EE123

Digital Signal Processing

Lecture 19

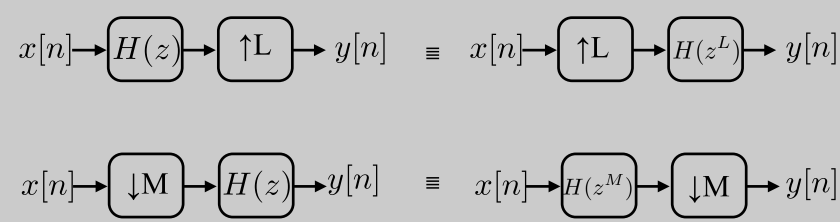
Topics

- Last time
  - Upsampling
  - Resampling by rational fraction
- Today
  - Interchanging Compressors/Expanders with filtering
  - Polyphase decomposition
  - Multi-rate processing

Multi-Rate Signal Processing

- What if we want to resample by 1.01T?
  - Expand by L=100
  - Filter π/101 ($$$$$)
  - Downsample by M=101
- Fortunately there are ways around it!
  - Called multi-rate
  - Uses compressors, expanders and filtering

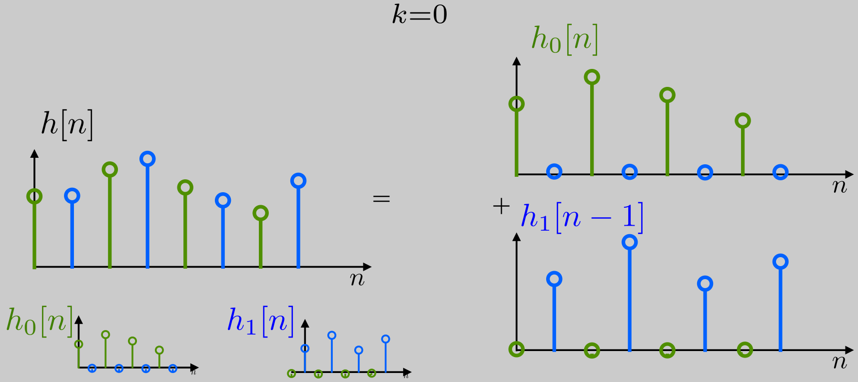
Interchanging Operations



Polyphase Decomposition

- We can decomposed an impulse response to:

$$h[n] = \sum_{k=0}^{M-1} h_k[n-k]$$

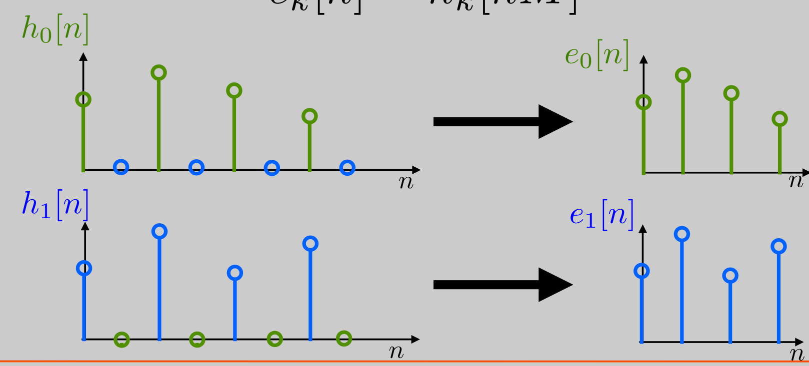


Polyphase Decomposition

- Define:

$$h_k[n] \rightarrow \boxed{\downarrow \mathrm{M}} \rightarrow e_k[n]$$

$$e_k[n] = h_k[nM]$$



Polyphase Decomposition

$$e_k[n] \rightarrow \boxed{\uparrow \mathrm{M}} \rightarrow h_k[n]$$

recall upsampling ⇒ scaling

$$H_k(z) = E_k(z^M)$$

Also, recall:

