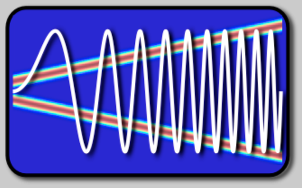
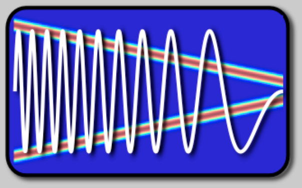


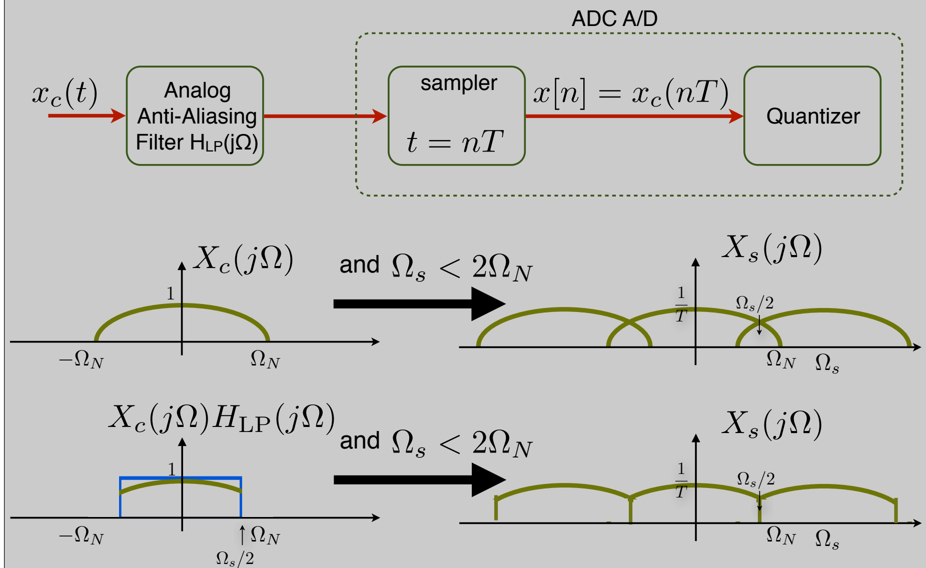
EE123



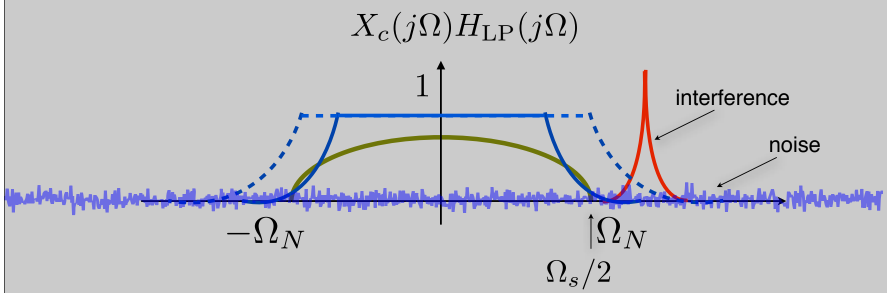
Digital Signal Processing

Lecture 20

Ideal Anti-Aliasing

