EE123

Digital Signal Processing

Lecture 22

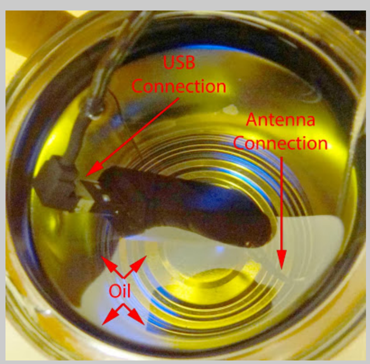
Ham Shack at Cory 532

- Will be run by IEEE (Roy Tu)
- Triple-band (2m/70cm/23cm) roof antenna
- 2M all mod 10w radio
- For keycard access: http://goo.gl/xxW9vt

Lab 2 Part II

- SDR crystal oscillator has often has offset
- Also drifts with temperature
- Cellphones do the same!

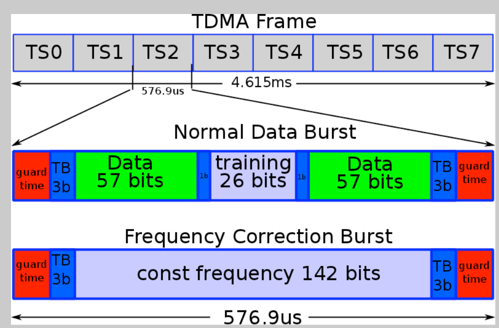
- GSM protocol has built in synchronizations



http://sdrformariners.blogspot.com/2013/12/cooling.html

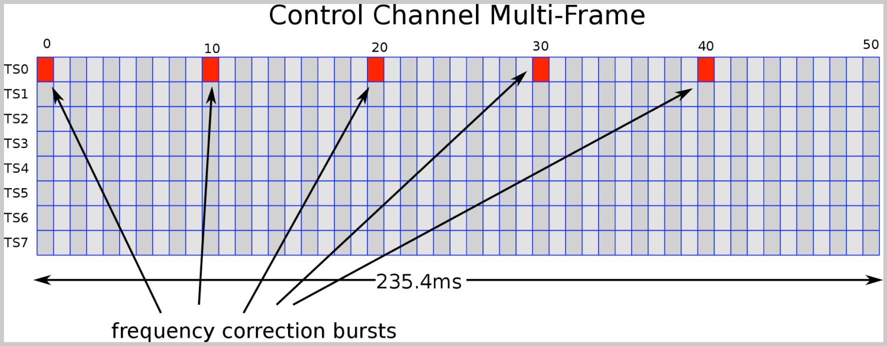
GSM-850

- Frequencies 200KHz channels
  - –Uplink 824-849
  - –Downlink 869-849
- TDMA: Time division multiple access

