EE123 Digital Signal Processing

Lecture 27

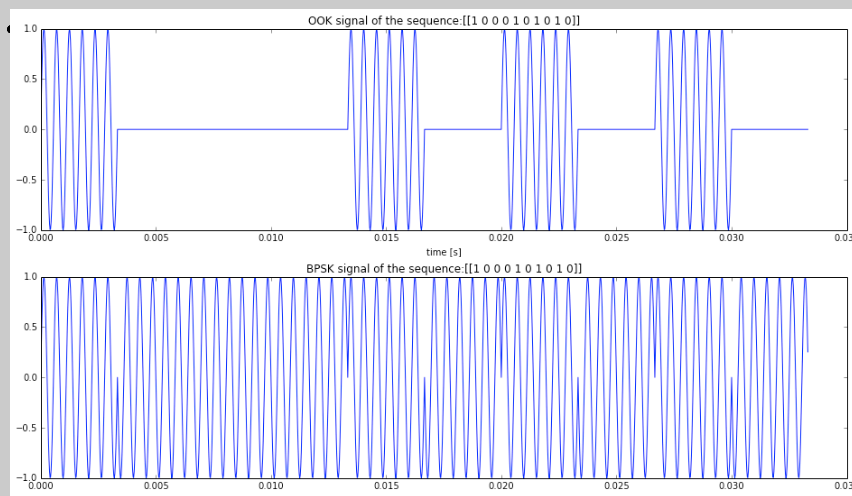
Based on lecture notes by Prof. Murat Arcak

Lab 3 - Part I

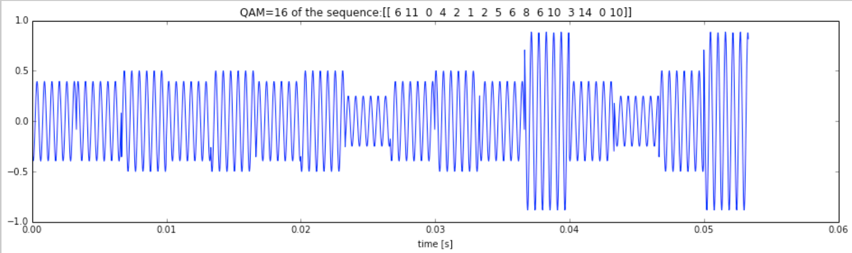
- Be careful! cables can become Antennas.......