$$h[n] = \sum_{k=0}^{M-1} h_k[n-k]$$

So,

$$H(z) = \sum_{k=0}^{M-1} E_k(z^M) z^{-k}$$

Polyphase Decomposition

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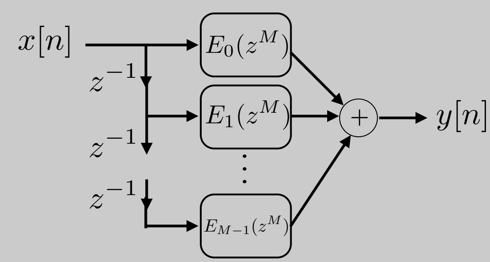
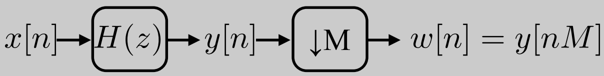


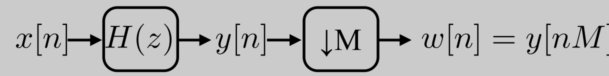
Why should you care?

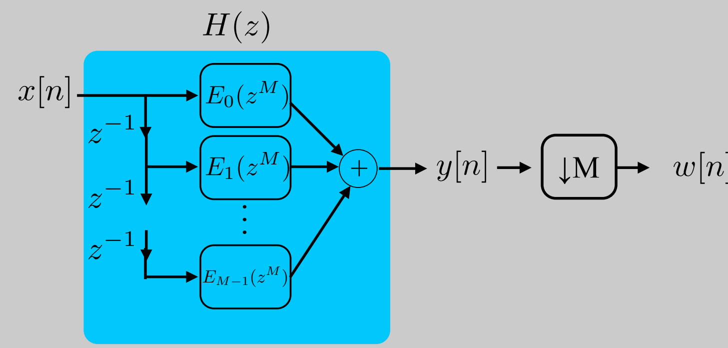
Polyphase Implementation of Decimation



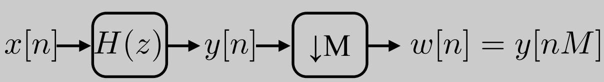
- Problem:
  - –Compute all y[n] and then throw away -- wasted computation!
    - For FIR length N ⇒ N mults/unit time
  - –Can interchange Filter with compressor?
    - Not in general!

Polyphase Implementation of Decimation

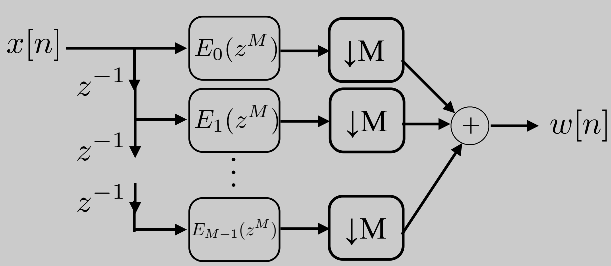




Polyphase Implementation of Decimation

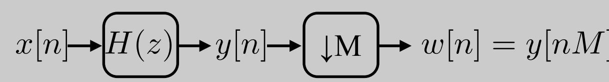


Interchange filter with decimation

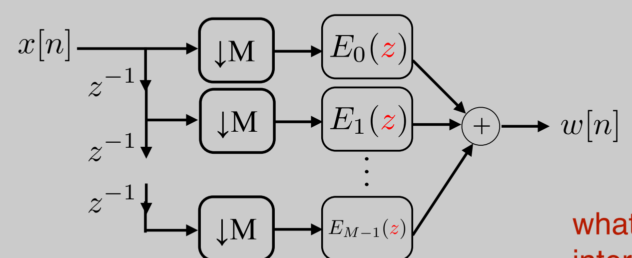


now, what can we do?

Polyphase Implementation of Decimation



Interchange filter with decimation

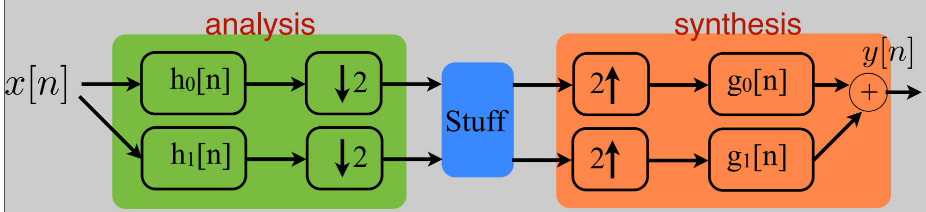


what about interpolation?

