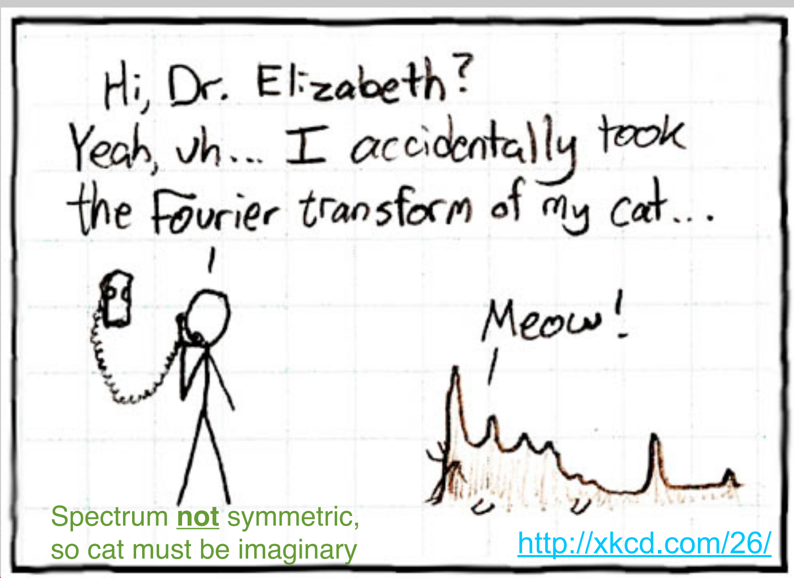
EE123
Digital Signal Processing

Lecture 10

Announcements

- Midterms: 02/28, 03/21, 04/25
- HAM:
  - –Obtain FRN number from FCC
  - –Fill form application for amateur radio operator and bring to class on Friday or before (to Frank)
    - Ink only, print carefully.
    - Fill phone and email and address please!
    - Section 2 BLANK!
    - Will post form on the website too.
  - – Lecture on ham tomorrow 6-7pm HP auditorium

How do you know this guy is insane?



Spectrum **not** symmetric, so cat must be imaginary

http://xkcd.com/26/

Last Time

- Started with STFT
- Heisenberg Boxes
- Continue and move to wavelets

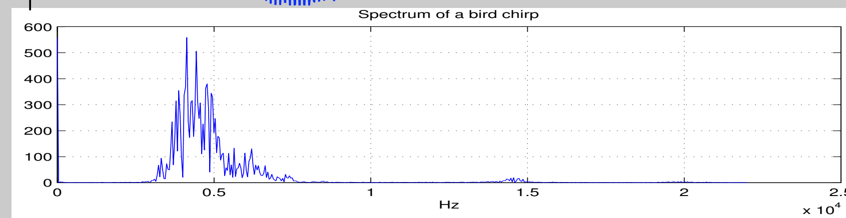
Discrete Transforms (Finite)

- DFT is only one out of a LARGE class of transforms
- Used for:
  - –Analysis
  - –Compression
  - –Denoising
  - –Detection
  - –Recognition
  - –Approximation (Sparse)

Sparse representation has been one of the hottest research topics in the last 15 years in sp

Example of spectral analysis

- Spectrum of a bird chirping
  - – Interesting,.... but...
  - – Does not tell the whole story
  - – No temporal information!



Time Dependent Fourier Transform

- To get temporal information, use part of the signal around every time point

$$X[n,\omega) = \sum_{m=-\infty}^{\infty} x[n+m]w[m]e^{-j\omega m}$$

*Also called Short-time Fourier Transform (STFT)

- Mapping from 1D ⇒ 2D, n discrete, *w* cont.
- Simply slide a window and compute DTFT

