EE123
Digital Signal Processing

Lecture 16

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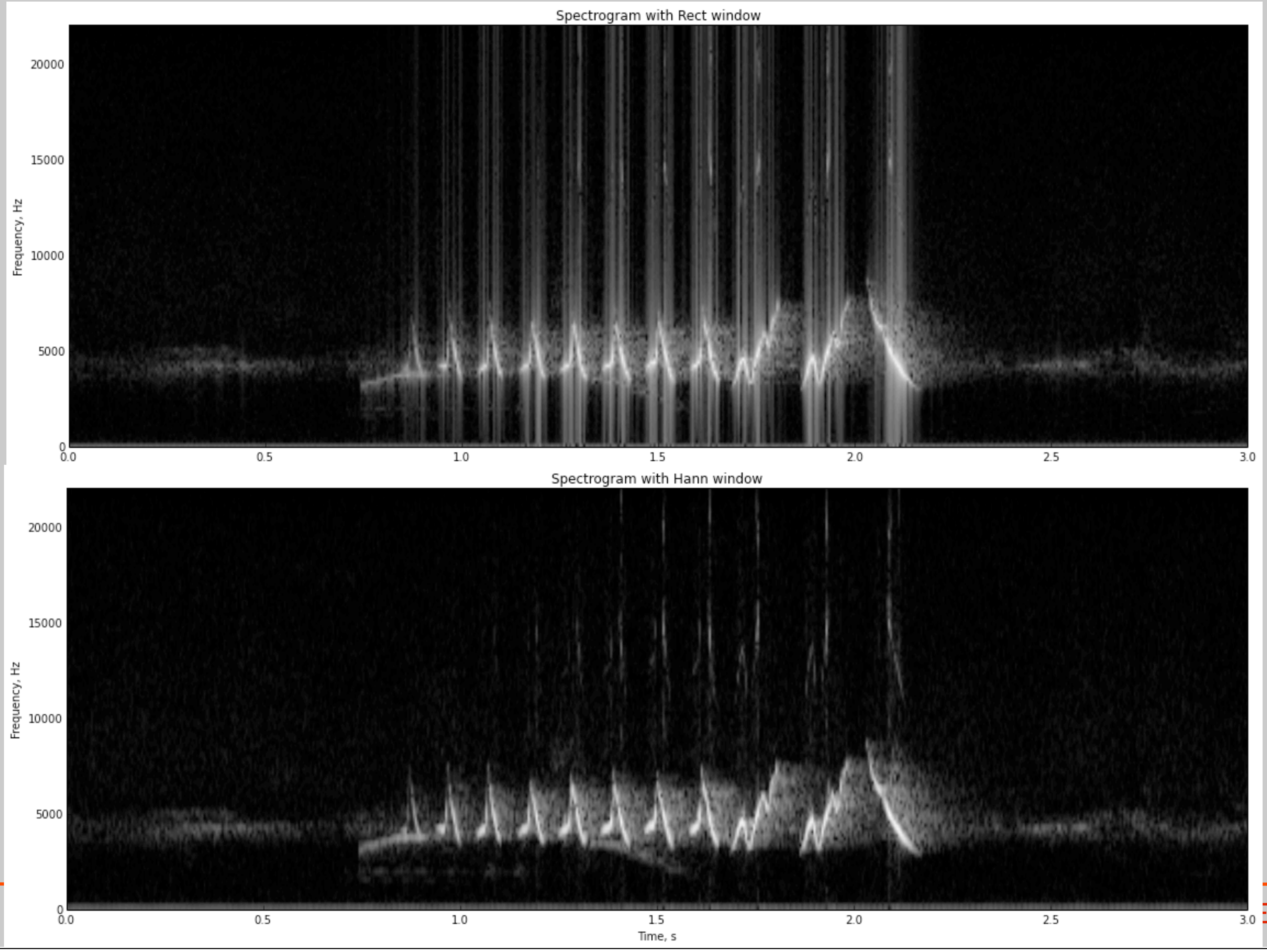
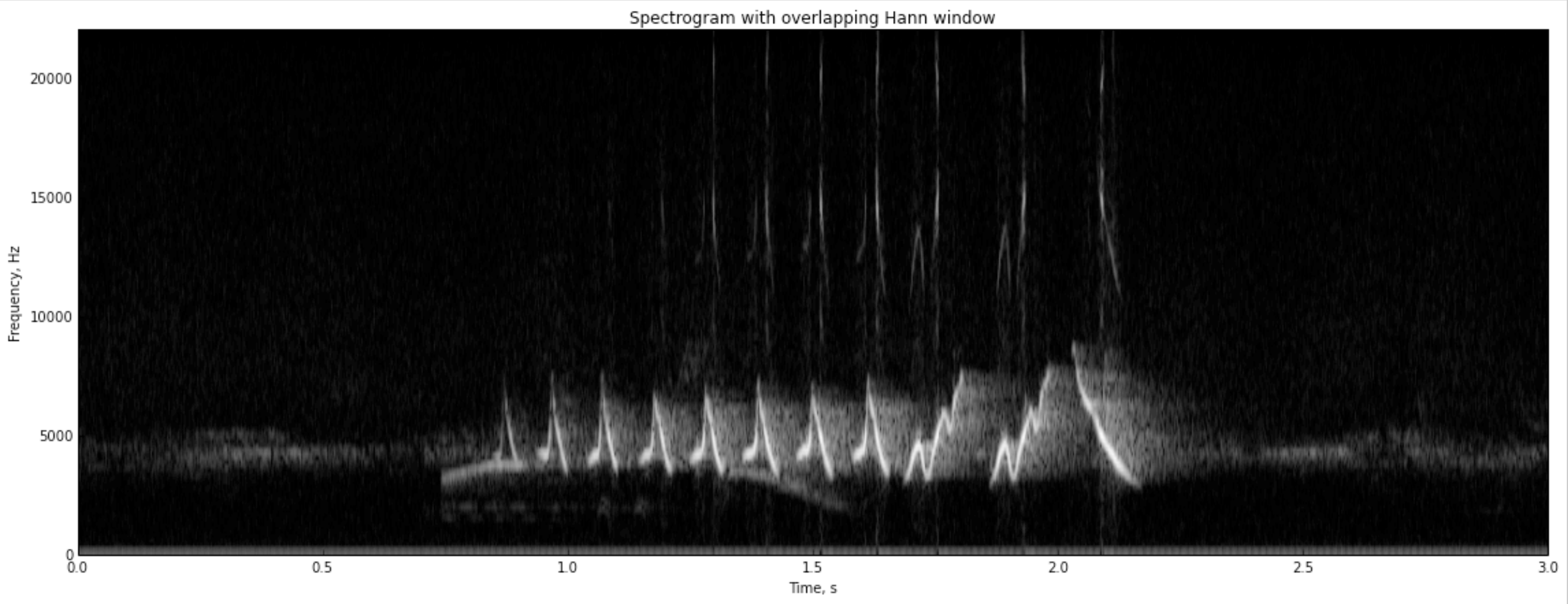
Lab II - Time-Frequency