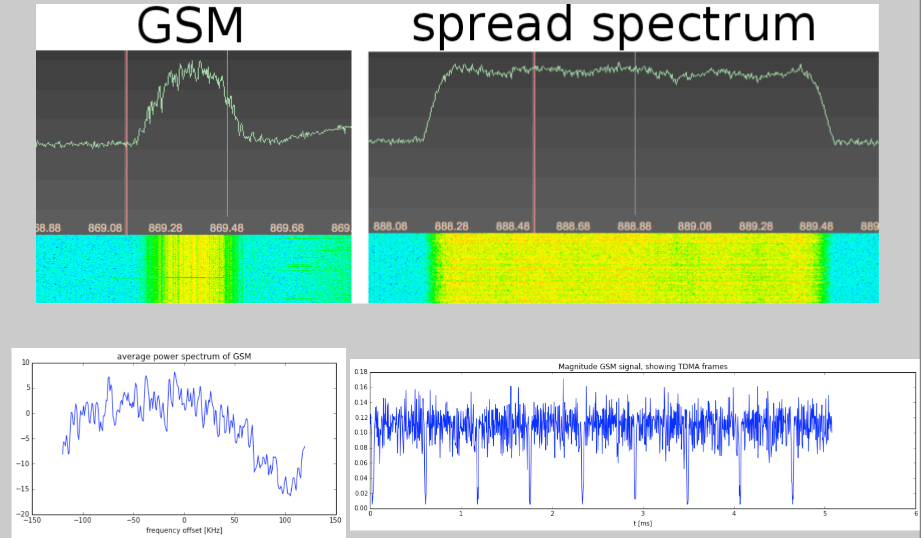


GSM Frequency Correction Channel

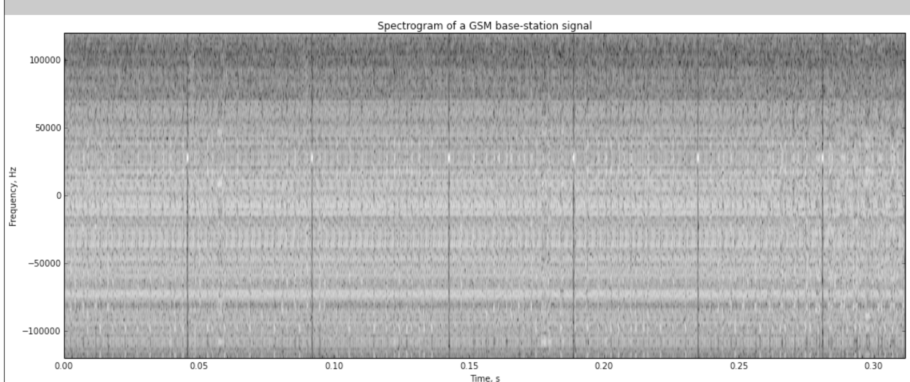
- Pure frequency bursts @67.7083KHz



How to find GSM Base Stations

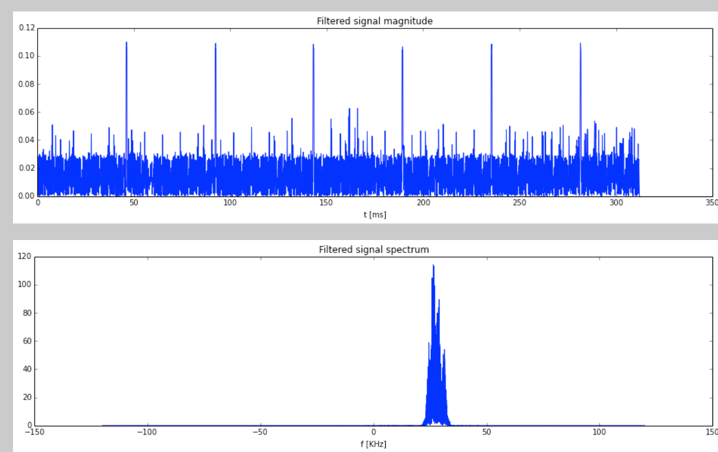


Spectrogram of GSM



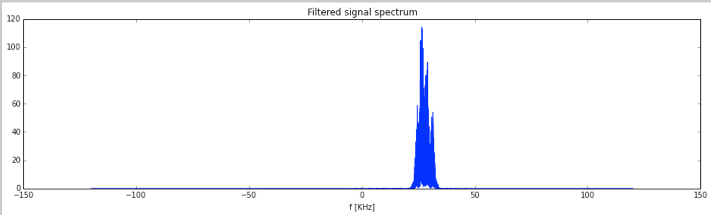
How to find Bursts?

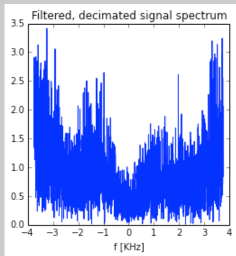
- Use Bandpass filter and compute magnitude of result



How to find Bursts?

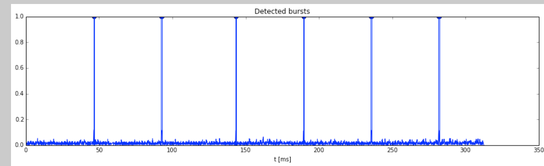
- Can process at lower rate!

