EE123

Digital Signal Processing

Lecture 18

Topics

- Last time
  - Upsampling
  - Resampling by rational fraction
- Today
  - Interchanging Compressors/Expanders with filtering
  - Polyphase decomposition
  - Multi-rate processing

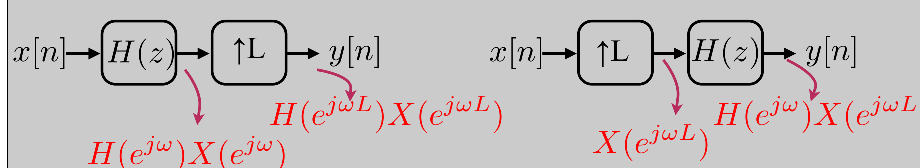
Multi-Rate Signal Processing

- What if we want to resample by 1.01T?
  - Expand by L=100
  - Filter π/101 ($$$$$)
  - Downsample by M=101
- Fortunately there are ways around it!
  - Called multi-rate
  - Uses compressors, expanders and filtering

Interchanging Operations

not LTI!

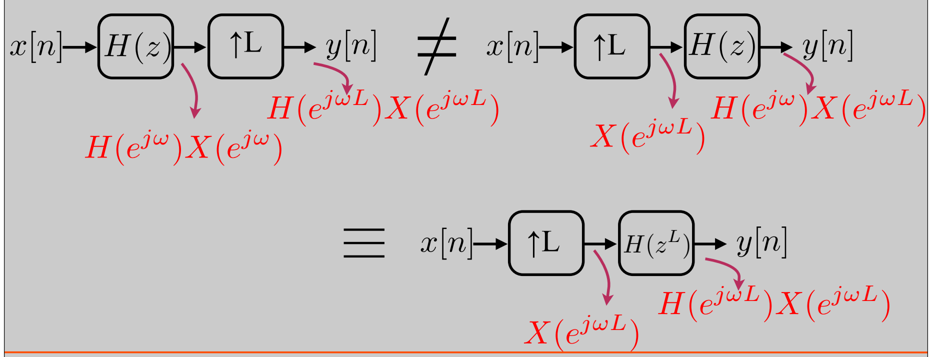
Note:



Interchanging Operations

"expander" "compressor" not LTI!

Note:

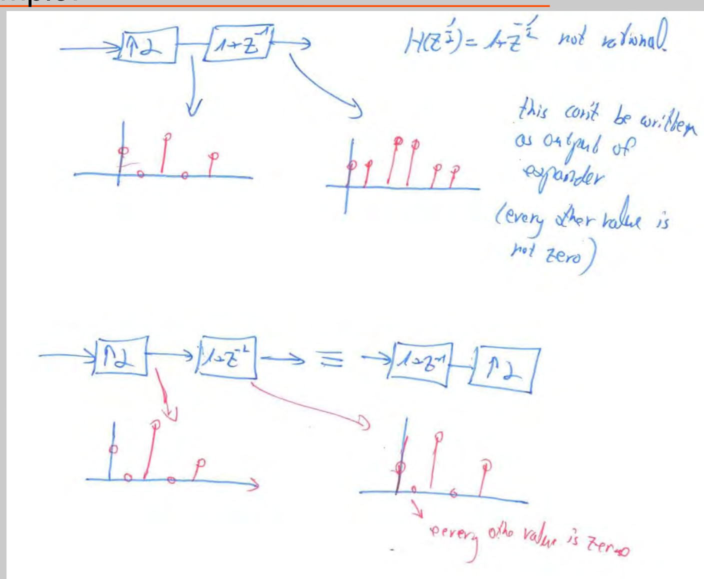


Interchanging Filter Expander

- Q: Can we move expander from Left to Right (with xform)?