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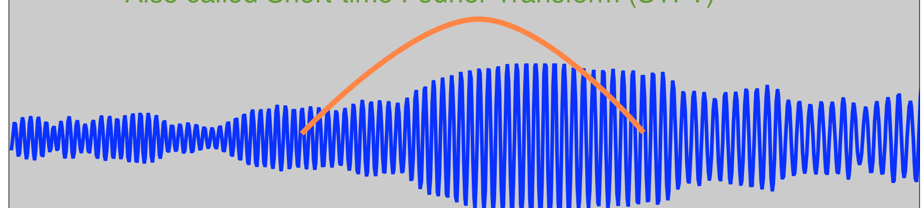
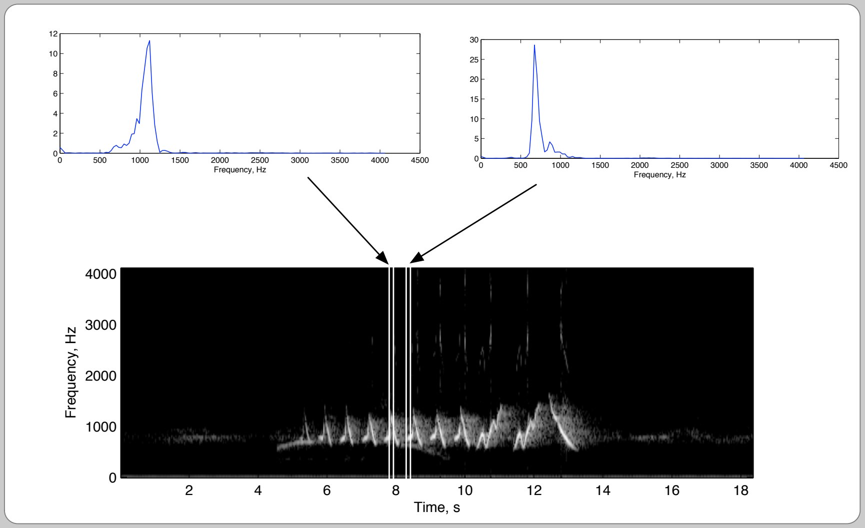


Spectrogram



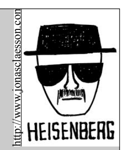
Discrete Time Dependent FT

$$X_r[k] = \sum_{m=0}^{L-1} x[rR+m]w[m]e^{-j2\pi km/N}$$

- L - Window length
- R - Jump of samples
- N - DFT length

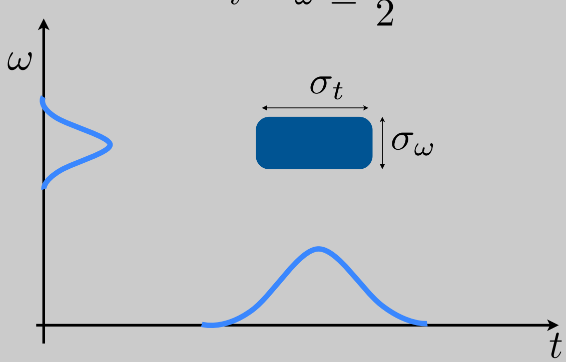
- Tradeoff between time and frequency resolution