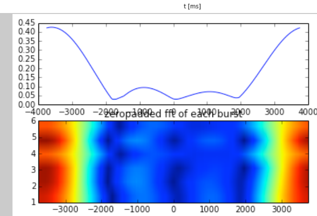




Detect Bursts and Compute Frequency

- Detect bursts at low rate sampling
- Compute frequency
- Calculate the original frequency!





MiniProject

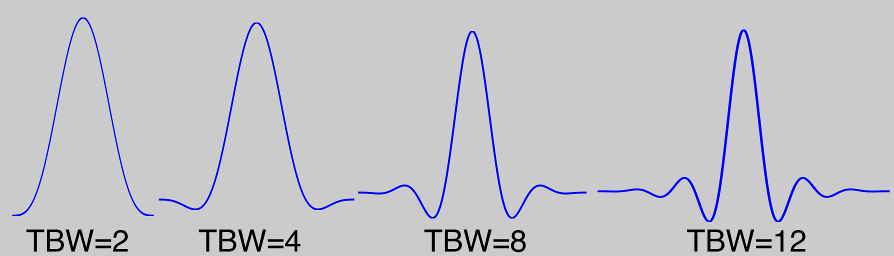
- Automate:
  - Run through a filter bank to detect shifts of 50ppm shifts
  - Process filter bank to find active bank
  - Find burst in active bank
  - Find frequency of FCCH

- Optional:
  - Scan GSM band to find GSM base-stations

Characterization of Filter Shape

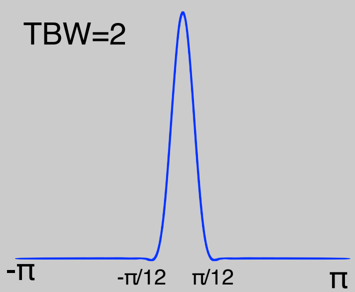
Time-Bandwidth Product

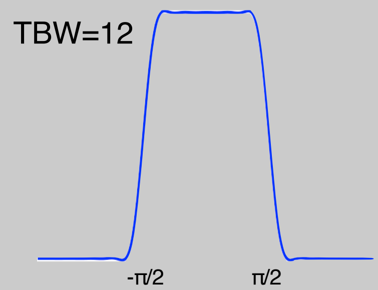
$T(BW) = (M+1)\omega/2\pi$ ⇒ also, total # of zero crossings



Frequency Response Profile

Q: What are the lengths of these filters in samples?





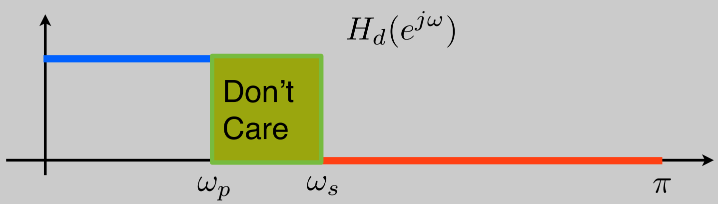
2 = (M+1)*(π/6) / (2π) ⇒ M=23   12= (M+1)*(π) / (2π) ⇒ M=23

Note that transition is the same!

Optimal Filter Design

- Last time:
  - Design Filters heuristically using windowed sinc functions
- Today: Optimal design
  - Design a filter h[n] with $H(e^{j\omega})$
  - Approximate $H_d(e^{j\omega})$ with some optimality criteria - or satisfies specs.