Computation:
Each Filter: N/M *(1/M) mult/unit time
Total: N/M mult/unit time

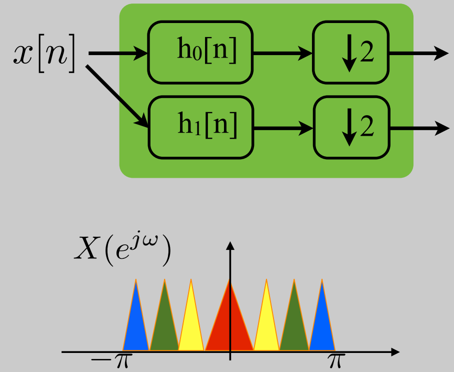
Multirate FilterBank

- $h_0[n]$ is low-pass, $h_1[n]$ is high-pass
- Often $h_1[n] = e^{j\pi n} h_0[n]$ or $H_1(e^{j\omega}) = H_0(e^{j(w-\pi)})$



Subtleties in Time-Freq Tiling

- Assume h_0, h_1 are ideal low,high pass filters



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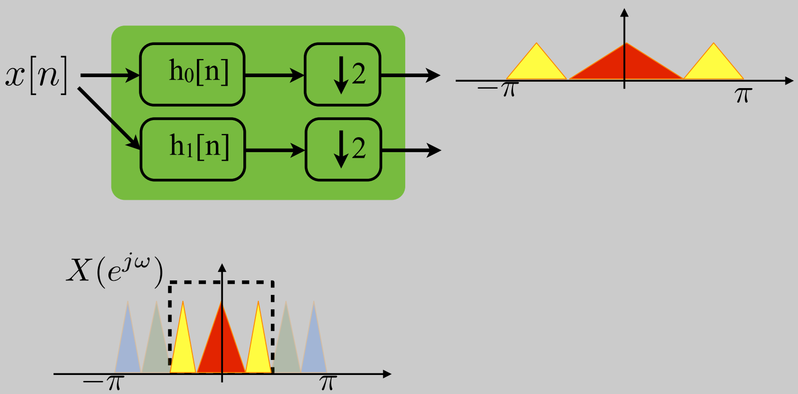

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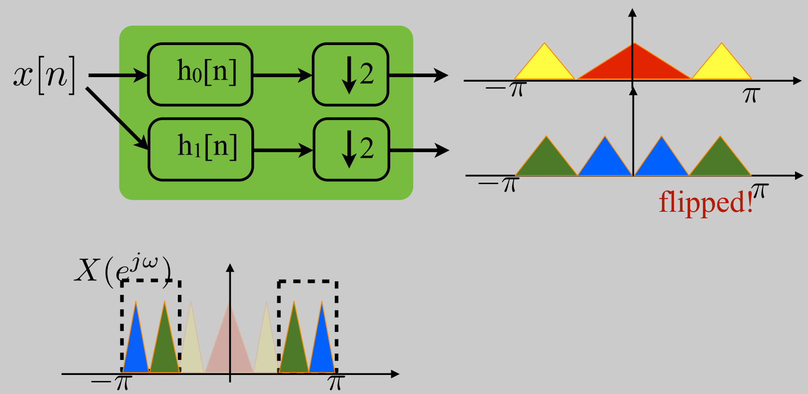