Heisenberg Boxes



- Time-Frequency uncertainty principle

$$\sigma_t \cdot \sigma_\omega \geq \frac{1}{2}$$



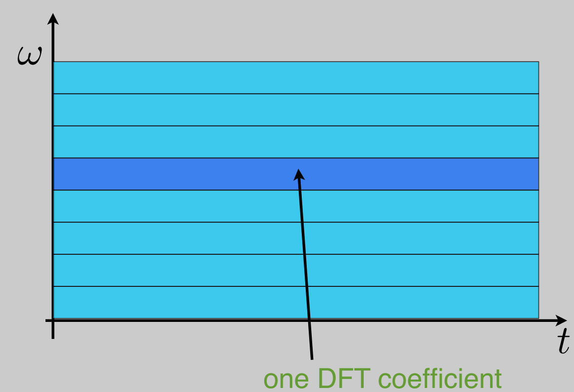
DFT

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j2\pi kn/N}$$

$$\Delta\omega = \frac{2\pi}{N}$$

$$\Delta t = N$$

$$\Delta\omega \cdot \Delta t = 2\pi$$



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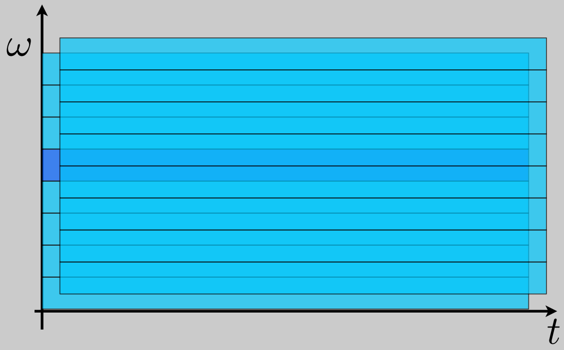


$$\Delta\omega \cdot \Delta t = 2\pi$$

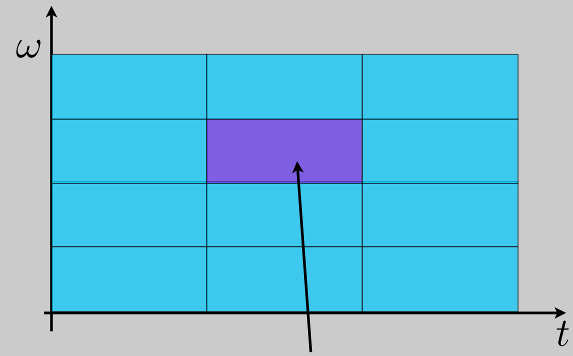
Question: What is the effect of zero-padding?
Answer: Overlapped Tiling!

Discrete STFT

$$X[r,k] = \sum_{m=0}^{L-1} x[r\underset{\text{optional}}{R} + m]w[m]e^{-j2\pi km/N}$$

$$\Delta\omega = \frac{2\pi}{L}$$

$$\Delta t = L$$



Discrete STFT

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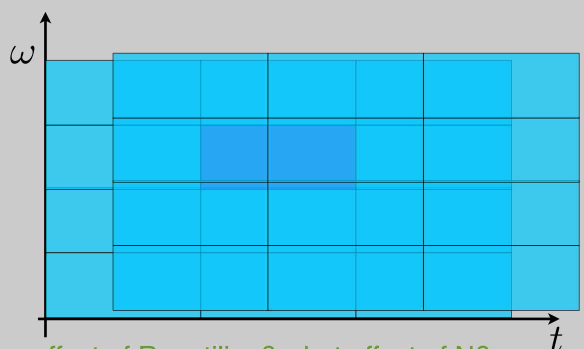
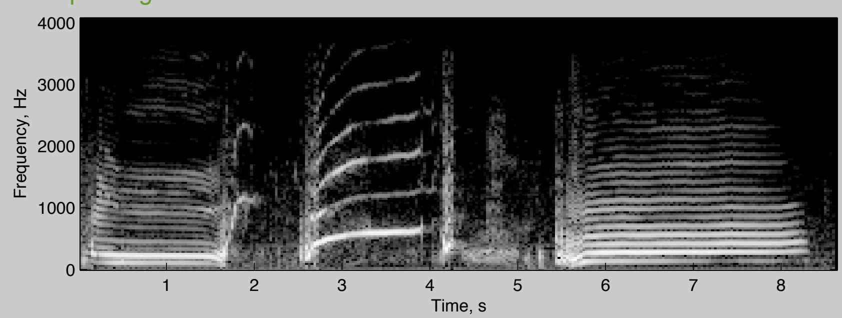


Question: What is the effect of R on tiling? what effect of N?
Answer: Overlapping in time of frequency or both!