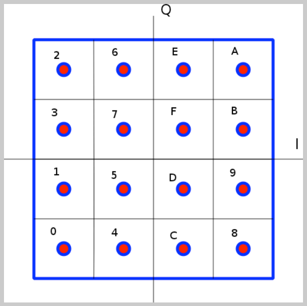
Last Part of Lab

- Learn about Digital Communication

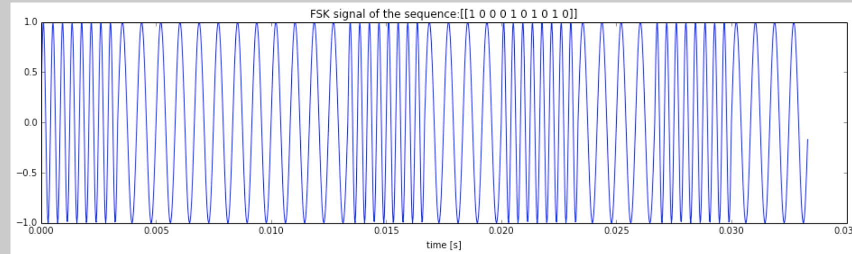


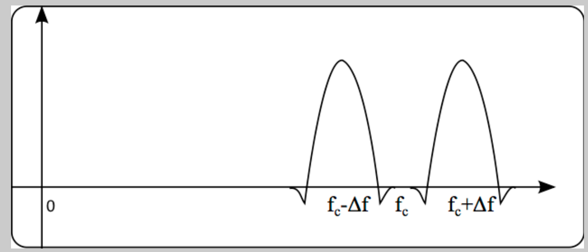
Digital Communication



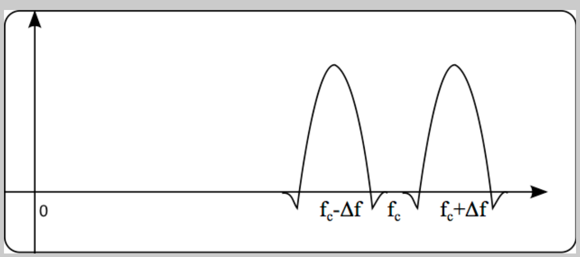


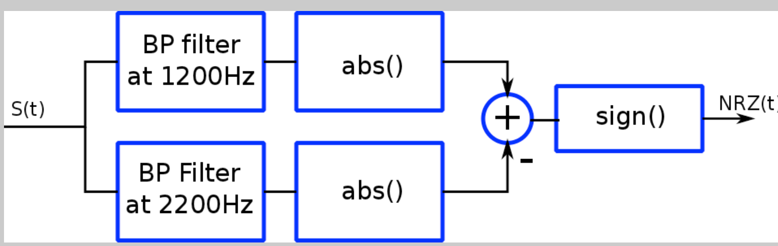
Frequency Shift Keying



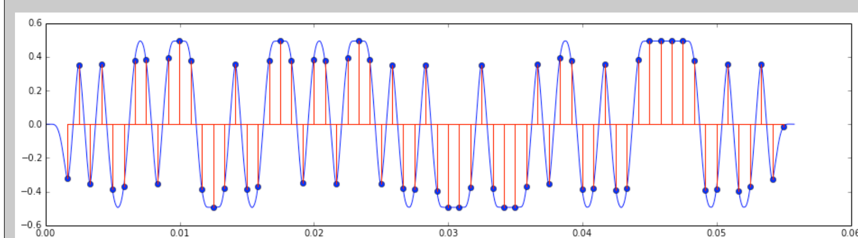


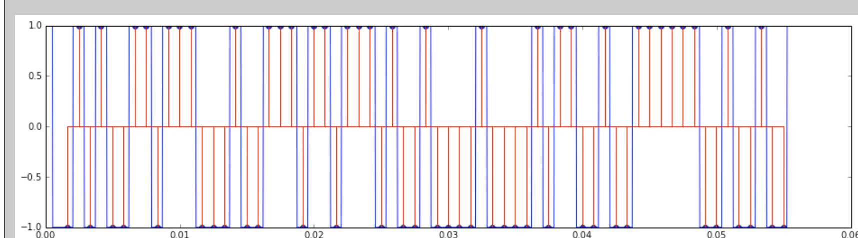
Demodulation





Demodulation





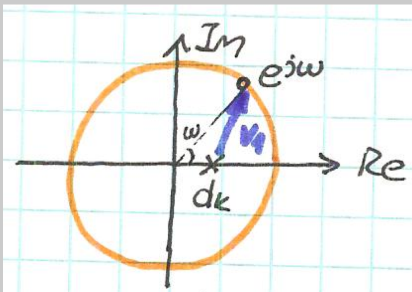
Magnitude Response

Magnitude of products is product of magnitudes

$$|H(e^{j\omega})| = |\frac{b_0}{a_0}| \cdot \frac{\prod_{k=0}^{M} |1 - c_k e^{-j\omega}|}{\prod_{k=0}^{N} |1 - d_k e^{-j\omega}|}$$

Consider one of the poles:

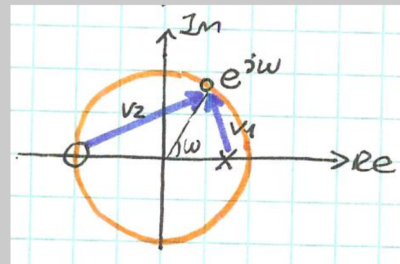
$$|1 - d_k e^{-j\omega}| = |e^{+j\omega} - d_k| \; = |v_1|$$

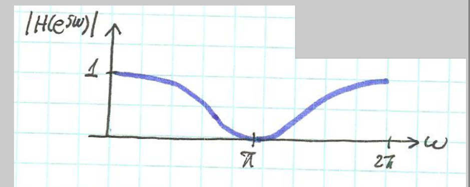


Magnitude Response Example

Example: $$H(z) = 0.05 \frac{1 + z^{-1}}{1 - 0.9z^{-1}}$$

$$|H(z)| = 0.05 \frac{|v_2|}{|v_1|}$$





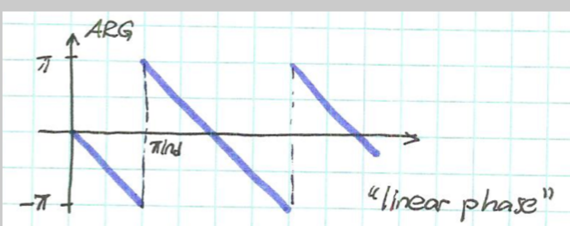
Phase response

Example: $H(e^{j\omega}) = e^{j\omega n_d} \quad \leftrightarrow \quad h[n] = \delta[n - n_d]$

