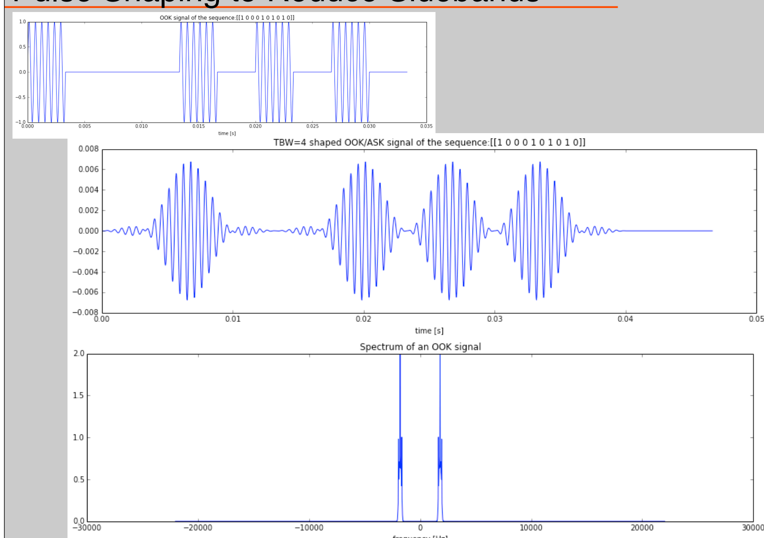
- Project Proposals Due Friday
- HW Due Friday
- End of the week is due Friday

Lab 3 Part II - Digital Communication

- Amplitude Shift Keying

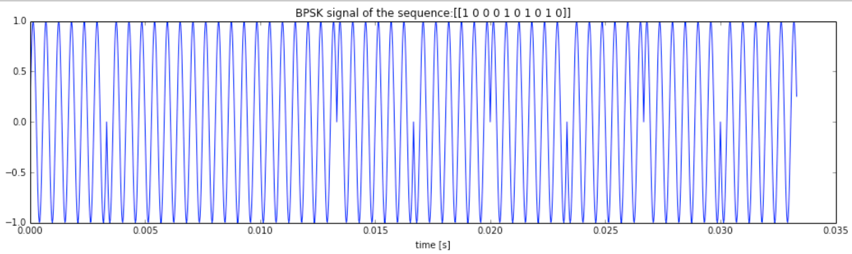


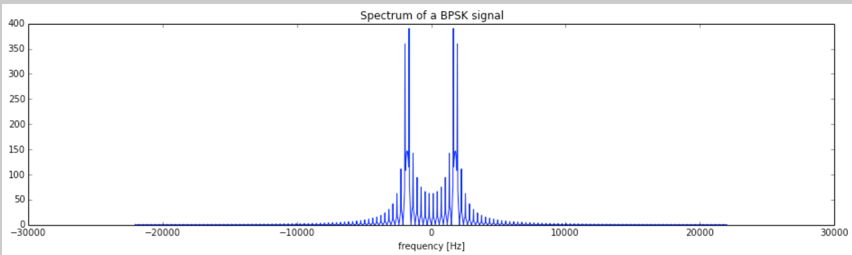
Pulse Shaping to Reduce Sidebands



- Narrow band, but inter-symbol interference!

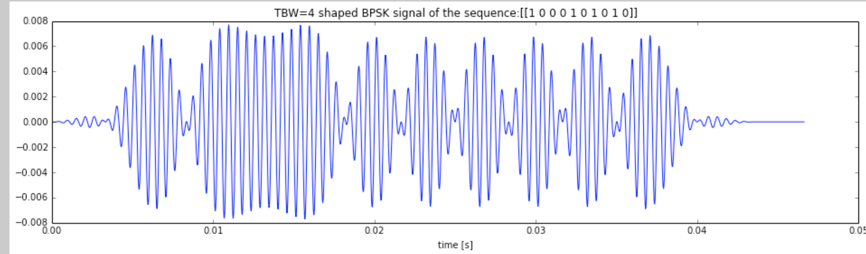
Phase Shift Keying

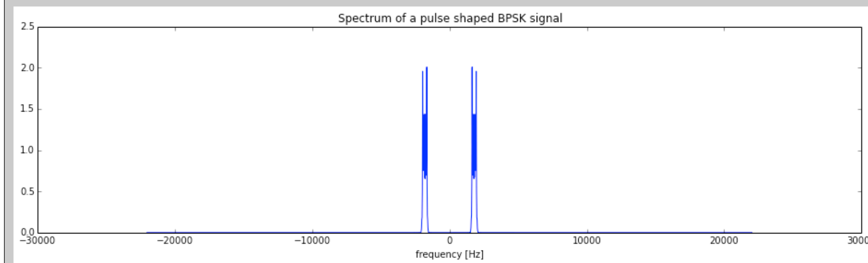




- Lots of sidelobes, but constant envelope!