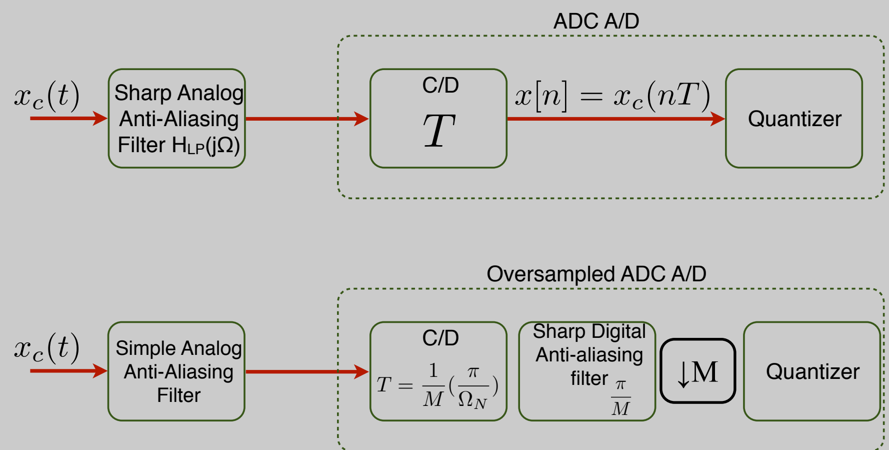


Non Ideal Anti-Aliasing

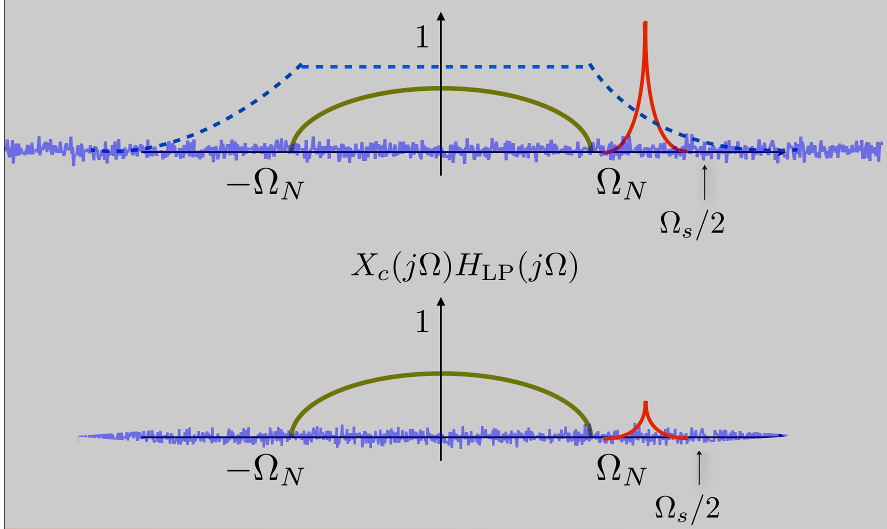


- Problem: Hard to implement sharp analog filter
- Tradeoff:
  - –Crop part of the signal
  - –Suffer from noise and interference (See lab II !)