$$|H(e^{j\omega})| = 1$$

$$\arg[H(e^{j\omega})] = -\omega n_d$$

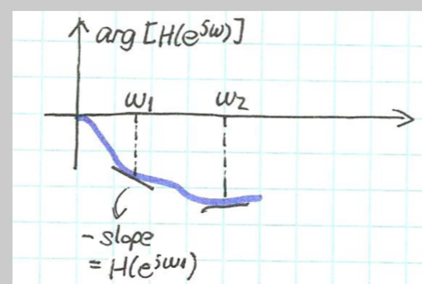
ARG is the wrapped phase
arg is the unwrapped phase



Group delay

To characterize general phase response, look at the group delay:

$$\mathrm{grd}[H(e^{j\omega})] = -\frac{d}{d\omega}\{\arg[H(e^{j\omega})]\}$$

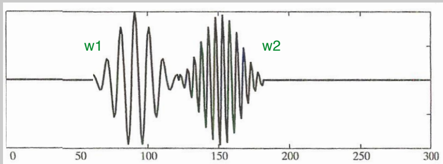


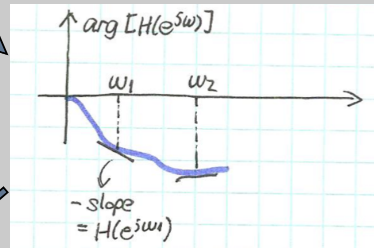
For linear phase system, the group delay is n_d

Group delay

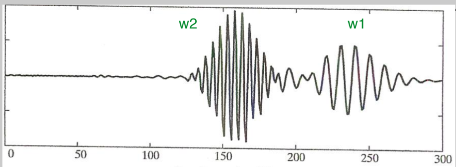
$$\mathrm{grd}[H(e^{j\omega})] = -\frac{d}{d\omega}\{\arg[H(e^{j\omega})]\}$$

Input





Output



For narrowband signals, phase response looks like a linear phase

Group delay math

$$H(z) = \frac{b_0}{a_0}\,\frac{\prod_{k=1}^{M}(1-c_k z^{-1})}{\prod_{k=1}^{N}(1-d_k z^{-1})}$$

arg of products is sum of args

$$\arg[H(e^{j\omega})] = -\sum_{k=1}^{N}\arg[1-d_k e^{-j\omega}] + \sum_{k=1}^{M}\arg[1-c_k e^{-j\omega}]$$

$$\mathrm{grd}[H(e^{j\omega})] = -\sum_{k=1}^{N}\mathrm{grd}[1-d_k e^{-j\omega}] + \sum_{k=1}^{M}\mathrm{grd}[1-c_k e^{-j\omega}]$$

Group delay math

$$\mathrm{grd}[H(e^{j\omega})] = -\sum_{k=1}^{N}\mathrm{grd}[1-d_k e^{-j\omega}] + \sum_{k=1}^{M}\mathrm{grd}[1-c_k e^{-j\omega}]$$

Look at each factor:

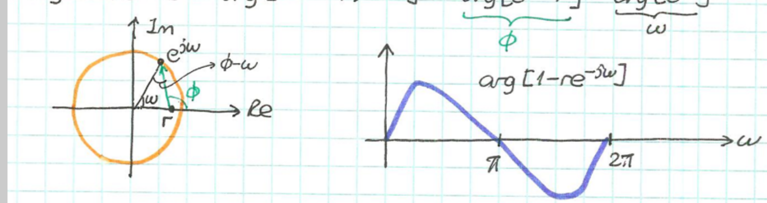
$$\arg[1-\underbrace{re^{j\theta}}_{c_k \text{ or } d_k}e^{-j\omega}] = \tan^{-1}\left(\frac{r\sin(\omega-\theta)}{1-r\cos(\omega-\theta)}\right)$$