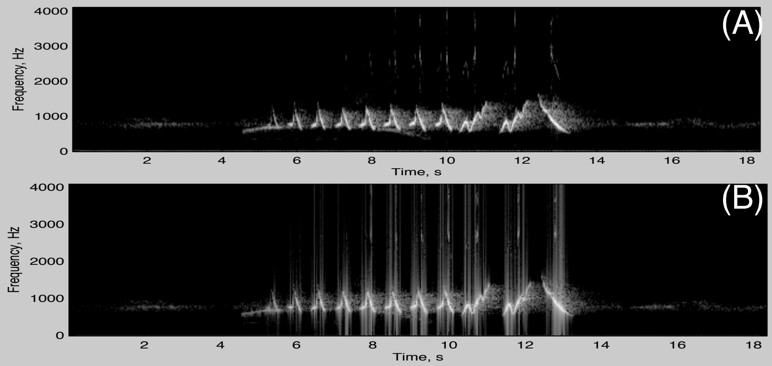
Applications

- Time Frequency Analysis

Spectrogram of Orca whale



Spectrogram



- What is the difference between the spectrograms?

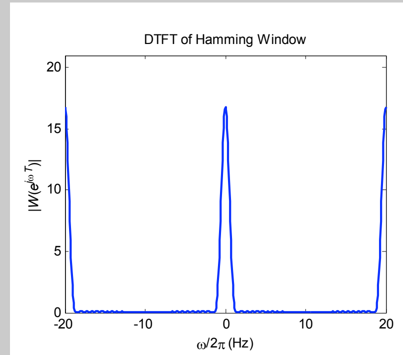
  a) Window size B<A

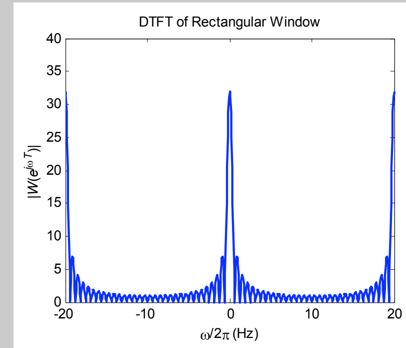
  b) Window size B>A

  c) Window type is different

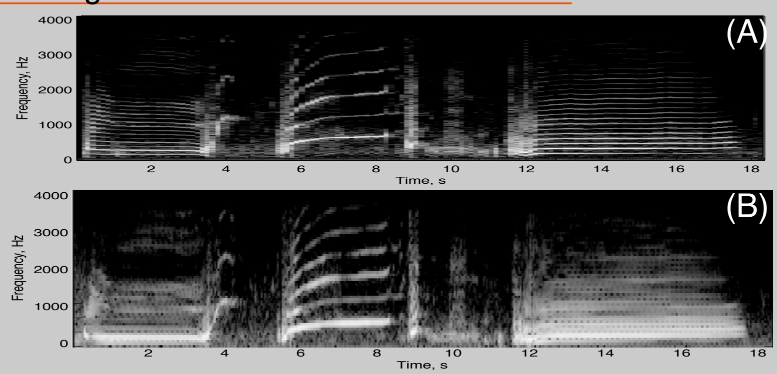
  d) (A) uses overlapping window

Sidelobes of Hann vs rectangular window





Spectrogram



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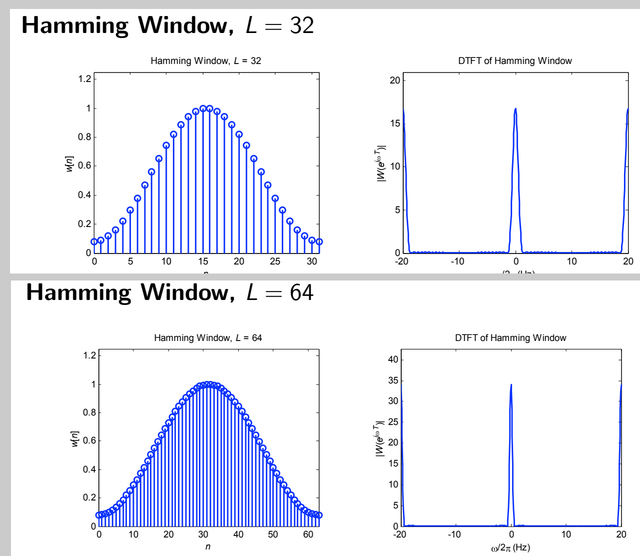
Spectrogram