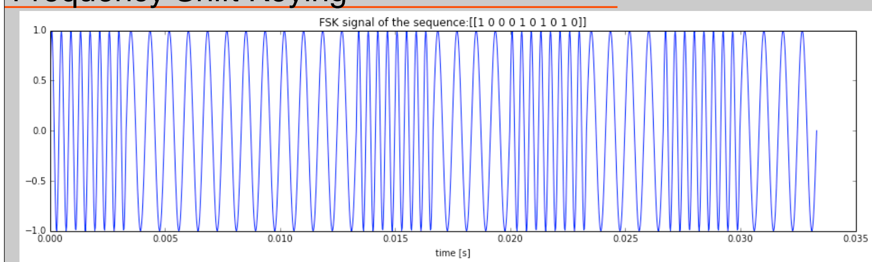
Pulse Shaping

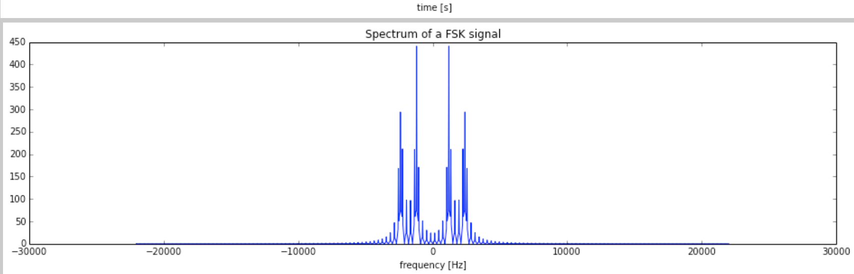




- Lost the sidelobes, but not constant envelope!

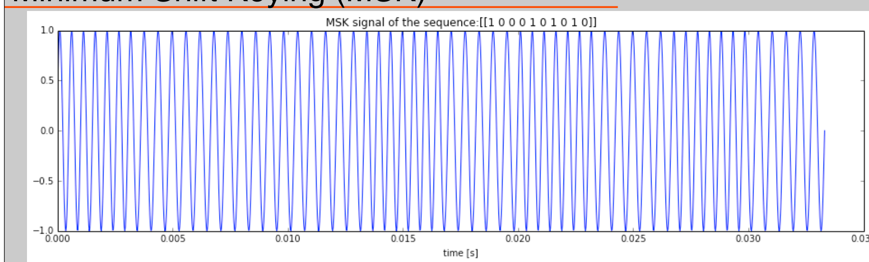
Frequency Shift Keying

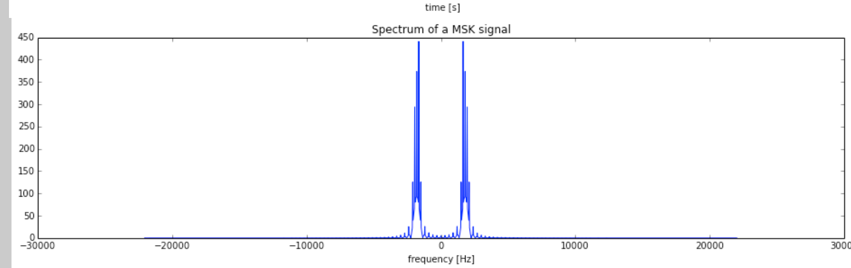




- 1200Hz/2400Hz 300 baud. 4/8 cycles/bit
- Constant envelope, wide band

Minimum Shift Keying (MSK)