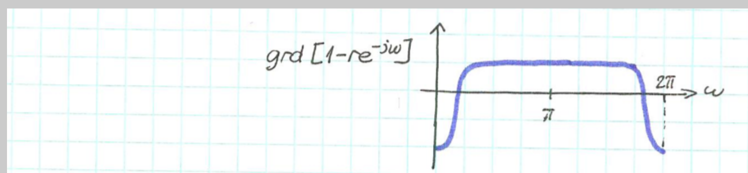
$$\mathrm{grd}[1-re^{j\theta}e^{-j\omega}] = \frac{r^2-r\cos(\omega-\theta)}{|1-re^{j\theta}e^{-j\omega}|^2}$$

Look at a zero lying on the real axis

Geometric Interpretation (for $\theta=0$)

$$\arg[1-re^{-j\omega}] = \arg[(e^{j\omega}-r)e^{-j\omega}] = \underbrace{\arg[e^{j\omega}-r]}_{\phi} - \underbrace{\arg[e^{j\omega}]}_{\omega}$$





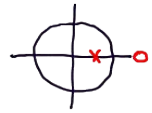
$\theta \neq 0 \Rightarrow$ shift to the right by θ

All-Pass Systems

(2)

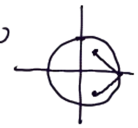
what is the magnitude response of

$$H(z) = \frac{z^{-1} - a^*}{1 - a z^{-1}}$$



$$|H(e^{j\omega})| = \frac{|e^{-j\omega} - a^*|}{|1 - a e^{-j\omega}|} = \frac{|e^{-j\omega}(1 - a^* e^{j\omega})|}{|1 - a e^{-j\omega}|} =$$

$$= \frac{|1 - a^* e^{j\omega}|}{|1 - (a^* e^{j\omega})^*|} = 1 \quad \forall \omega$$



A generall all-pass system: (3)

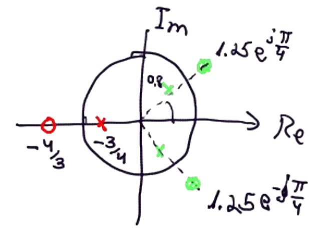
$$H_{ap}(z) = \prod_{k=1}^{\overset{\text{Real}}{M_r}} \frac{z^{-1} - d_k}{1 - d_k z^{-1}} \cdot \prod_{k=1}^{\overset{\text{complex}}{M_c}} \frac{z^{-1} - e_k^*}{1 - e_k z^{-1}} \cdot \frac{z^{-1} - e_k}{1 - e_k^* z^{-1}}$$

d_k: real Poles

e_k: complex poles paired w/ conjugate e_k^*

$$|H_{ap}(e^{j\omega})| \equiv 1$$

Example



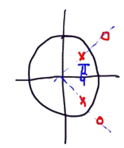
phase response of an all-pass: (4)

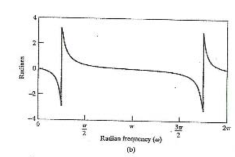
$$\arg\left[\frac{e^{-j\omega} - \overbrace{re^{-j\theta}}^{a^*}}{1 - \underbrace{re^{j\theta}}_{a} e^{-j\omega}}\right] = \arg\left[\frac{e^{-j\omega}(1 - re^{-j\theta}e^{j\omega})}{1 - re^{j\theta}e^{-j\omega}}\right] =$$

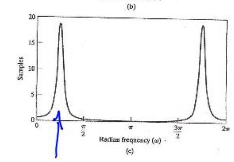
$$= \underbrace{\arg[e^{-j\omega}]}_{-\omega} - 2\arg[1 - re^{j\theta}e^{-j\omega}]$$

$$\operatorname{grd}\left[\frac{e^{-j\omega} - re^{-j\theta}}{1 - re^{j\theta}e^{-j\omega}}\right] = 1 - 2\operatorname{grd}[1 - re^{j\theta}e^{-j\omega}]$$

Example: <figure 5.20> (5)







can be used to compensate phase distortion.

Claim: for a stable ap system $H_{ap}(z)$: (6)

(i) $\operatorname{grd}\left[H_{ap}(e^{j\omega})\right] > 0$

(ii) $\arg\left[H_{ap}(e^{j\omega})\right] \leq 0$

Delay positive → causal

phase negative → phase lag.

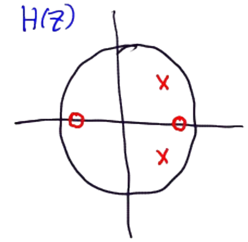
proof in book.