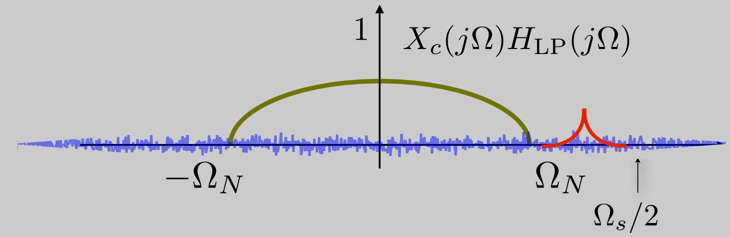
Oversampled ADC



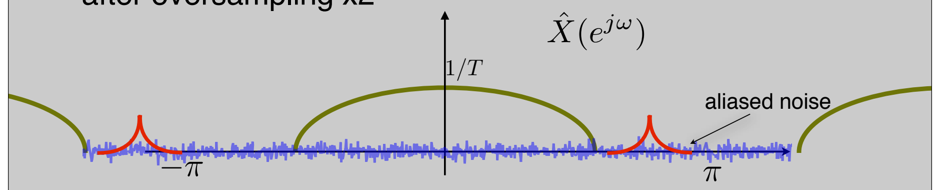
Oversampled ADC



Oversampled ADC

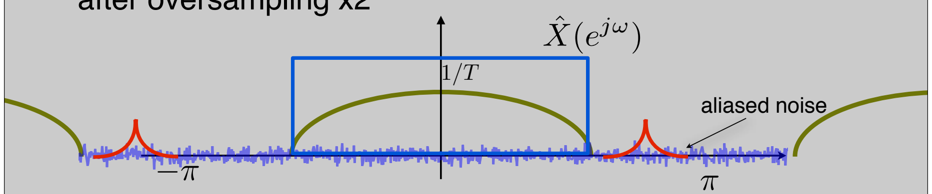


after oversampling x2

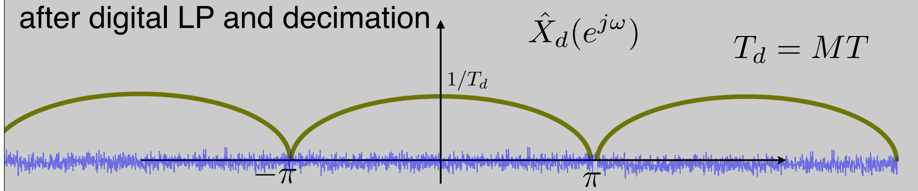


Oversampled ADC

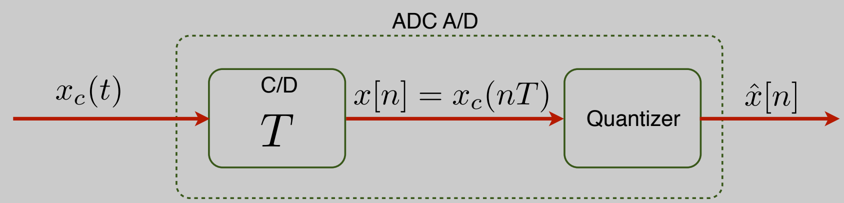
after oversampling x2

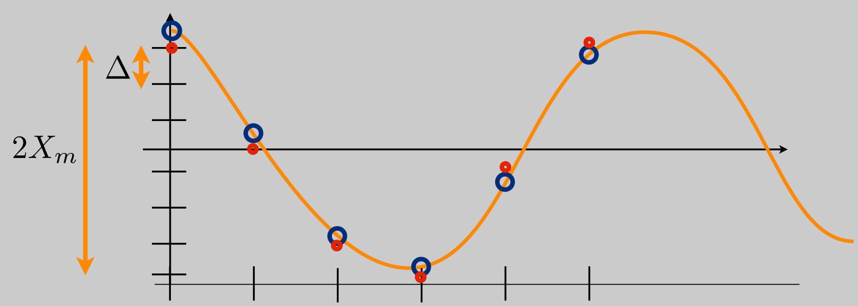


after digital LP and decimation



Sampling and Quantization



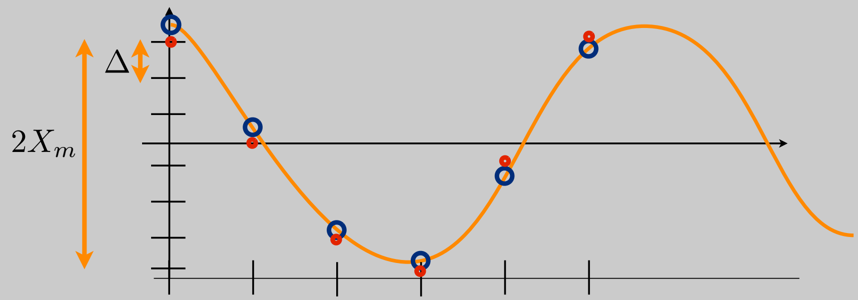


Sampling and Quantization

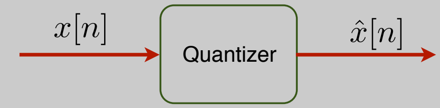
- for 2's complement with B+1 bits $-1 \le \hat{x}_B[n] < 1$

$$\Delta = \frac{2X_m}{2^{B+1}} = \frac{X_m}{2^B}$$

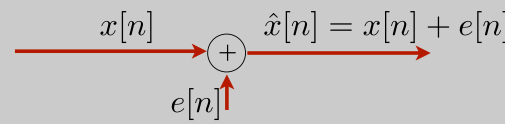
$$\hat{x}[n] = X_m \hat{x}_B[n]$$



Quantization Error



- Model quantization error as noise



- In that case:

$$-\Delta/2 \le e[n] < \Delta/2$$

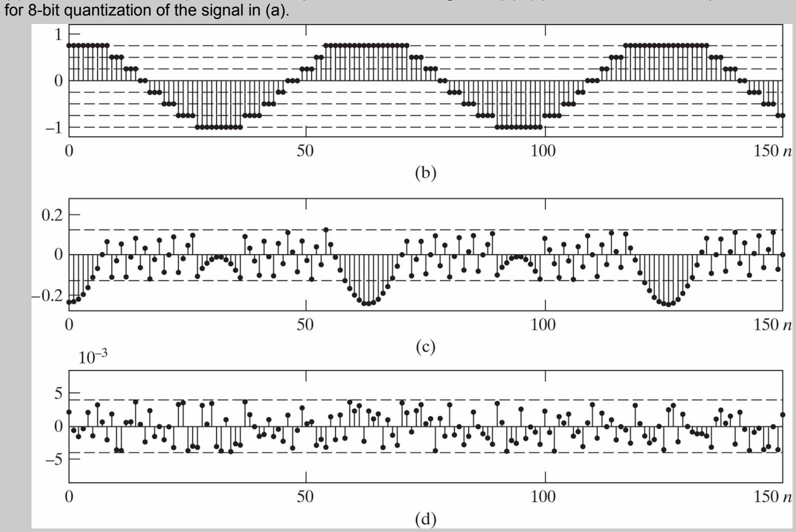
$$(-X_m - \Delta/2) < x[n] \le (X_m - \Delta/2)$$

Noise Model for Quantization Error

- Assumptions:
  - Model e[n] as a sample sequence of a stationary random process
  - e[n] is not correlated with x[n]
  - e[n] not correlated with e[m], e.g., white noise
  - e[n] ~ U[-Δ/2, Δ/2]
- Result:
  - Variance is: $\sigma_e^2 = \frac{\Delta^2}{12}$, or $\sigma_e^2 = \frac{2^{-2B}X_m^2}{12}$ since $\Delta = 2^{-B}X_m$
  - Assumptions work well for signals that change rapidly, are not clipped and for small Δ

Quantization Noise

Figure 4.57 (*continued*) (b) Quantized samples of the cosine waveform in part (a) with a 3-bit quantizer. (c) Quantization error sequence for 3-bit quantization of the signal in (a). (d) Quantization error sequence for 8-bit quantization of the signal in (a).



SNR of Quantization Noise

- For uniform B+1 bits quantizer: $\sigma_e^2 = \frac{2^{-2B} X_m^2}{12}$

$$
\begin{aligned}
SNR_Q &= 10\log_{10}\left(\frac{\sigma_x^2}{\sigma_e^2}\right) \\
&= 10\log_{10}\left(\frac{12 \cdot 2^{2B}\sigma_x^2}{X_m^2}\right)
\end{aligned}
$$

$$\mathrm{SNR}_Q = 6.02B + 10.8 - 20\log_{10}\left(\frac{X_m}{\sigma_x}\right)$$

Quantizer range (X_m), rms of amp (σ_x)

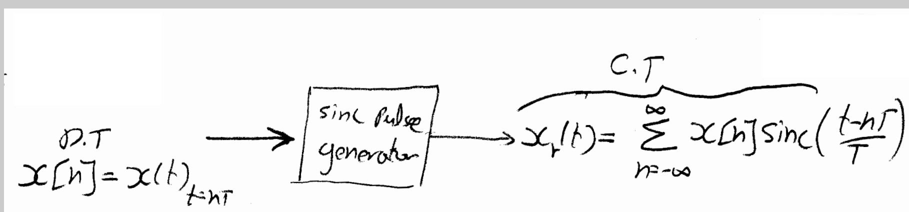
SNR of Quantization Noise

$$\mathrm{SNR}_Q = 6.02B + 10.8 - 20\log_{10}\left(\frac{X_m}{\sigma_x}\right)$$

Quantizer range (X_m), rms of amp (σ_x)