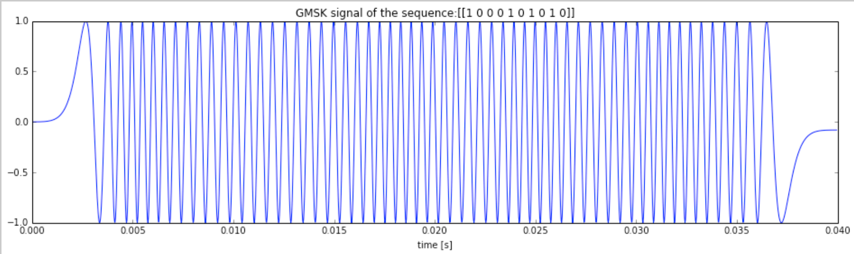


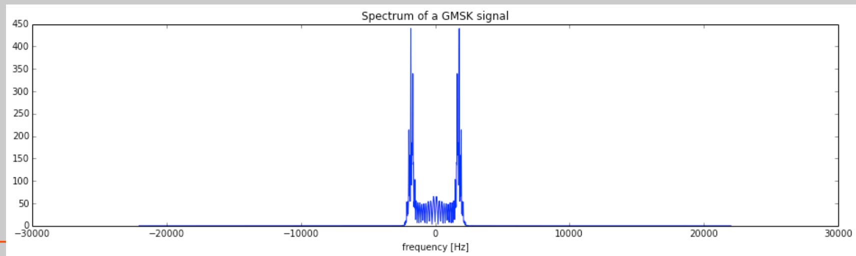


- 1200Hz/2400Hz 2400 baud. 0.5/1 cycles/bit
- Much more narrow-band

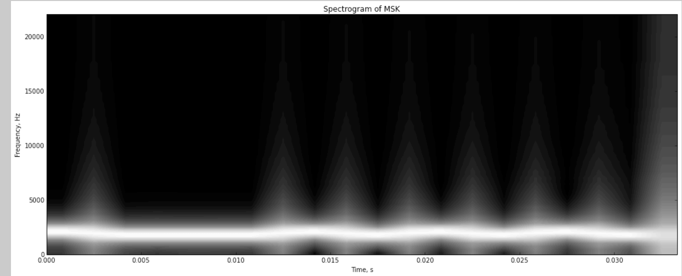
Gaussian Minimum Shift Keying

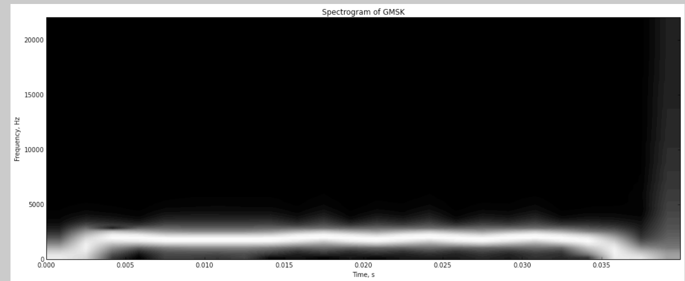
- Transition between bits sharp -- still lots of sidelobs. Reduce by filtering
- Used in many telecom apps, including GSM





GMSK vs MSK

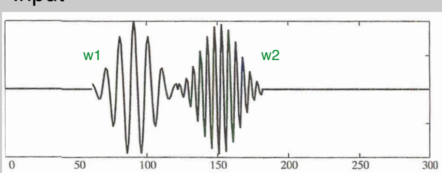


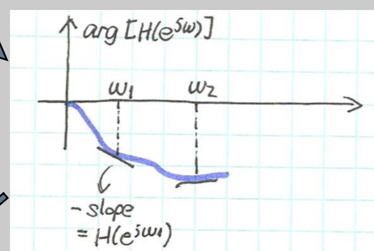


Group delay

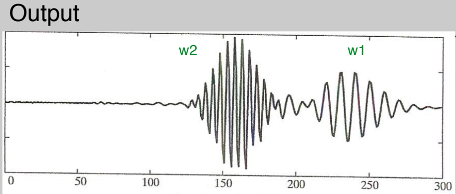
$$\mathrm{grd}[H(e^{j\omega})] = -\frac{d}{d\omega}\{\arg[H(e^{j\omega})]\}$$

Input





Output



For narrowband signals, phase response looks like a linear phase

Group delay math

$$H(z) = \frac{b_0}{a_0} \frac{\prod_{k=1}^{M}(1-c_k z^{-1})}{\prod_{k=1}^{N}(1-d_k z^{-1})}$$

arg of products is sum of args

$$\arg[H(e^{j\omega})] = -\sum_{k=1}^{N} \arg[1-d_k e^{-j\omega}] + \sum_{k=1}^{M} \arg[1-c_k e^{-j\omega}]$$

$$\mathrm{grd}[H(e^{j\omega})] = -\sum_{k=1}^{N} \mathrm{grd}[1-d_k e^{-j\omega}] + \sum_{k=1}^{M} \mathrm{grd}[1-c_k e^{-j\omega}]$$