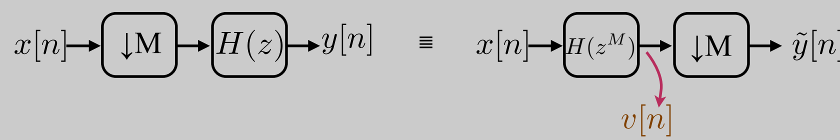
- A: Yes, if $H(z^{1/L})$ is rational
  No, otherwise

Example:



Compressor

Claim:



Proof:

Compressor

Proof:

$$Y(e^{j\omega}) = H(e^{j\omega})\left(\frac{1}{M}\sum_{i=0}^{M-1} X\left(e^{j\left(\frac{\omega}{M}-\frac{2\pi i}{M}\right)}\right)\right) =$$

$$= \frac{1}{M}\sum_{i=0}^{M-1} \underbrace{H\left(e^{j(\omega-2\pi i)}\right)}_{=H(e^{j\omega})} X\left(e^{j\left(\frac{\omega}{M}-\frac{2\pi i}{M}\right)}\right)$$

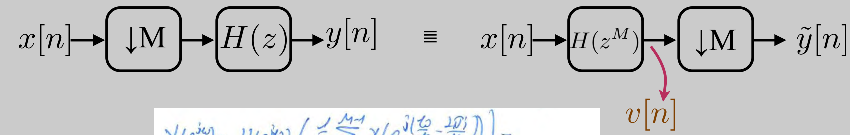
$$= \frac{1}{M}\sum_{i=0}^{M-1} H\left(e^{jM\left(\frac{\omega}{M}-\frac{2\pi i}{M}\right)}\right) X\left(e^{j\left(\frac{\omega}{M}-\frac{2\pi i}{M}\right)}\right)$$

$$V(e^{j\omega}) = H(e^{j\omega M})\, X(e^{j\omega})$$

after compressor

Compressor

Claim:



Proof:

$$Y(e^{j\omega}) = H(e^{j\omega})\left(\frac{1}{M}\sum_{i=0}^{M-1} X\left(e^{j\left(\frac{\omega}{M}-\frac{2\pi i}{M}\right)}\right)\right) =$$

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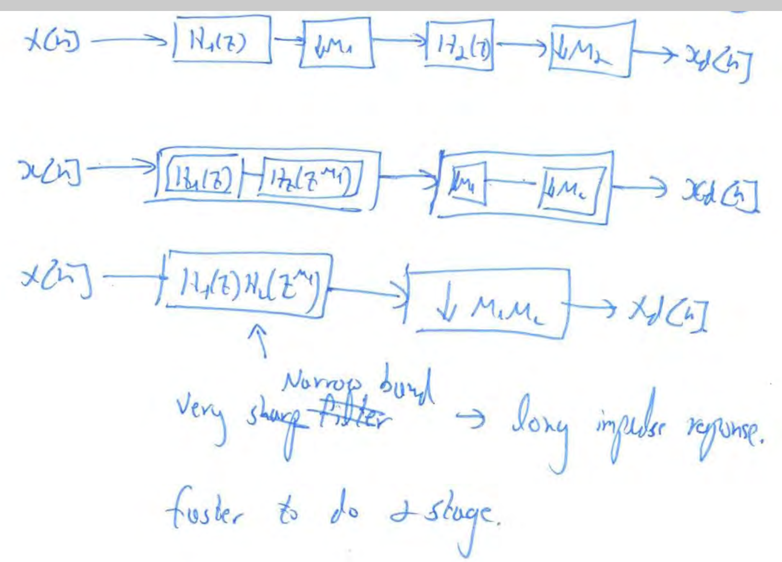
$$= \frac{1}{M}\sum_{i=0}^{M-1} H\left(e^{jM\left(\frac{\omega}{M}-\frac{2\pi i}{M}\right)}\right) X\left(e^{j\left(\frac{\omega}{M}-\frac{2\pi i}{M}\right)}\right)$$