- compute spectrograms with w/o windowing



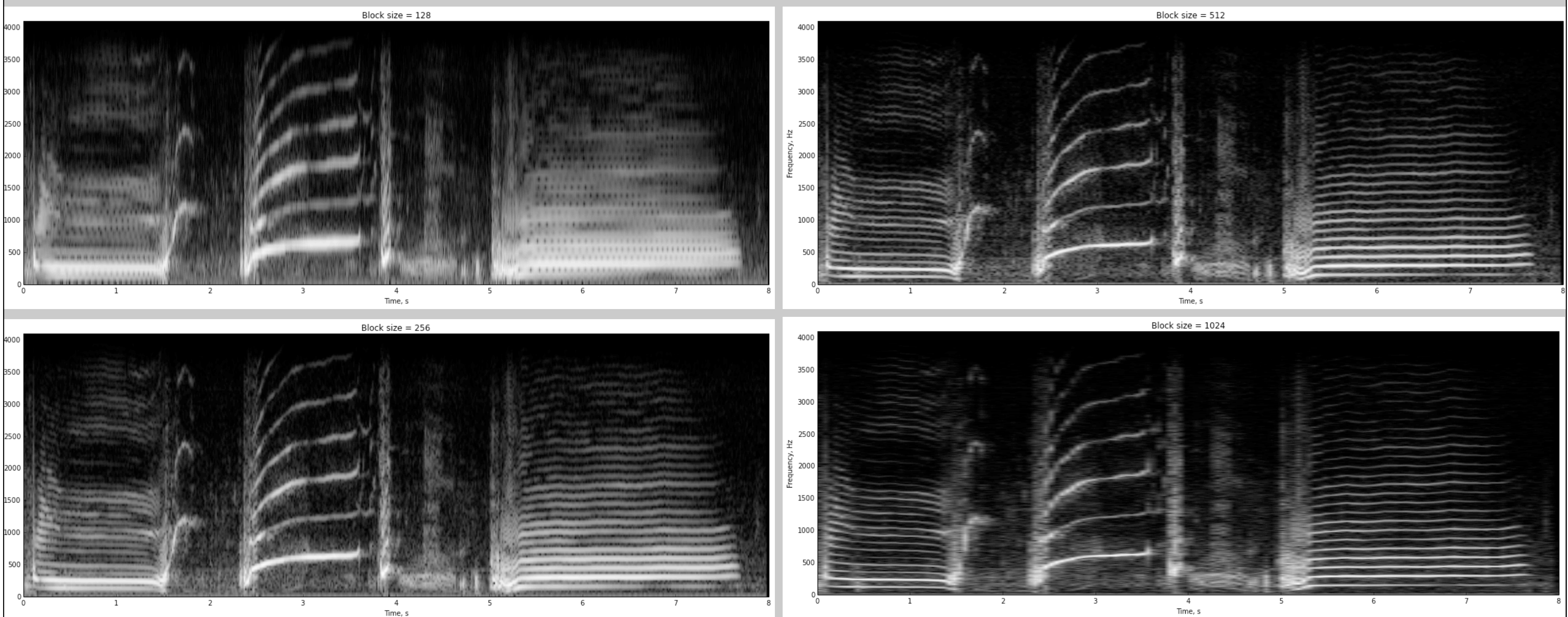
Lab II - Time-Frequency

- Compute with overlapping window

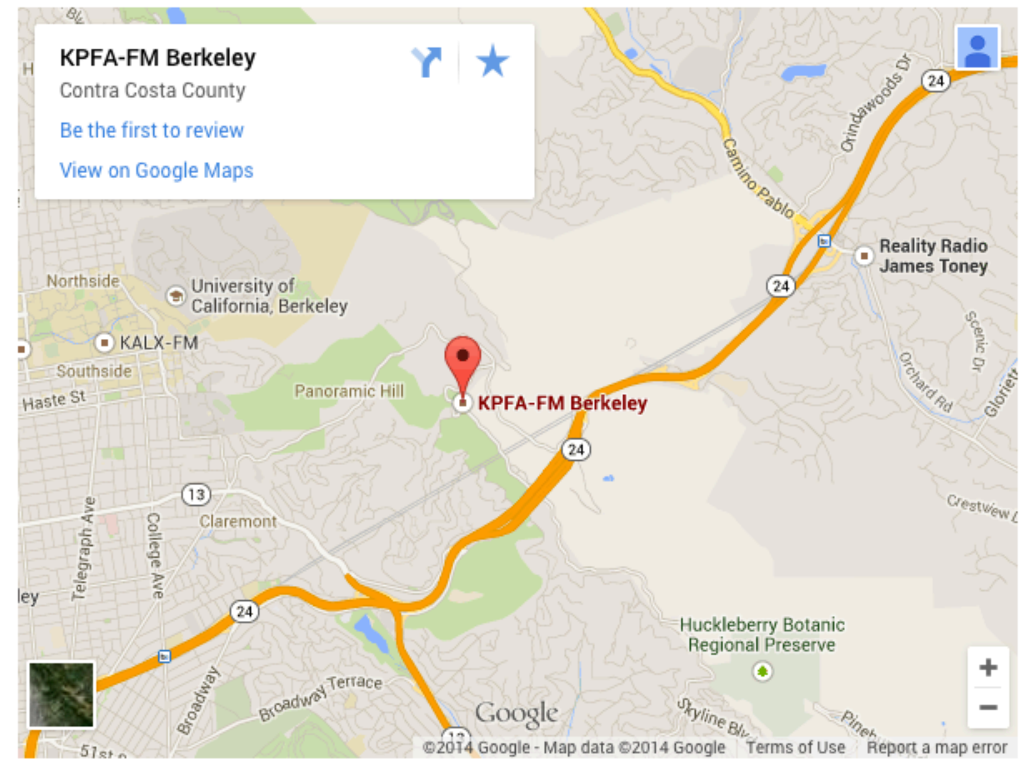


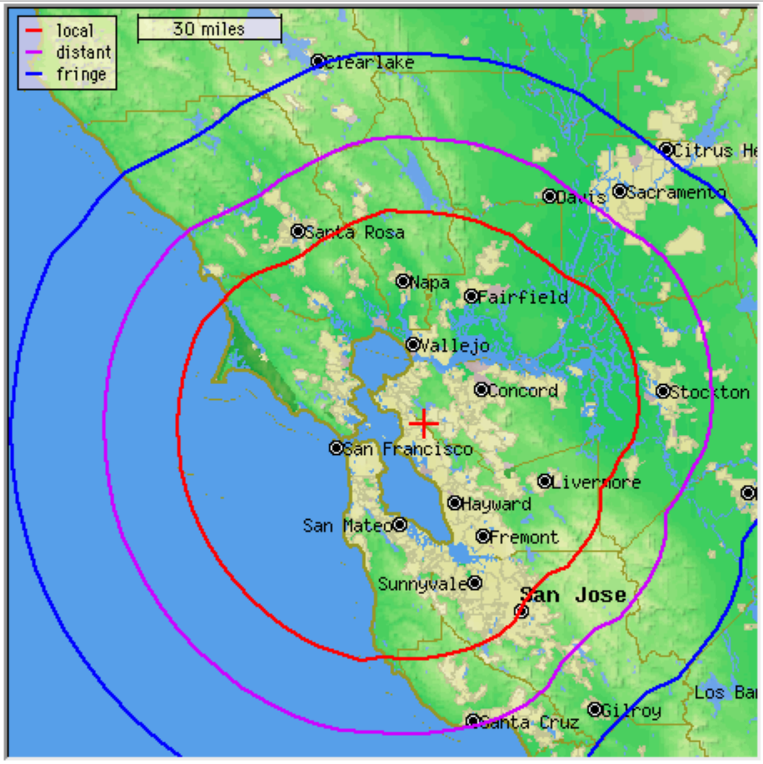
Lab II - Time-Frequency

- Look at temporal/frequency resolution tradeoffs:

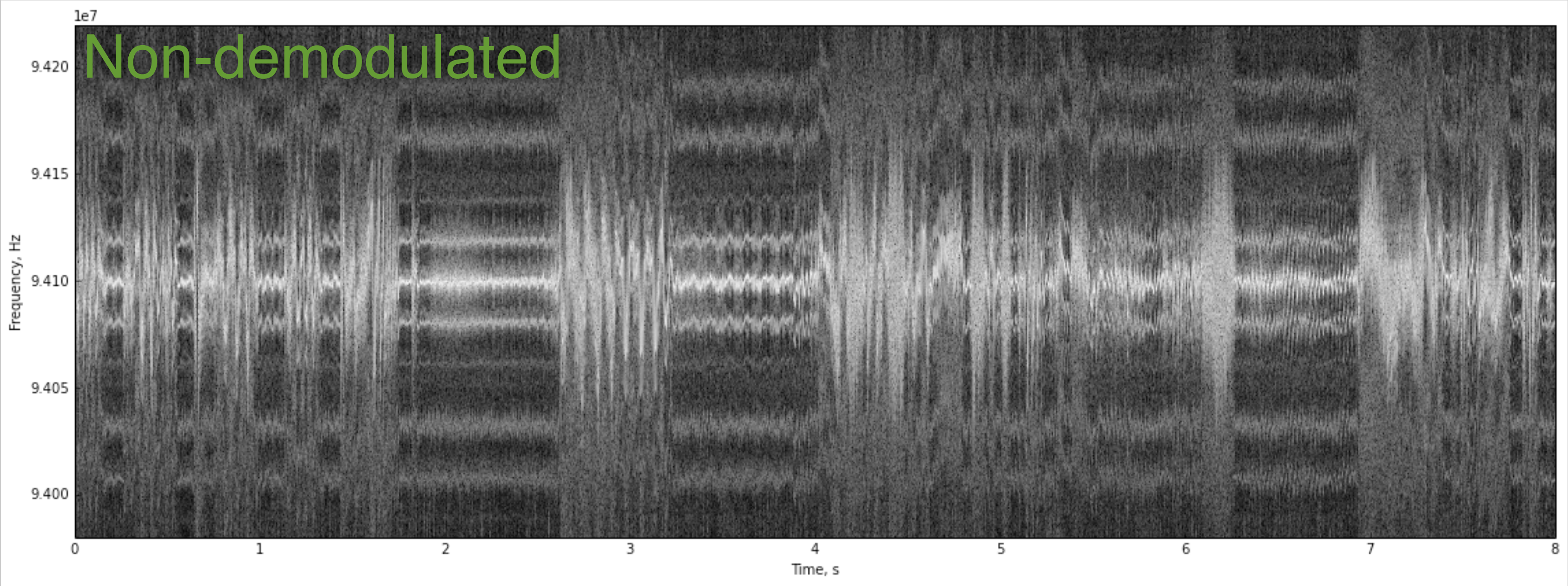


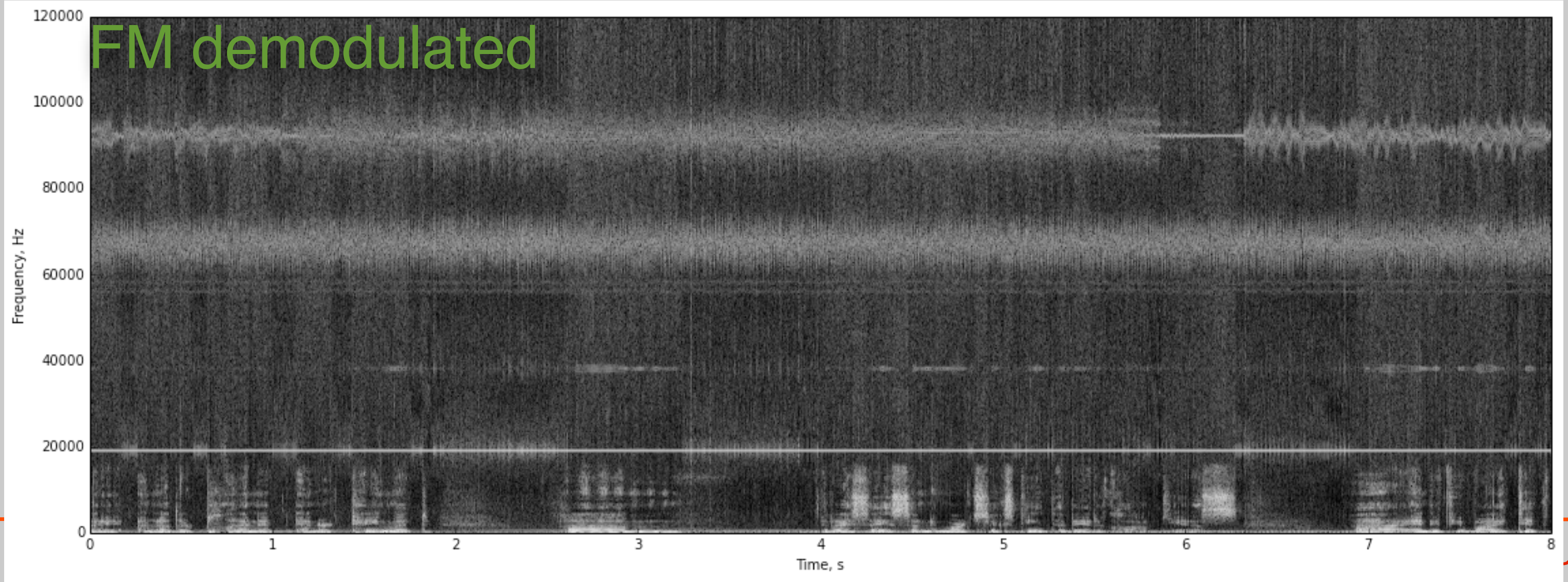
FM Broadcast Radio - KPFA 94.1MHz



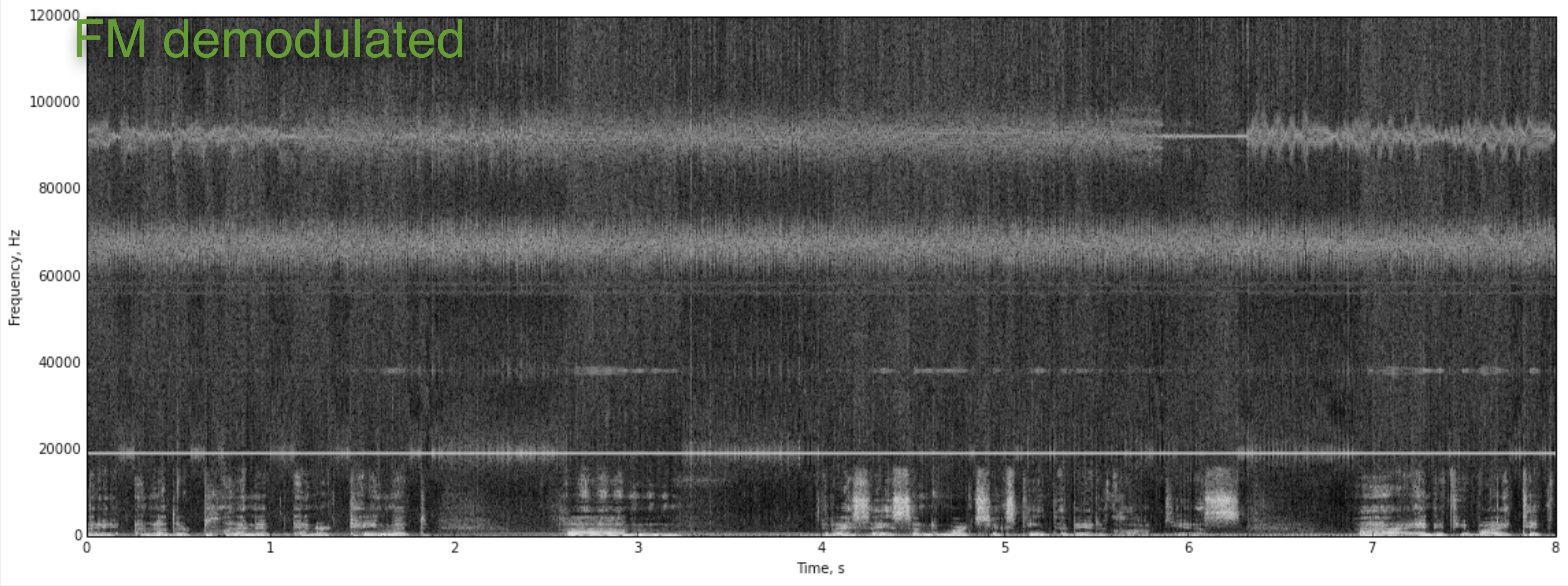


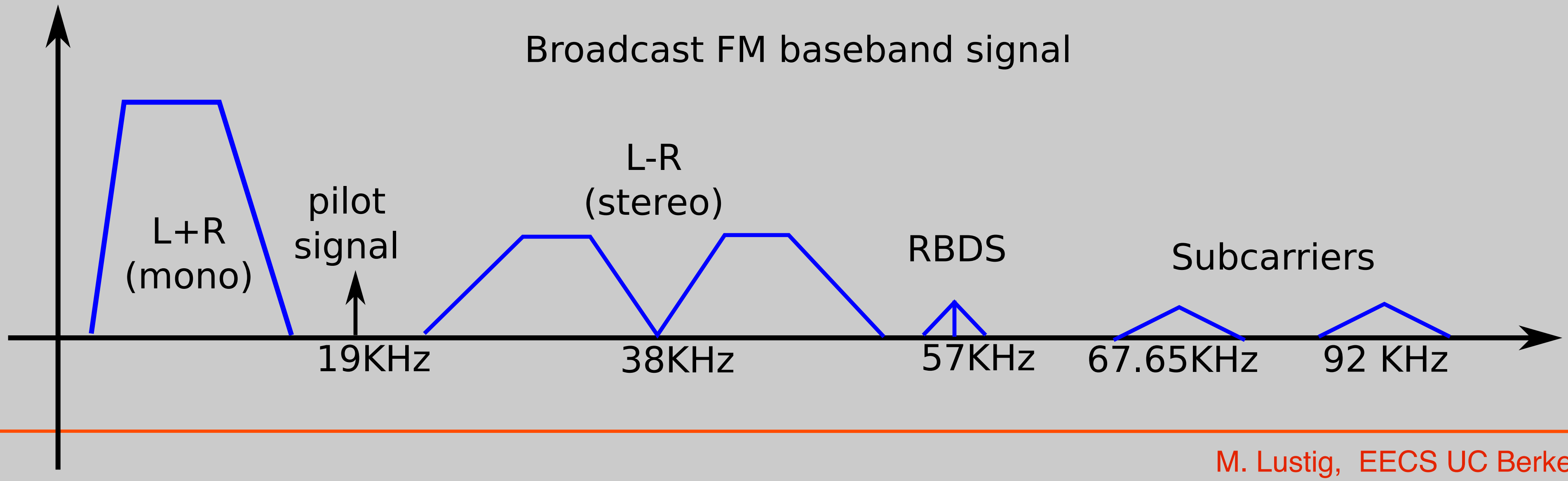
Spectrogram of Broadcast FM