Group delay math

$$\mathrm{grd}[H(e^{j\omega})] = -\sum_{k=1}^{N} \mathrm{grd}[1-d_k e^{-j\omega}] + \sum_{k=1}^{M}\mathrm{grd}[1-c_k e^{-j\omega}]$$

Look at each factor:

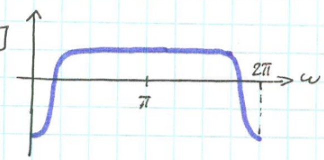
$$\arg[1-\underbrace{re^{j\theta}}_{c_k \text{ or } d_k} e^{-j\omega}] = \tan^{-1}\left(\frac{r\sin(\omega-\theta)}{1-r\cos(\omega-\theta)}\right)$$

$$\mathrm{grd}[1-re^{j\theta}e^{-j\omega}] = \frac{r^2 - r\cos(\omega-\theta)}{|1-re^{j\theta}e^{-j\omega}|^2}$$

Look at a zero lying on the real axis

Geometric Interpretation (for $\theta=0$)

$$\arg[1-re^{-j\omega}] = \arg[(e^{j\omega}-r)e^{-j\omega}] = \underbrace{\arg[e^{j\omega}-r]}_{\phi} - \underbrace{\arg[e^{j\omega}]}_{\omega}$$



$\theta \neq 0 \Rightarrow$ shift to the right by θ

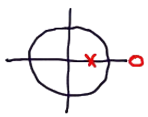
M. Lustig, EECS UC Berkeley

All-Pass Systems

②

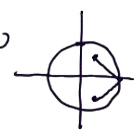
what is the magnitude response of

$$H(z) = \frac{z^{-1}-a^*}{1-az^{-1}}$$



$$|H(e^{j\omega})| = \frac{|e^{-j\omega}-a^*|}{|1-ae^{-j\omega}|} = \frac{|e^{-j\omega}(1-a^*e^{j\omega})|}{|1-ae^{-j\omega}|} =$$

$$= \frac{|1-a^*e^{j\omega}|}{|1-(a^*e^{j\omega})^*|} = 1 \quad \forall\omega$$



A generall all-pass system:

③

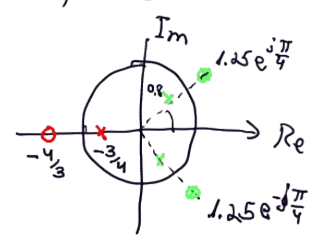
$$H_{ap}(z) = \prod_{k=1}^{M_r \,(\text{Real})} \frac{z^{-1}-d_k}{1-d_k z^{-1}} \cdot \prod_{k=1}^{M_c\,(\text{complex})} \frac{z^{-1}-e_k^*}{1-e_k z^{-1}} \cdot \frac{z^{-1}-e_k}{1-e_k^* z^{-1}}$$

d_k: real Poles

e_k: complex poles paired w/ conjugate e_k^*

$$|H_{ap}(e^{j\omega})| \equiv 1$$

Example



phase response of an all-pass: (4)

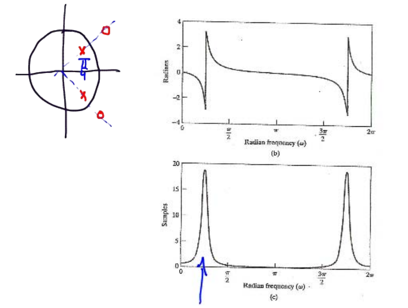
$$\arg\left[\frac{e^{-j\omega} - \overbrace{re^{-j\theta}}^{a^*}}{1 - \underbrace{re^{j\theta}}_{a} e^{-j\omega}}\right] = \arg\left[\frac{e^{-j\omega}(1 - re^{-j\theta}e^{-j\omega})}{1 - re^{j\theta}e^{-j\omega}}\right] =$$

$$= \underbrace{\arg[e^{-j\omega}]}_{-\omega} - 2\arg[1 - re^{j\theta}e^{-j\omega}]$$

$$\text{grd}\left[\frac{e^{-j\omega} - re^{-j\theta}}{1 - re^{j\theta}e^{-j\omega}}\right] = 1 - 2\,\text{grd}[1 - re^{j\theta}e^{-j\omega}]$$

(5)

Example: < figure 5.20 >



can be used to compensate phase distortion.