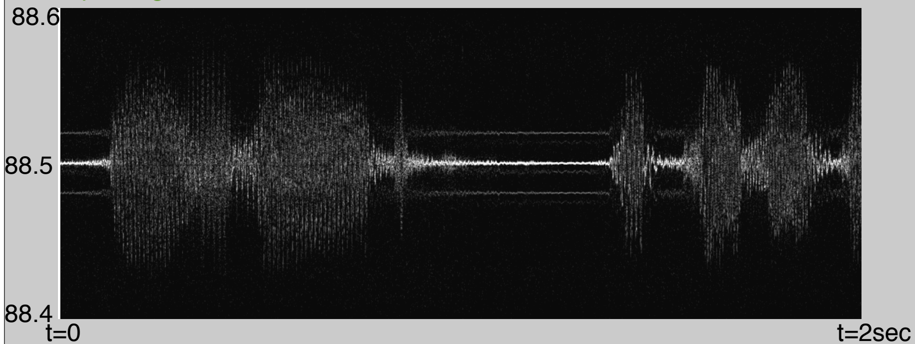


Spectrogram of FM

$$y_c(t) = A\cos\left(2\pi f_c t + 2\pi\Delta f\int_0^t x(\tau)d\tau\right)$$

$$y[n] = y(nT) = A\exp\left(j2\pi\Delta f\int_0^{nT} x(\tau)d\tau\right)$$

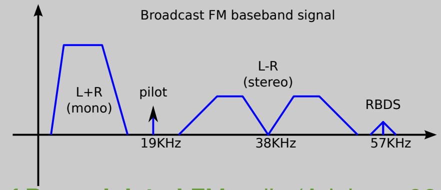
Spectrogram of FM radio



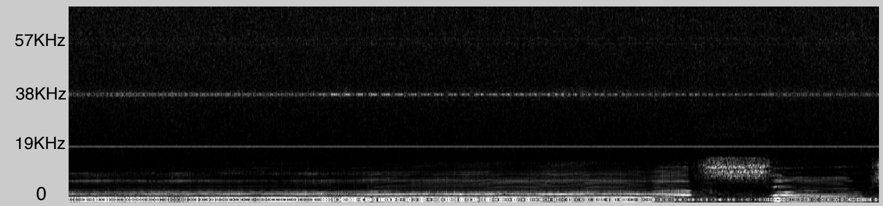
Spectrogram of FM radio Baseband

$$y[n] = y(nT) = A\exp\left(j2\pi\Delta f\int_0^{nT} x(\tau)d\tau\right)$$

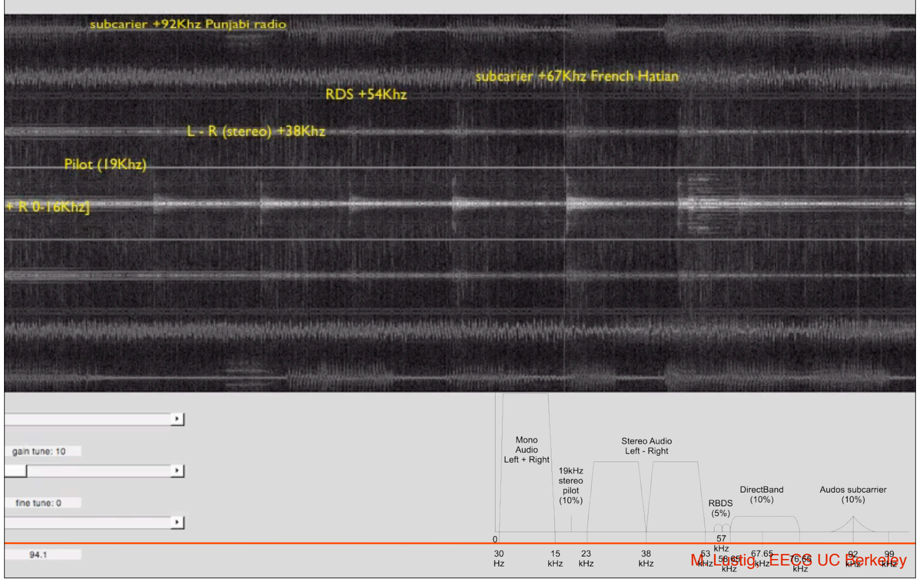
$$x(t) = \underbrace{(L+R)}_{\text{mono}} + \underbrace{0.1\cdot\cos(2\pi f_p t)}_{\text{pilot}} + \underbrace{(L-R)\cos(2\pi(2f_p)t)}_{\text{stereo}} + \underbrace{0.05\cdot\text{RBDS}(t)\cos(2\pi(3f_p)t)}_{\text{digital RBDS}}.$$