$$V(e^{j\omega}) = H(e^{j\omega M})\, X(e^{j\omega})$$

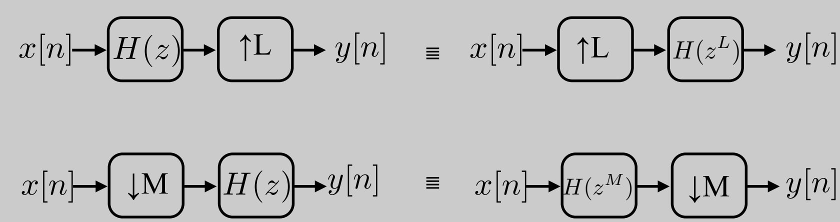
after compressor

Q: How compress from right to left?
A: only if $H(z^{1/M})$ rational.

Multi-Rate Filtering



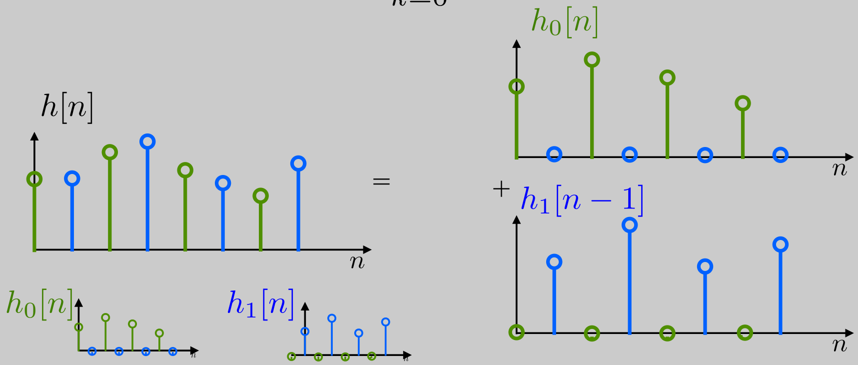
Interchanging Operations



Polyphase Decomposition

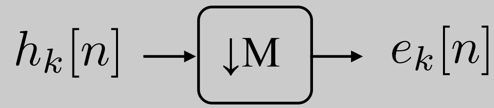
- We can decomposed an impulse response to:

$$h[n] = \sum_{k=0}^{M-1} h_k[n-k]$$

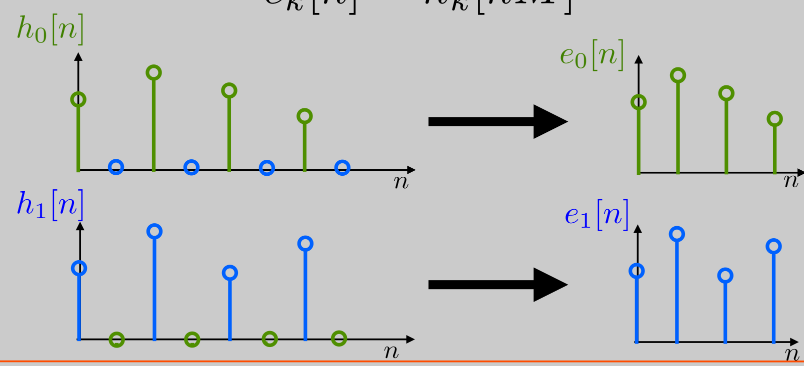


Polyphase Decomposition

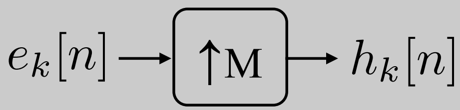
- Define:



$$e_k[n] = h_k[nM]$$



Polyphase Decomposition



recall upsampling ⇒ scaling

$$H_k(z) = E_k(z^M)$$

Also, recall:

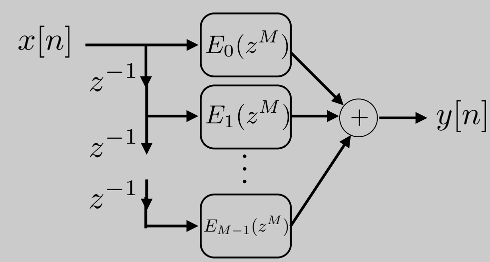
$$h[n] = \sum_{k=0}^{M-1} h_k[n-k]$$

So,

$$H(z) = \sum_{k=0}^{M-1} E_k(z^M) z^{-k}$$

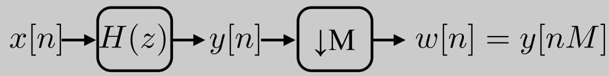
Polyphase Decomposition

$$H(z) = \sum_{k=0}^{M-1} E_k(z^M) z^{-k}$$



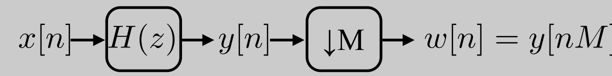
Why should you care?

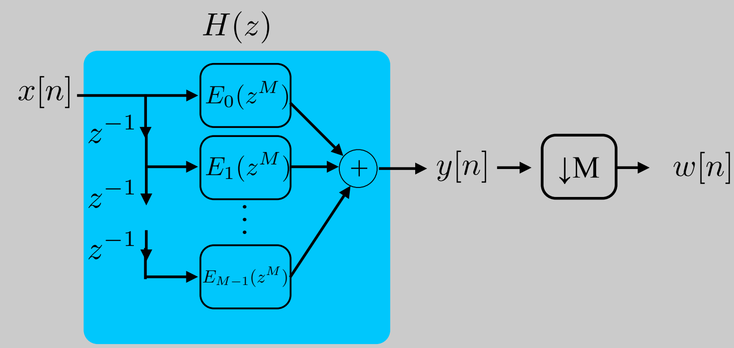
Polyphase Implementation of Decimation



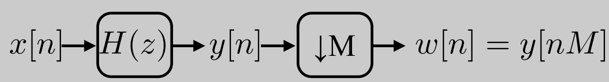
- Problem:
  - –Compute all y[n] and then throw away -- wasted computation!
    - For FIR length N ⇒ N mults/unit time
  - –Can interchange Filter with compressor?
    - Not in general!

Polyphase Implementation of Decimation

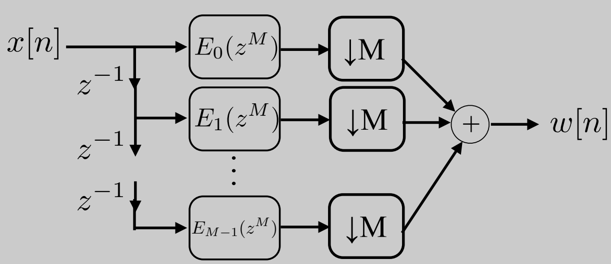




Polyphase Implementation of Decimation

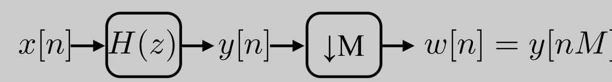


Interchange sum with decimation

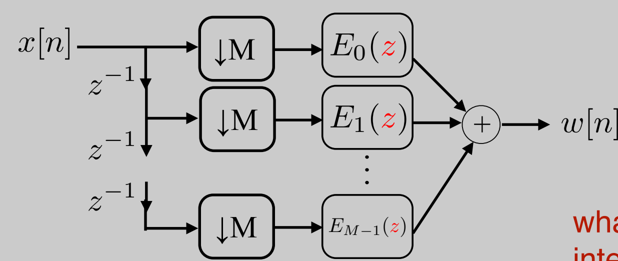


now, what can we do?

Polyphase Implementation of Decimation



Interchange filter with decimation



what about interpolation?

Computation:
Each Filter: N/M *(1/M) mult/unit time
Total: N/M mult/unit time