Claim: for a stable ap system $H_{ap}(z)$: (6)

(i) $\text{grd}[H_{ap}(e^{j\omega})] > 0$

(ii) $\arg[H_{ap}(e^{j\omega})] \leq 0$

Delay positive → causal
phase negative → phase lag.

proof in book.

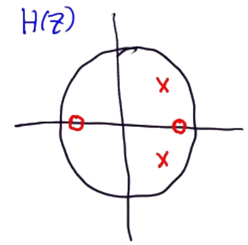
Minimum-Phase Systems (7)

Definition: a $\underbrace{\text{stable}}_{\text{poles insid unit circle}}$ and causal system $H(z)$

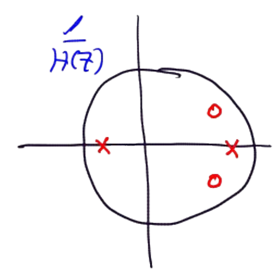
who's inverse $\frac{1}{H(z)}$ is also stable & causal

zeros are insid unit circle.

$H(z)$



$\frac{1}{H(z)}$



AP-Min-Phase decomposition: (8)

stable, causal system can be decomposed to:

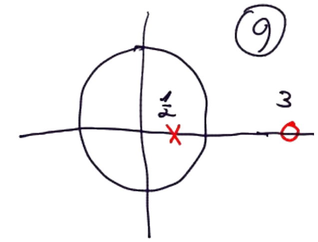
$$H(z) = \underbrace{H_{min}(z)}_{\text{min phase}} \cdot \underbrace{H_{ap}(z)}_{\text{all pass}}$$

Approach ① first construct H_{ap} with all zeros outside unit circle

② compute $H_{min}(z) = \dfrac{H(z)}{H_{ap}(z)}$

Example (9)

$$H(z) = \frac{1-3z^{-1}}{1-\frac{1}{2}z^{-1}}$$



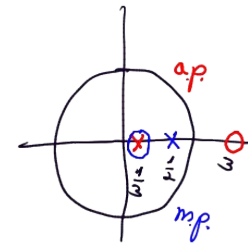
set:

$$H_{ap} = \frac{z^{-1}-\frac{1}{3}}{1-\frac{1}{3}z^{-1}}$$

$$H_{min}(z) = \frac{1-3z^{-1}}{1-\frac{1}{2}z^{-1}} \cdot \frac{1-\frac{1}{3}z^{-1}}{z^{-1}-\frac{1}{3}} =$$

-3 =

$$= -3\,\frac{1-\frac{1}{3}z^{-1}}{1-\frac{1}{2}z^{-1}}$$



why m.p. property important? (10)

for ex. communication chan.

If $H_d(z)$ is minimum phase, design $H_c(z) = \dfrac{1}{H_d(z)}$ (stable!)

If not m.p., decompose: $H_d(z) = H_{d,mp}(z) \cdot H_{d,ap}(z)$

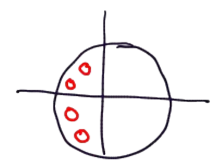
$$H_c(z) = \frac{1}{H_{d,min}(z)} \Rightarrow H_d H_c = H_{d,ap}(z)$$

only compensate for mag.

Why called "minimum phase"? (11)

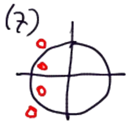
Different systems can have same mag. response.

$H_1(z)$ min phase:



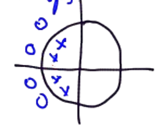
$H_2(z)$ (max phase)

$H_3(z)$

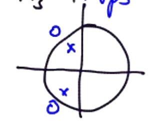


$H_4(z)$

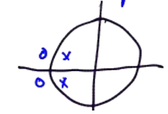
$H_2 = H_1 H_{ap,2}$



$H_3 = H_1 H_{ap3}$



$H_4 = H_1 H_{ap4}$



of all, $H_1(z)$ has minimum phase lag (12)
because:

$$\arg[H_i(e^{j\omega})] = \arg[H_1(e^{j\omega})] + \arg[H_{ap_i}]$$

≤ 0

other properties:

<u>minimum group delay</u>:

$$\mathrm{grd}[H(e^{j\omega})] = \mathrm{grd}[H_{min}] + \mathrm{grd}[H_{ap}]$$

≥ 0

<u>minimum energy delay</u>:

Problem 5.72