Spectrogram of Broadcast FM

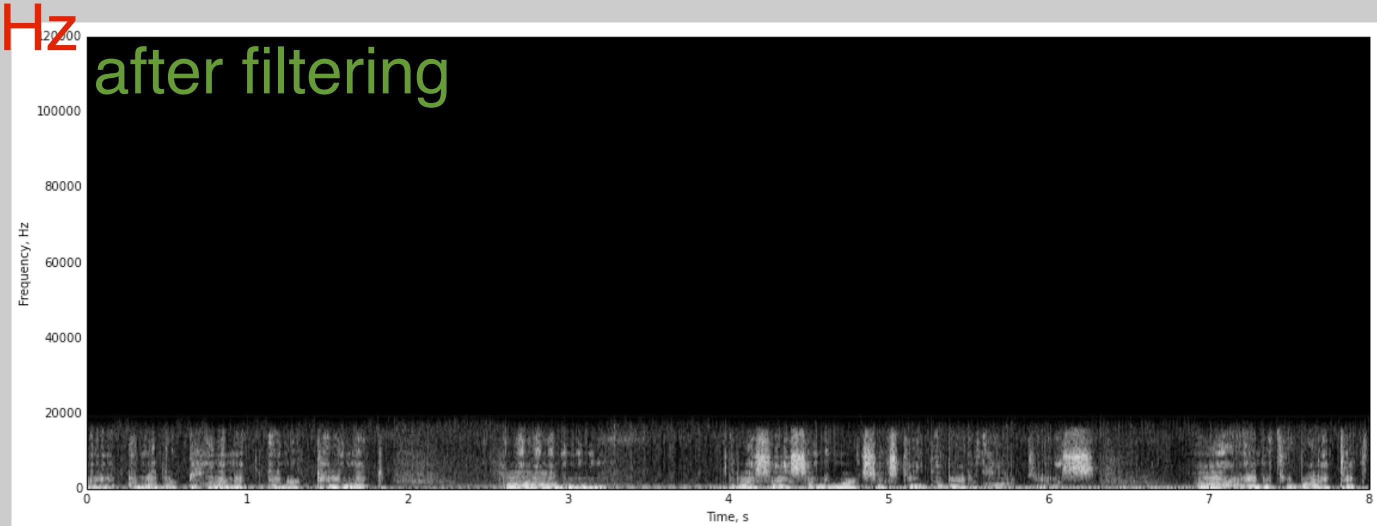




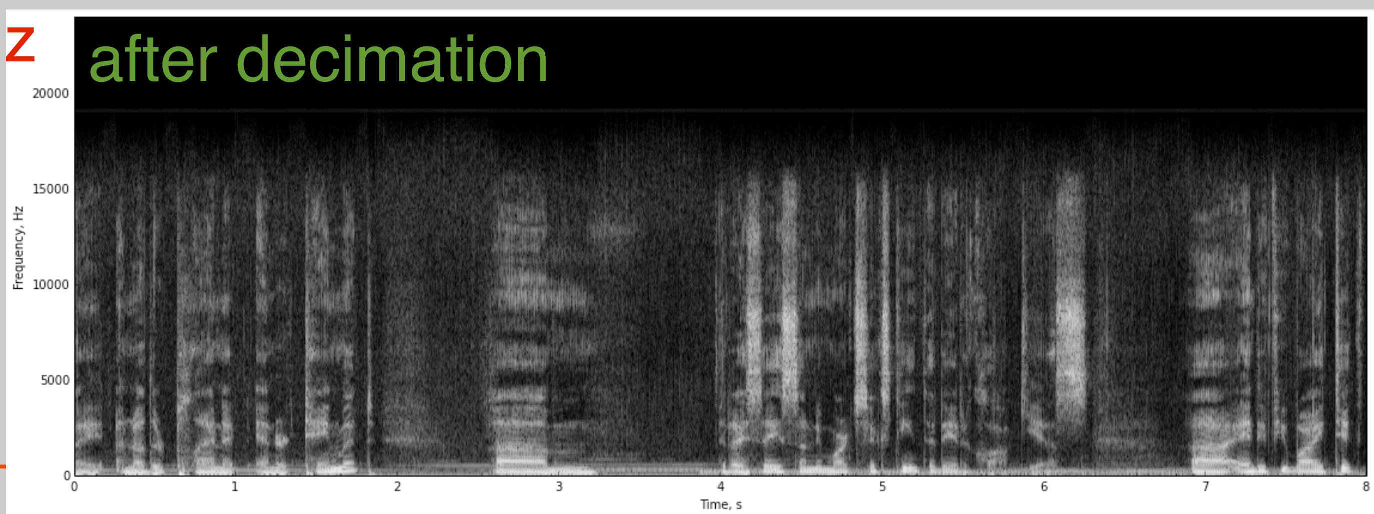
Filter Mono and down

- To play we need to filter the right signal
- Downsample to 48KHz so we can play on the computer