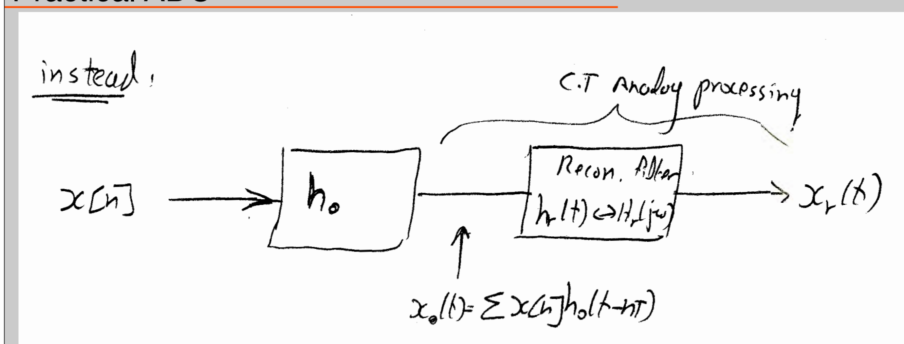
- Improvement of 6dB with every bit
- The range of the quantization must be adapted to the rms amplitude of the signal
  - Tradeoff between clipping and noise!
  - If $\sigma_x = X_m/4$ then $\mathrm{SNR}_Q \approx 6B - 1.25\mathrm{dB}$ so SNR of 90-96 dB requires 16-bits (audio)

Practical ADC (Ch. 4.8.4)

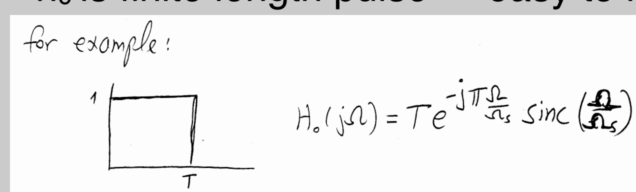


- Scaled train of sinc pulses
- Difficult to generate sinc ⇒ Too long!

Practical ADC

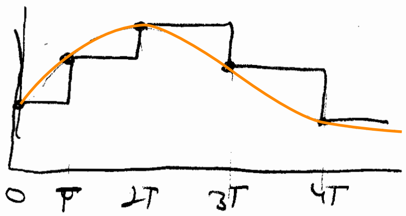


- h_0 is finite length pulse ⇒ easy to implement



Practical ADC

output zero-order-hold



$$x_0(t) = \sum_{n=-\infty}^{\infty} x[n]\, h_0(t-nT) = h_0(t) * x_s(t)$$

taking FT:

$$X_0(j\Omega) = H_0(j\Omega)\cdot X_s(j\Omega) =$$

$$= H_0(j\Omega)\cdot \frac{1}{T}\sum_k X(j(\Omega - k\Omega_s))$$

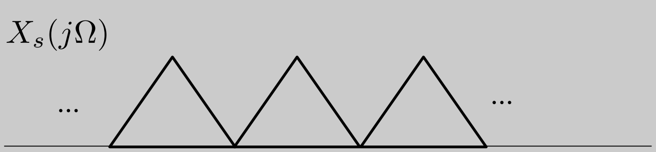
Practical ADC

output of recon filter:

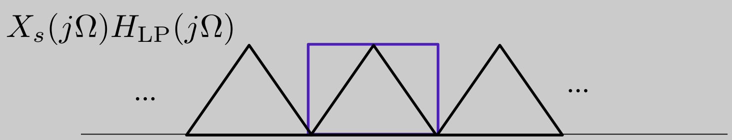
$$X_r(j\Omega) = H_r(j\Omega)\cdot H_0(j\Omega)\cdot X_s(j\Omega) =$$

$$= \underbrace{H_r(j\Omega)}_{\text{recon filter}} \cdot \underbrace{T e^{-j\pi\frac{\Omega}{\Omega_s}} \mathrm{sinc}\left(\frac{\Omega}{\Omega_s}\right)}_{\text{from zero order hold}} \cdot \underbrace{\frac{1}{T}\sum X(j(\Omega - k\Omega_s))}_{\text{shifted copies from sampling}}$$