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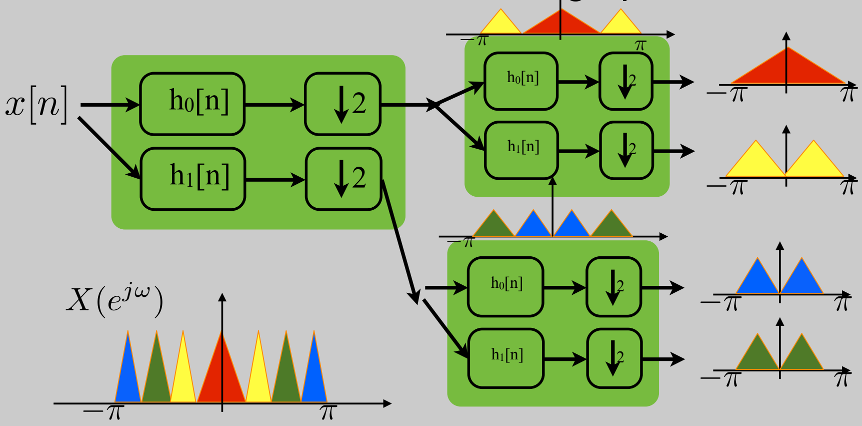
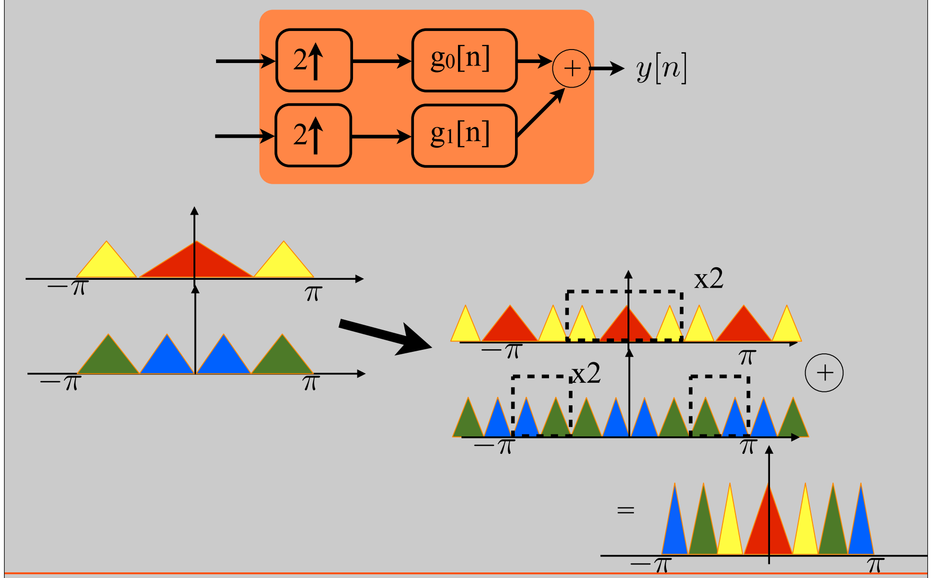
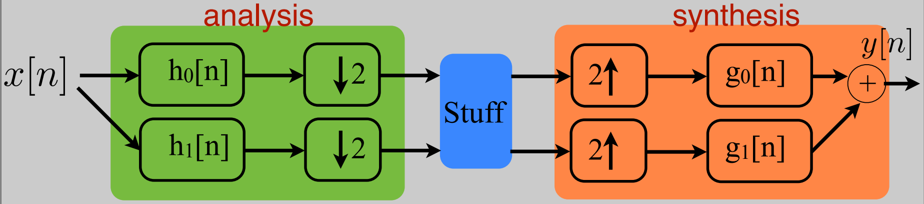


Perfect Reconstruction Ideal Filters



Perfect Reconstruction non-Ideal Filters

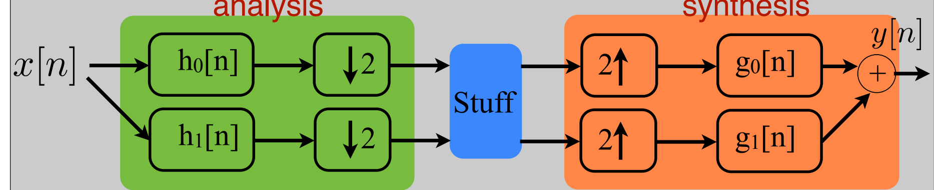


$$Y(e^{j\omega}) = \frac{1}{2}\left[G_0(e^{j\omega})H_0(e^{j\omega}) + G_1(e^{j\omega})H_1(e^{j\omega})\right] X(e^{j\omega})$$
$$+\frac{1}{2}\left[G_0(e^{j\omega})H_0(e^{j(\omega-\pi)}) + G_1(e^{j\omega})H_1(e^{j(\omega-\pi)})\right] X(e^{j(\omega-\pi)})$$

need to cancel!