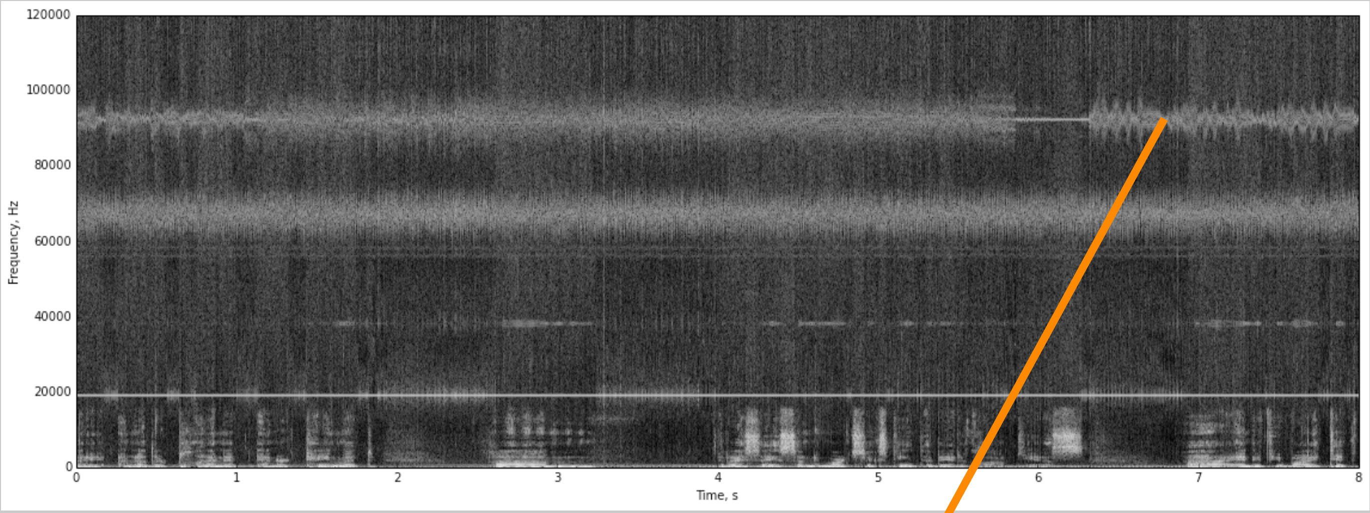
120KHz



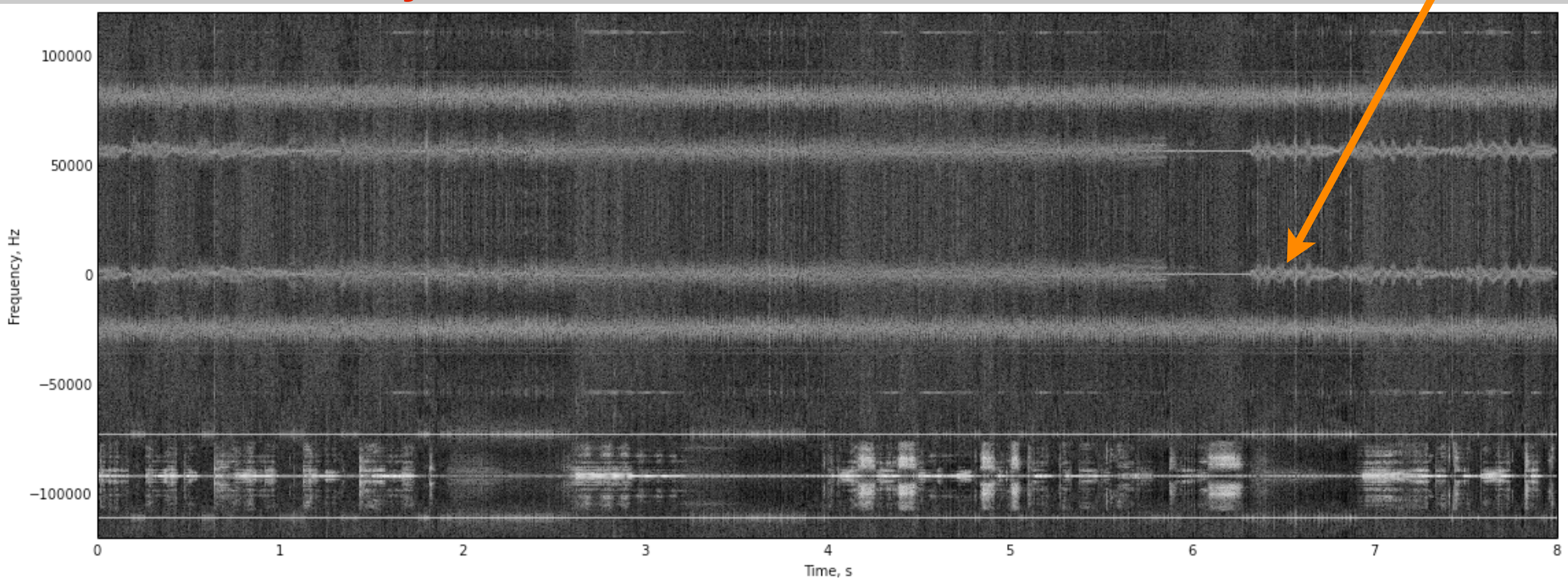
24KHz



Demodulate subcarriers: Example 92KHz

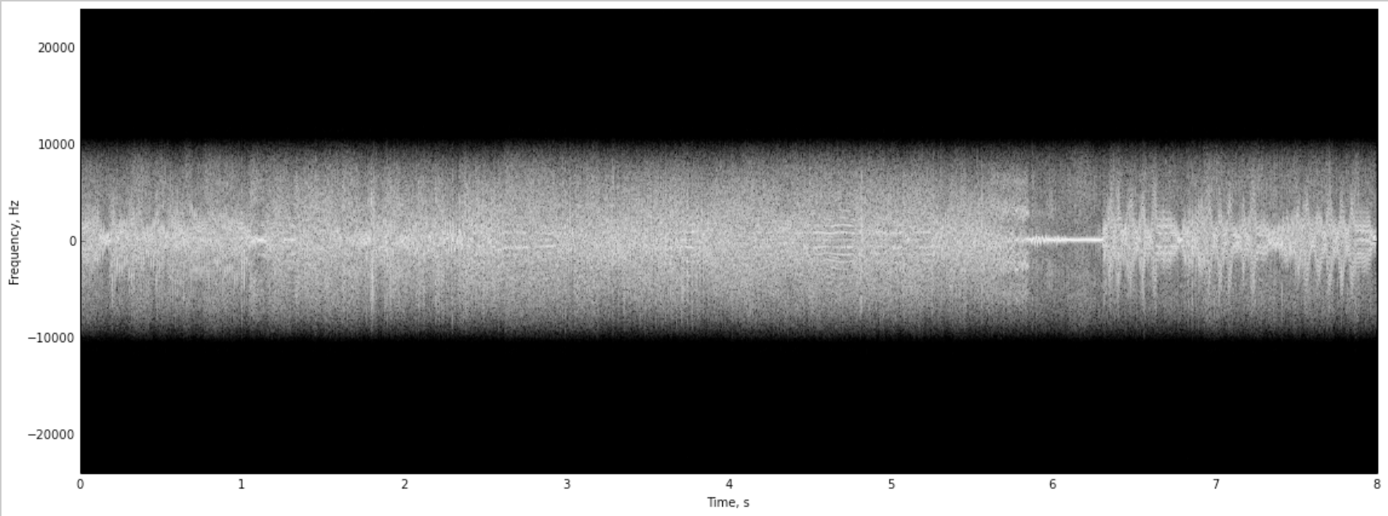


demodulate by 92KHz

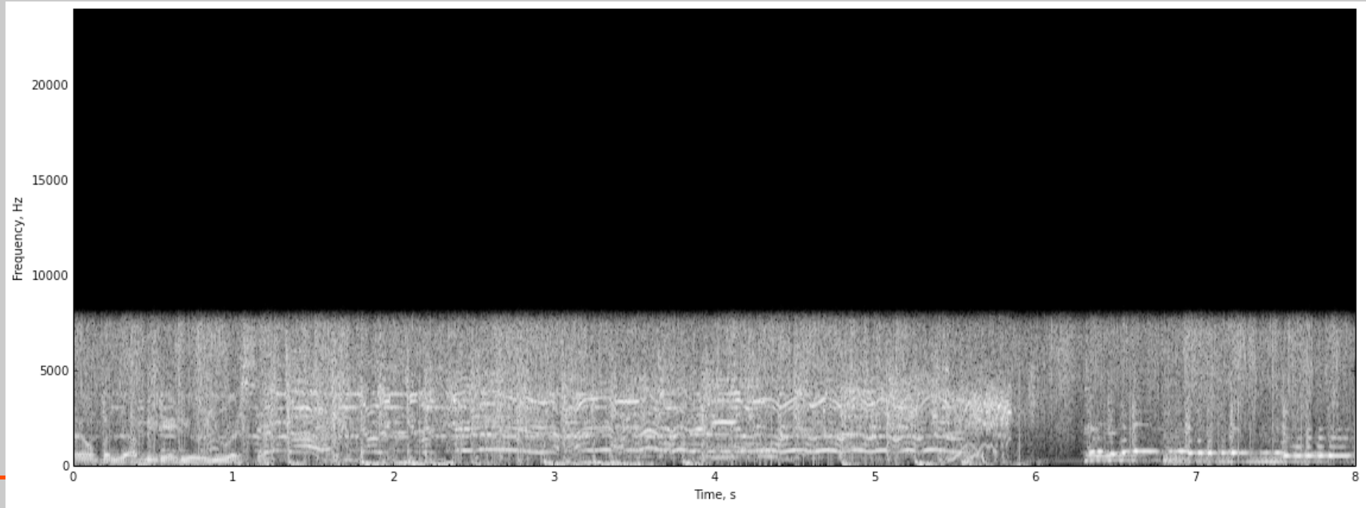


Demodulate subcarriers: Example 92KHz

- Filter and decimate



- FM demodulate and filter



Topics