Optimality



- Least Squares:

$$\text{minimize} \int_{\omega \in \text{care}} |H(e^{j\omega}) - H_d(e^{j\omega})|^2 d\omega$$

Variation: weighted least-squares

$$\text{minimize} \int_{-\pi}^{\pi} W(\omega)|H(e^{j\omega}) - H_d(e^{j\omega})|^2 d\omega$$

Optimality

- Chebychev Design (min-max)

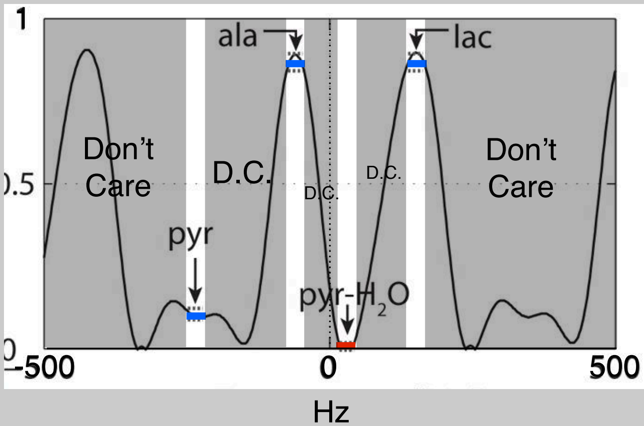
$$\text{minimize}_{\omega \in \text{care}} \quad \max |H(e^{j\omega}) - H_d(e^{j\omega})|$$

  - Parks-McClellan algorithm - equi-ripple
  - Also known as Remez exchange algorithms (signal.remez)

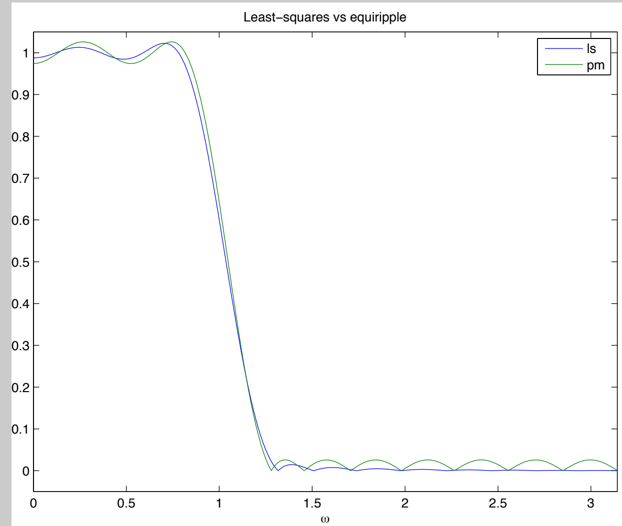
Example of Complex Filter

Larson et. al, “Multiband Excitation Pulses for Hyperpolarized 13C Dynamic Chemical Shift Imaging” JMR 2008;194(1):121-127

Need to design 11 taps filter with following frequency response:



Least-Squares v.s. Min-Max



Design Through Optimization

- Idea: Sample/discretize the frequency response

$$H(e^{j\omega}) \Rightarrow H(e^{j\omega_k})$$

  - Sample points are fixed $\omega_k = k\frac{\pi}{P}$

$$-\pi \leq \omega_1 < \cdots < \omega_p \leq \pi$$

  - M+1 is the filter order
  - P >> M + 1 (rule of thumb P=15M)
  - Yields a (good) approximation of the original problem

Example: Least Squares

- Target: Design M+1= 2N+1 filter
- First design non-causal $\tilde{H}(e^{j\omega})$ and hence $\tilde{h}[n]$
- Then, shift to make causal

$$h[n] = \tilde{h}[n - M/2]$$

$$H(e^{j\omega}) = e^{-j\frac{M}{2}}\tilde{H}(e^{j\omega})$$

Example: Least Squares

- Matrix formulation:

$$\tilde{h} = \left[\tilde{h}[-N], \tilde{h}[-N+1], \cdots, \tilde{h}[N]\right]^T$$

$$b = \left[H_d(e^{j\omega_1}), \cdots, H_d(e^{j\omega_P})\right]^T$$

$$A = \begin{bmatrix} e^{-j\omega_1(-N)} & \cdots & e^{-j\omega_1(+N)} \\ \vdots & & \\ e^{-j\omega_P(-N)} & \cdots & e^{-j\omega_P(+N)} \end{bmatrix}$$