aliasing

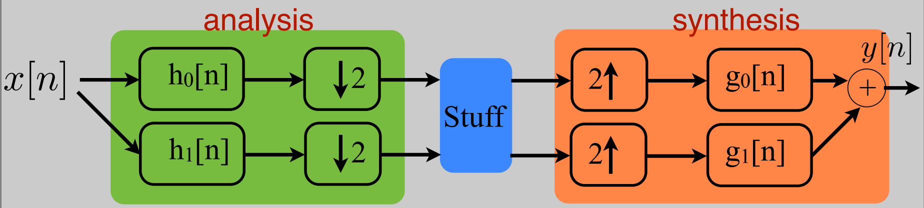
Quadrature Mirror Filters - perfect recon



QMF - mirror around pi/2

$$H_1(e^{j\omega}) = H_0(e^{j(\omega-\pi)})$$
$$G_0(e^{j\omega}) = 2H_0(e^{j\omega})$$
$$G_1(e^{j\omega}) = -2H_1(e^{j\omega})$$

Quadrature Mirror Filters - perfect recon



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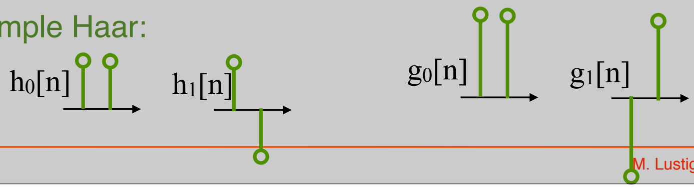
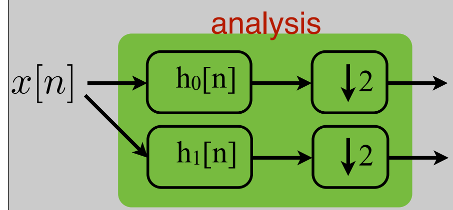
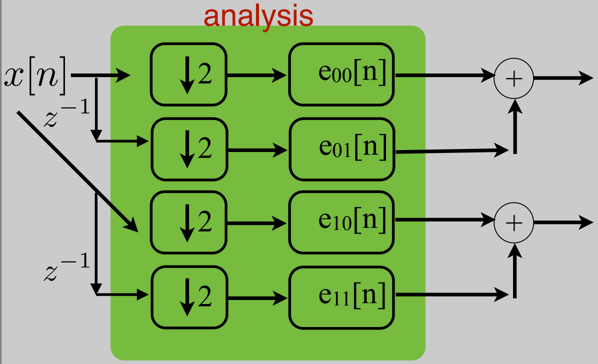
Example Haar:





$$e_{00} = h_0[2n]$$
$$e_{01} = h_0[2n+1]$$
$$e_{10} = h_1[2n] = e^{j2\pi n}h_0[2n] = e_{00}[n]$$
$$e_{11} = h_1[2n+1] = e^{j2\pi n}e^{j\pi}h_0[2n+1] = -e_{01}[n]$$

Polyphase Filter-Bank



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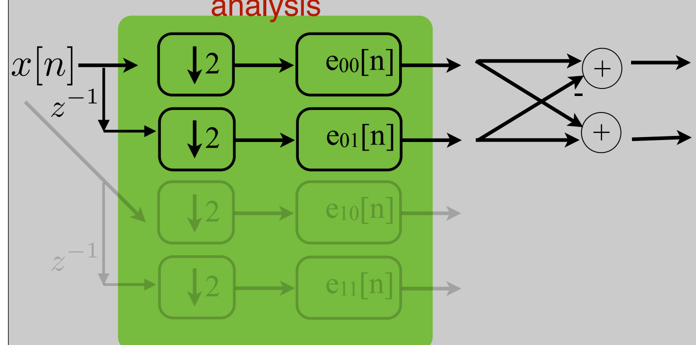
Polyphase Filter-Bank



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