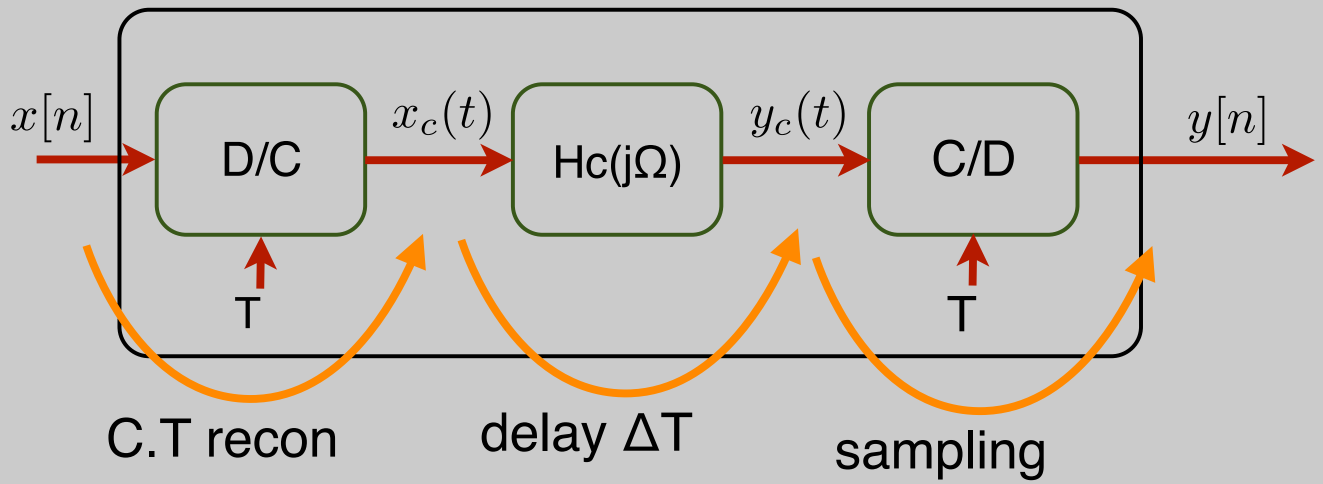
- Last time
  - Ideal reconstruction D/C
  - D.T processing of C.T signals
  - C.T processing of D.T signals (ha?????)
- Today
  - Changing Sampling Rate via DSP
  - Downsampling
  - Upsampling

Example:

Non-integer delay: $$H(e^{j\omega}) = e^{-j\omega\Delta}$$

- What is the time-domain operation when Δ is not an integer (Δ=1/2)?