$$\text{argmin}_{\tilde{h}} \quad ||A\tilde{h} - b||^2$$

Least Squares

$$\text{argmin}_{\tilde{h}} \quad ||A\tilde{h} - b||^2$$

Solution:

$$\tilde{h} = (A^*A)^{-1}A^*b$$

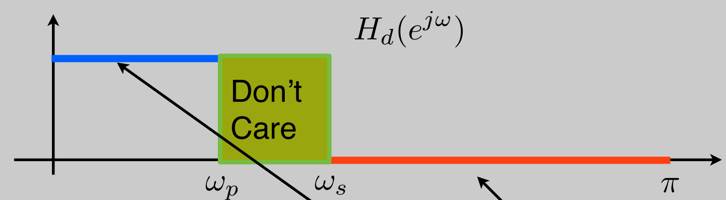
- Result will generally be non-symmetric and complex valued.
- However, if $\tilde{H}(e^{j\omega})$ is real, $\tilde{h}[n]$ should have symmetry!

Design of Linear-Phase L.P Filter

- Suppose:
  - $\tilde{H}(e^{j\omega})$ is real-symmetric
  - M is even (M+1 taps)
- Then:
  - $\tilde{h}[n]$ is real-symmetric around midpoint
- So:

$$\begin{aligned} \tilde{H}(e^{j\omega}) &= \tilde{h}[0] + \tilde{h}[1]e^{-j\omega} + \tilde{h}[-1]e^{+j\omega} \\ &\quad + \tilde{h}[2]e^{-j2\omega} + \tilde{h}[-2]e^{+j2\omega} \cdots \\ &= \tilde{h}[0] + 2\cos(\omega)\tilde{h}[1] + 2\cos(2\omega)\tilde{h}[2] + \cdots \end{aligned}$$

Least-Squares Linear-Phase Filter



Given M, ω_P, ω_s find the best LS filter:

$$A = \begin{bmatrix} 1 & \cdots & 2\cos(\frac{M}{2}\omega_1) \\ \vdots & & \\ 1 & \cdots & 2\cos(\frac{M}{2}\omega_p) \\ 1 & \cdots & 2\cos(\frac{M}{2}\omega_s) \\ \vdots & & \\ 1 & \cdots & 2\cos(\frac{M}{2}\omega_P) \end{bmatrix} \qquad b = [1, 1, \cdots, 1, 0, 0, \cdots, 0]^T$$

Least-Squares Linear-Phase Filter

Given M, ω_P, ω_S find the best LS filter:

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$$\tilde{h}_+ = [\tilde{h}[0], \cdots, \tilde{h}[\frac{M}{2}]]^T = (A^*A)^{-1}A^*b$$

$$\tilde{h} = \begin{cases} \tilde{h}_+[n] & n \geq 0 \\ \tilde{h}_+[-n] & n < 0 \end{cases}$$

$$h[n] = \tilde{h}[n - M/2]$$

Extension:

- LS has no preference for pass band or stop band
- Use weighting of LS to change ratio

want to solve the discrete version of:

$$\text{minimize} \quad \int_{-\pi}^{\pi} W(\omega)|H(e^{j\omega}) - H_d(e^{j\omega})|^2 d\omega$$

where W(ω) is δp in the pass band and δs in stop band

Similarly: W(ω) is 1 in the pass band and δp/δs in stop band

Weighted Least-Squares

$$\text{argmin}_{\tilde{h}_+} \quad (A\tilde{h}_+ - b)^* W^2 (A\tilde{h} - b)$$

Solution:

$$\tilde{h}_+ = (A^*W^2A)^{-1}W^2A^*b$$

$$W = \begin{bmatrix} 1 & & & & & 0 \\ & 1 & & & & \\ & & \cdots & & & \\ & & & \frac{\delta_p}{\delta_s} & & \\ & & & & \cdots & \\ 0 & & & & & \frac{\delta_p}{\delta_s} \end{bmatrix}$$