Spectrogram of **Demodulated** FM radio (Adele on 96.5 MHz)



Subcarrier FM radio (Hidden Radio Stations)



Applications

- Time Frequency Analysis

Spectrogram of digital communications - Frequency Shift Keying

STFT Reconstruction

$$x[rR+m]w_L[m] = \frac{1}{N}\sum_{k=0}^{N-1} X[n,k]e^{j2\pi km/N}$$

- For non-overlapping windows, R=L :

$$x[n] = \frac{x[n-rL]}{w_L[n-rL]}$$

$$rL \le n \le (r+1)R-1$$

- What is the problem?

STFT Reconstruction

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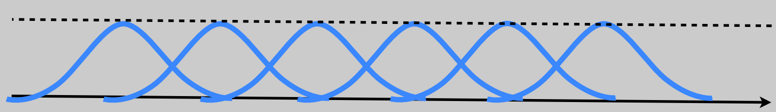
$$rL \le n \le (r+1)R-1$$

- For stable reconstruction must overlap window 50% (at least)

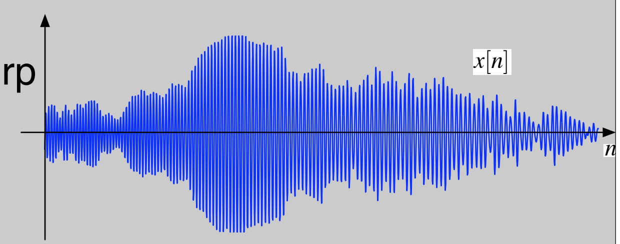
STFT Reconstruction

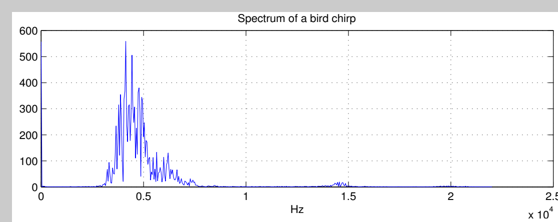
- For stable reconstruction must overlap window 50% (at least)
- For Hann, Bartlett reconstruct with overlap and add. No division!



Applications

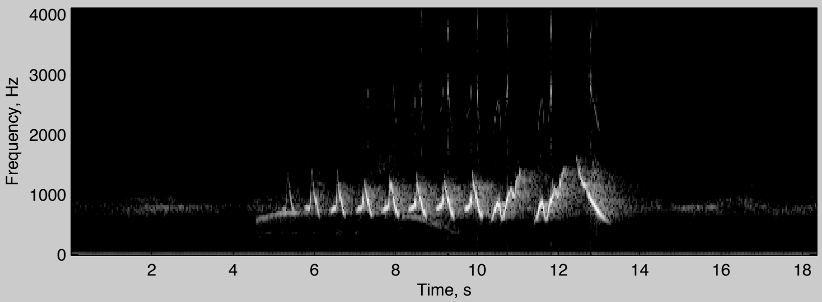
- Noise removal
- Recall bird chirp





Application

- Denoising of Sparse spectrograms



- Spectrum is sparse! can implement adaptive filter, or just threshold!

Limitations of Discrete STFT

- Need overlapping ⇒ Not orthogonal
- Computationally intensive O(MN log N)
- Same size Heisenberg boxes

From STFT to Wavelets

- Basic Idea:
  - low-freq changes slowly - fast tracking unimportant
  - Fast tracking of high-freq is important in many apps.
  - Must adapt Heisenberg box to frequency
- Back to continuous time for a bit.....