Let: $H_c(j\Omega) = e^{-j\Omega\Delta T}$ <u>delay of ΔT in time</u>



Example: Non Integer Delay

- The block diagram is only for interpretation!

$$y_c(t) = x_c(t - \Delta)$$

$$\begin{aligned} y[n] &= y_c(nT) = x_c(nT - T\Delta) \\ &= \sum_k x[k]\mathrm{sinc}\left(\frac{t - kT - T\Delta}{T}\right)\Bigg|_{t=nT} \end{aligned}$$

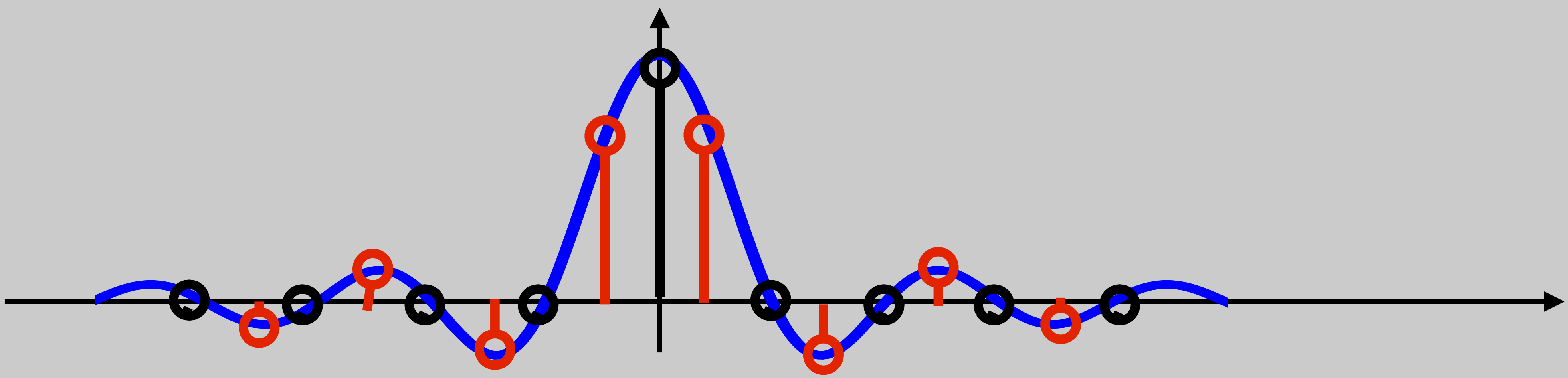
T's cancel!

$$= \sum_k x[k]\mathrm{sinc}(n - k - \Delta)$$

Example: Non Integer Delay

$$h[n] = \mathrm{sinc}(n - \Delta)$$

Example: a discrete delta is a representation of a sampled sinc



shifted by partial samples results in many coefficients!

DownSampling

- Much like C/D conversion
- Expect similar effects:
  - –Aliasing
  - –mitigate by antialiasing filter
- Finely sampled signal ⇒ almost continuous
  - –Downsample in that case is like sampling!

Changing Sampling-rate via D.T Processing

Downsampling:

$$x[n] \longrightarrow \boxed{\downarrow M} \longrightarrow x_d[n] = x[nM]$$

$$x[n] = x_c(nT) \qquad x_d[n] = x_c(n\underbrace{MT}_{T'})$$

The DTFT:

$$X(e^{j\omega}) = \frac{1}{T}\sum_k X_c\left(j\left(\underbrace{\frac{\omega}{T}}_{\Omega} - \underbrace{\frac{2\pi}{T}}_{\Omega_s}k\right)\right)$$

$$X_d(e^{j\omega}) = \frac{1}{MT}\sum_k X_c\left(j\left(\frac{\omega}{MT} - \frac{2\pi}{MT}k\right)\right)$$

Changing Sampling-rate via D.T Processing

The DTFT:

$$X(e^{j\omega}) = \frac{1}{T}\sum_k X_c\left(j\left(\underbrace{\frac{\omega}{T}}_{\Omega} - \underbrace{\frac{2\pi}{T}}_{\Omega_s}k\right)\right)$$

$$X_d(e^{j\omega}) = \frac{1}{MT}\sum_k X_c\left(j\left(\frac{\omega}{MT} - \frac{2\pi}{MT}k\right)\right)$$

we would like to bypass X_c and go from $X(e^{j\omega}) \Rightarrow X_d(e^{j\omega})$

substitute $r = kM + i$ $\quad i=0,1,\ldots,M-1$

$k=-\infty,\ldots,\infty$

two counters

e.g., k: hours, i: minutes

Changing Sampling-rate via D.T Processing

$$X_d(e^{j\omega}) = \frac{1}{MT}\sum_k X_c\left(j\left(\frac{\omega}{MT} - \frac{2\pi}{MT}k\right)\right)$$

$$= \frac{1}{M}\sum_{i=0}^{M-1} \underbrace{\frac{1}{T}\sum_{-\infty}^{\infty} X_c\left(j\left(\frac{\omega}{MT} - \frac{2\pi}{MT}i - \frac{2\pi}{T}k\right)\right)}_{X(e^{j(\frac{\omega}{M} - \frac{2\pi}{M}i)})}$$

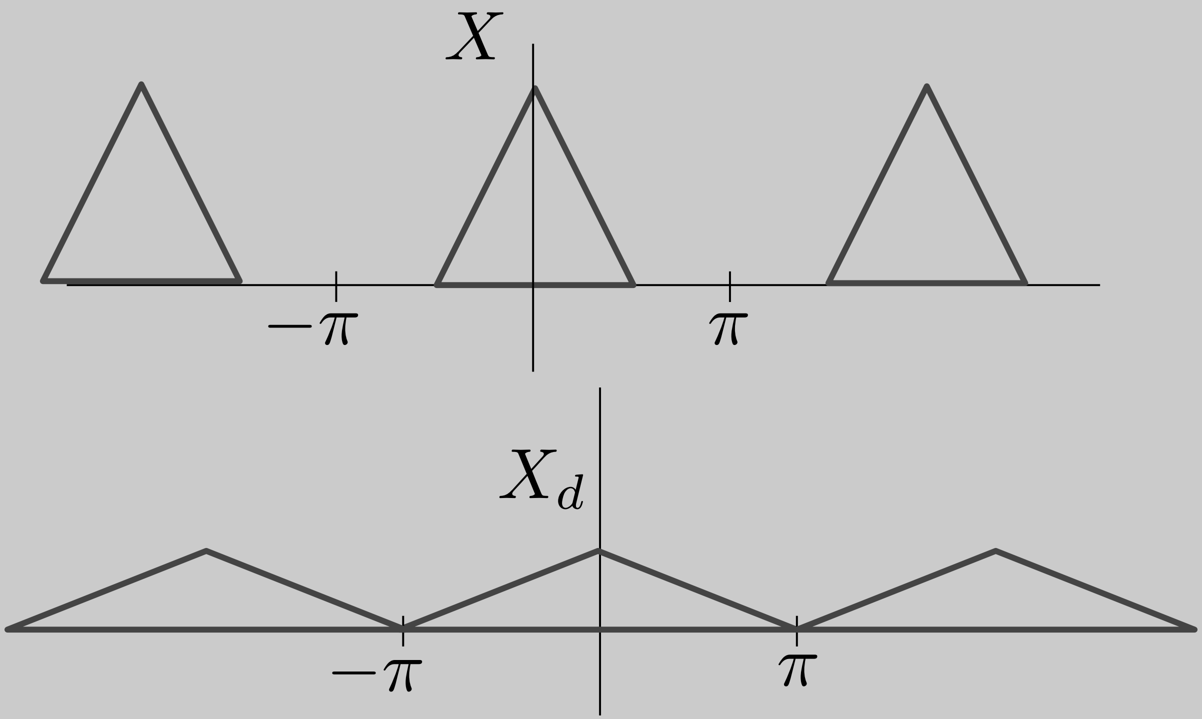
$$X_d(e^{j\omega}) = \frac{1}{M}\sum_{i=0}^{M-1} X(e^{j(\frac{\omega}{M} - \frac{2\pi}{M}i)})$$

