EE123 Digital Signal Processing

Lecture 21

FIR Design by Windowing

Given desired freq response $H_d(e^{j\omega})$, find impulse response:

$$\boxed{h_d[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}\,d\omega} \rightarrow \text{ideal}$$

obtain M^{th} order causal FIR filter by truncating:

$$h[n] = \begin{cases} h_d[n] & 0 \le n \le M \\ \phi & \text{otherwise} \end{cases}$$

Equivalently, $h[n] = h_d[n]w[n]$ where $w[n] = \begin{cases} 1 & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$

We already saw that:

$$H(e^{j\omega}) = H_d(e^{j\omega}) \circledast W(e^{j\omega})$$

for Boxcar $\Rightarrow W(e^{j\omega}) = e^{-j\omega\frac{M}{2}} \frac{\sin(\omega(M+1)/2)}{\sin(\omega/2)}$

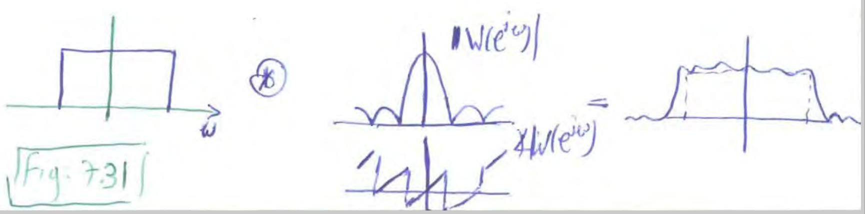


Fig. 7.31

"Tapered" windows:

Bartlett (triangular): $\Delta[n] = \begin{cases} \frac{n}{M/2} & 0 \le n \le M/2 \\ 2 - \frac{n}{M/2} & \frac{M}{2} \le n \le M \end{cases}$

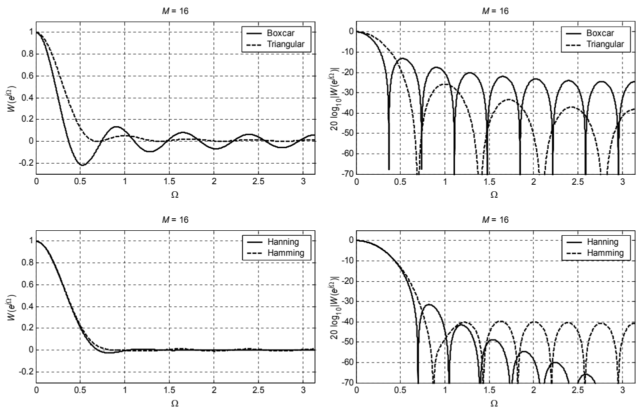
Hann $w[n] = \begin{cases} 0.5 - 0.5\cos(2\pi n/M) & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$

Hamming $w[n] = \begin{cases} 0.54 - 0.46\cos(2\pi n/M) & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$

< Figures 7.29 7.30 >

Note tradeoff between main lobe width and side-lobe amplitude

$\underbrace{\text{lobe width}}_{\text{transition}}$, $\underbrace{\text{amplitude}}_{\text{ripple}}$



Python: scipy.filter.firwin

FIR Filter Design

Frequency response

(-) Choose a desired frequency response $H_d(e^{j\omega})$

[Non causal and infinite impulse response]

zero delay

(If is derived from CT choose T and use:

$$H_d(e^{j\omega}) = H_c\left(j\frac{\Omega}{T}\right)$$)

FIR Filter Design

window

(-) Choose window

(-) Length M+1 → sharpness of transition

(-) Type → transition & ripple

(-) modulat desired freq. Reponse to shift impulse response

$$\boxed{H_d(e^{j\omega})e^{-j\omega\frac{M}{2}}}$$

FIR Filter Design

Determin truncated impulse response $h_1[n]$

$$h_1[n] = \begin{cases} \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{-j\omega\frac{M}{2}}e^{jn\omega} & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$$

Apply window

$$h_w[n] = w[n]\cdot h_1[n]$$

FIR Filter Design

Check

compute $H_w(e^{j\omega})$ if does not meet specs, increase M or use a different window.

Example FIR Low-Pass Filter Design (5)

$$H_d(e^{j\omega}) = \begin{cases} 1 & |\omega| \le \omega_c \\ 0 & \omega_c < |\omega| \le \pi \end{cases}$$

choose $M \rightarrow$ window length $\Rightarrow H_d(e^{j\omega}) \cdot e^{-j\omega\frac{M}{2}}$

$$h_1[n] = \begin{cases} \dfrac{\sin(\omega_c(n-\frac{M}{2}))}{\pi(n-\frac{M}{2})} = \text{sinc}\left(\frac{\omega_c}{\pi}\left(n-\frac{M}{2}\right)\right)\cdot\frac{\omega_c}{\pi} & 0 \le n \le M \\ \phi & \text{otherwise} \end{cases}$$

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$$\boxed{h_w[n] = h_1[n] \cdot w[n]} \Rightarrow \text{windowed sinc}$$

High-Pass: (1) design low pass $h_w[n]$

(2) transform $h_w[n] \cdot (-1)^n = e^{-j\pi n}$

general band pass

transform $h_w[n] \cdot \cos(\omega_0 n)$

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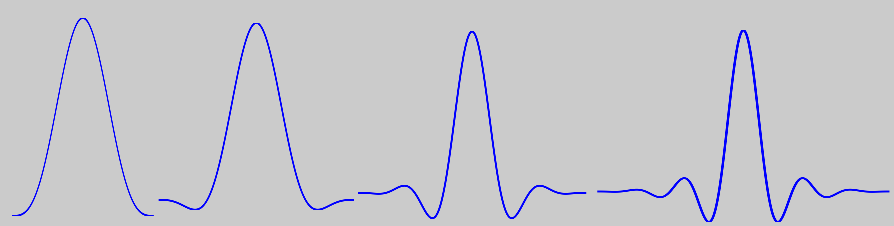
Characterization of Filter Shape

Time-Bandwidth Product

$T(BW) = (M+1)\omega/2\pi$ ⇒ also, total # of zero crossings



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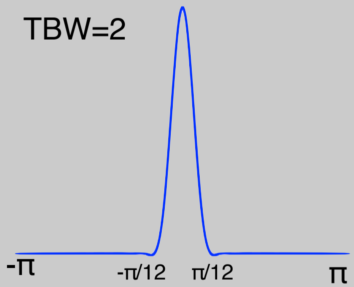
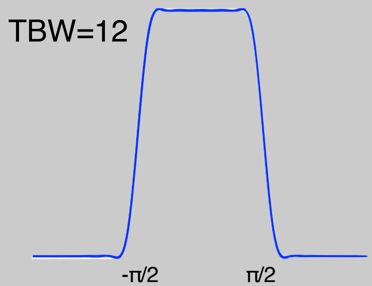
Frequency Response Profile

Q: What are the lengths of these filters in samples?





$2 = (M+1)*(\pi/6) / (2\pi) \Rightarrow M=23$ $\quad$ $12= (M+1)*(\pi) / (2\pi) \Rightarrow M=23$

Note that transition is the same!