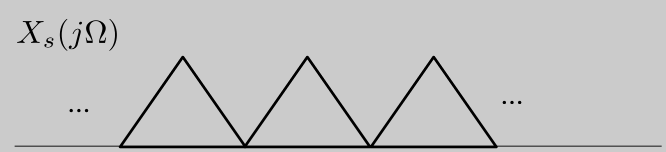
Practical ADC



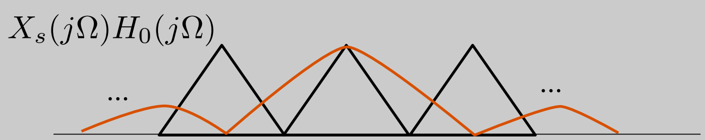
Ideally:



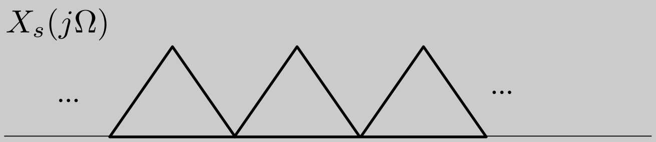
Practical ADC



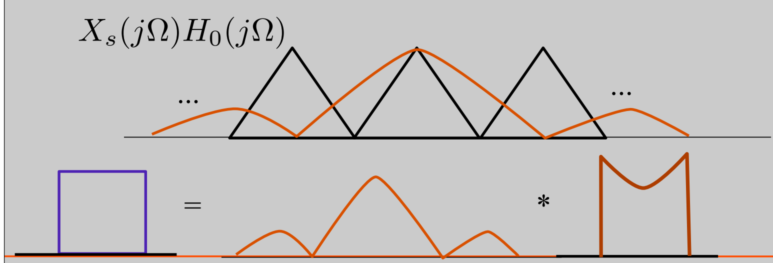
Practically:



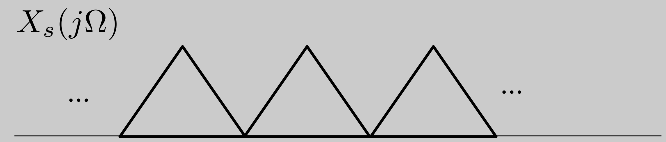
Practical ADC



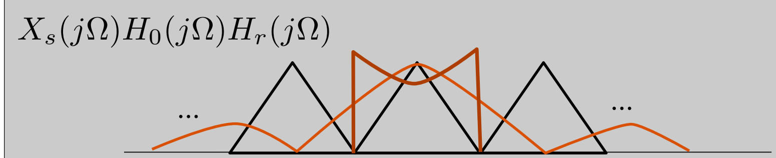
Practically:



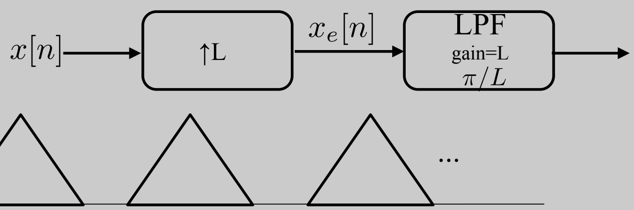
Practical ADC



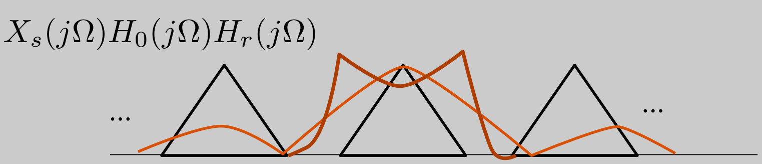
Practically:



Easier Implementation with Digital upsampling



Practically:



Easier Implementation with Digital upsampling