stretch by M

replicate

Changing Sampling-rate via D.T Processing

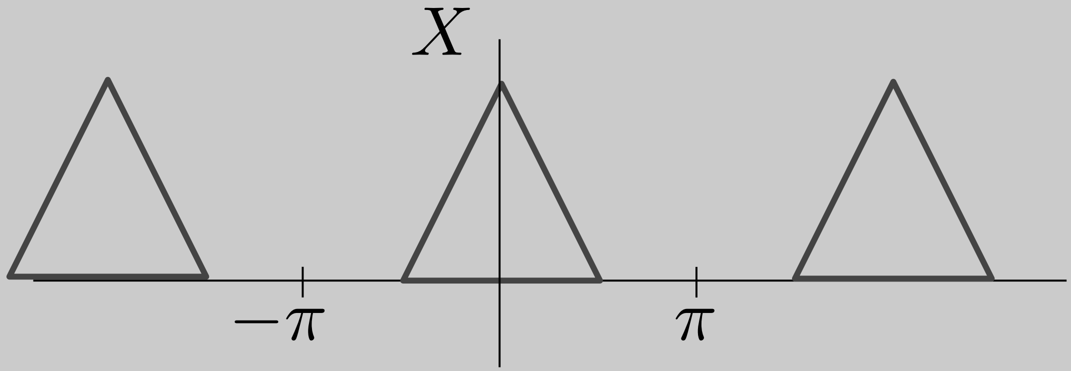
$$X_d(e^{j\omega}) = \frac{1}{M}\sum_{i=0} X\left(e^{j(w/M - 2\pi i/M)}\right)$$

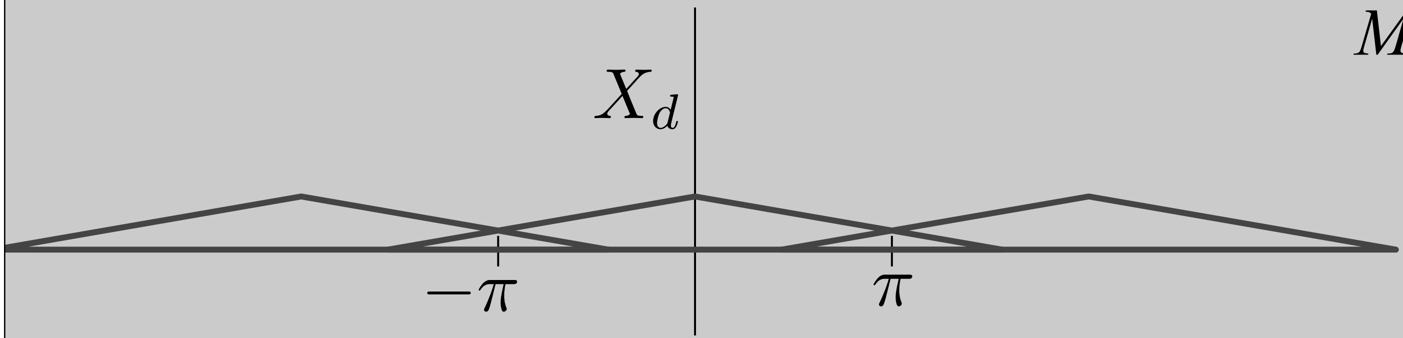


M=2

Changing Sampling-rate via D.T Processing

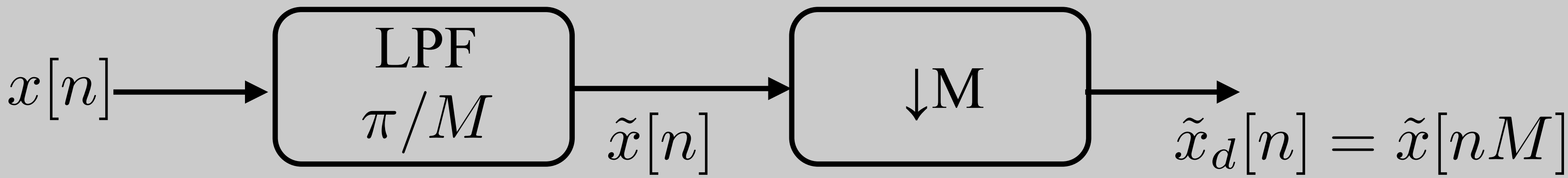
$$X_d(e^{j\omega}) = \frac{1}{M}\sum_{i=0} X\left(e^{j(w/M - 2\pi i/M)}\right)$$

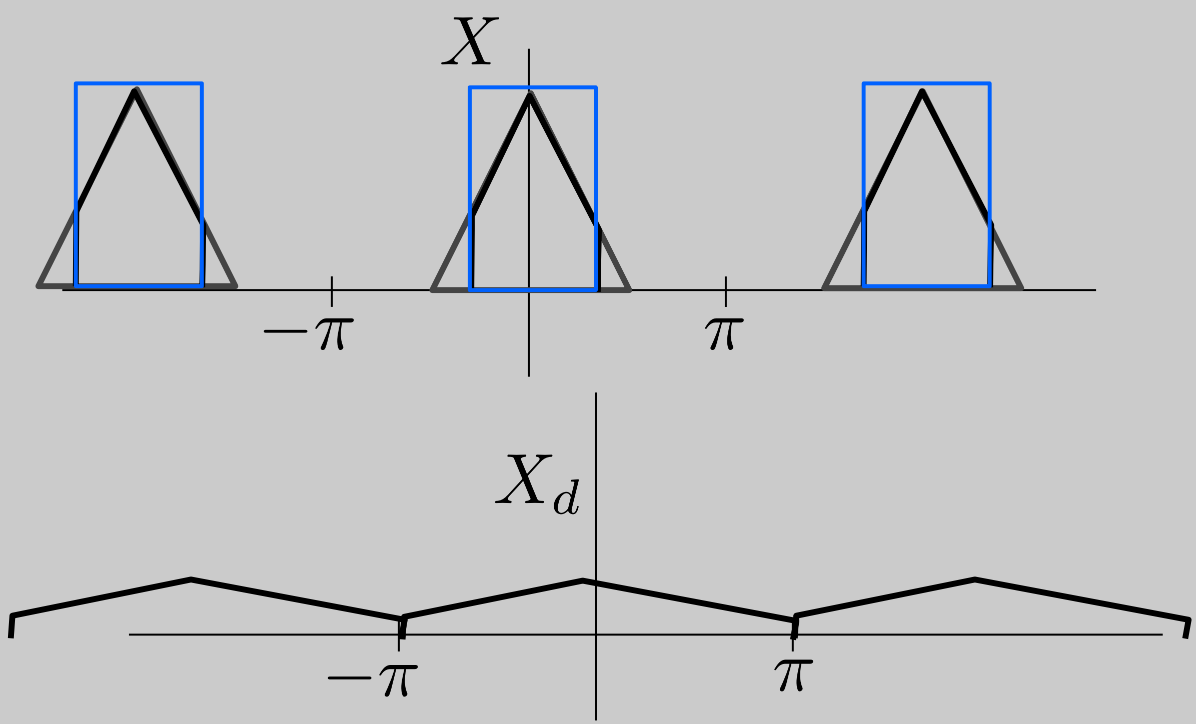




M=3

Anti-Aliasing





M=3