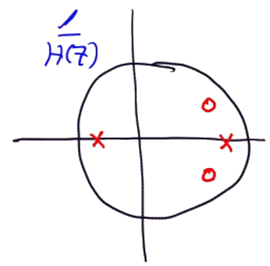
Minimum-Phase Systems (7)

Definition: a $\underbrace{\text{stable and causal}}_{\text{poles insid unit circle}}$ system $H(z)$

who's inverse $\frac{1}{H(z)}$ is also stable & causal

zeros are insid unit circle.





AP-Min-Phase decomposition: (8)

stable, causal system can be decomposed to:

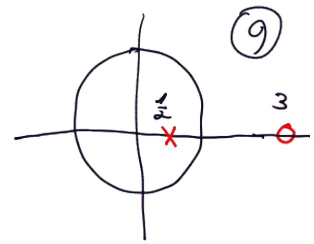
$$H(z) = \underbrace{H_{min}(z)}_{\text{min phase}} \cdot \underbrace{H_{ap}(z)}_{\text{all pass}}$$

Approach: ① first construct H_{ap} with all zeros outside unit circle

② compute $H_{min}(z) = \frac{H(z)}{H_{ap}(z)}$

Example (9)

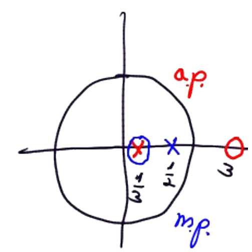
$$H(z) = \frac{1-3z^{-1}}{1-\frac{1}{2}z^{-1}}$$



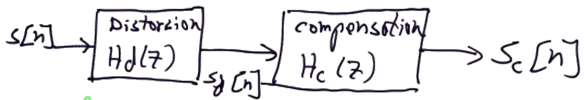
Set: $H_{ap} = \frac{z^{-1}-\frac{1}{3}}{1-\frac{1}{3}z^{-1}}$

-3

$$H_{min}(z) = \frac{1-3z^{-1}}{1-\frac{1}{2}z^{-1}} \cdot \frac{1-\frac{1}{3}z^{-1}}{z^{-1}-\frac{1}{3}} = -3\,\frac{1-\frac{1}{3}z^{-1}}{1-\frac{1}{2}z^{-1}}$$



why m.p. property important? (10)



for ex. communication chan.

If $H_d(z)$ is minimum phase, design

$$H_c(z) = \frac{1}{H_d(z)} \quad \text{(stable!)}$$

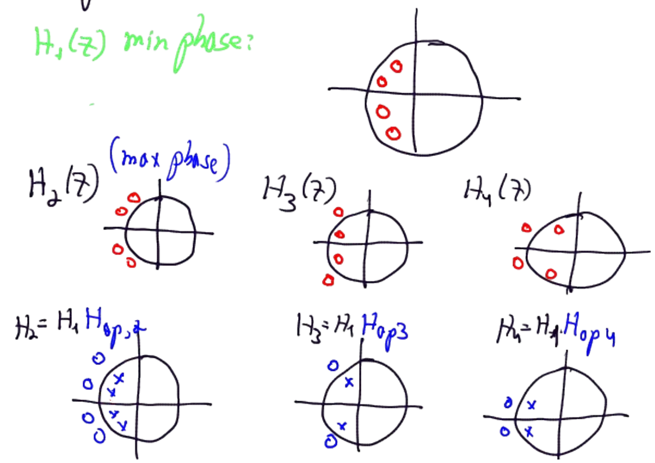
If not m.p., decompose: $H_d(z) = H_{d,mp}(z) \cdot H_{d,ap}(z)$

$$H_c(z) = \frac{1}{H_{d,min}(z)} \Rightarrow H_d H_c = H_{d,ap}(z)$$

only compensate for mag.

Why called "minimum phase"? (11)

Different systems can have same mag. response.



of all, $H_1(z)$ has minimum phase lag (12)

because:

$$\arg[H_i(e^{j\omega})] = \arg[H_1(e^{j\omega})] + \arg[H_{ap,i}]$$

$\arg[H_{ap,i}] \leq 0$

other properties:

minimum group delay:

$$\text{grd}[H(e^{j\omega})] = \text{grd}[H_{min}] + \text{grd}[H_{ap}]$$

$\text{grd}[H_{ap}] \geq 0$

minimum energy delay:

Problem 5.72