# EE123 Digital Signal Processing

Lecture 9

based on slides by J.M. Kahn



## Announcements

- Lab 01 part I and II posted will post III today or tomorrow
- Lab-bash Tuesday 2-3pm 521 Cory
- Three shorter Midterms:
  - 02/26 in class
  - 04/02 in class
  - 04/30 (or 28 TBD) in class
  - 05/05 or 05/06 (TBD) project presentations.
    - Posters and demos



## Announcements

- Last time:
  - –Frequency analysis with DFT
  - –Windowing
- Today:
  - Continue
  - Effect of zero-padding
  - Start Short-time Fourier Transform



## Windows Properties

These are characteristic of the window type

| Window | Main-lobe | Sidelobe $\delta_s$ | Sidelobe $-20\log_{10}\delta_s$ |
|---|---|---|---|
| Rect | $\frac{4\pi}{M+1}$ | 0.09 | 21 |
| Bartlett | $\frac{8\pi}{M+1}$ | 0.05 | 26 |
| Hann | $\frac{8\pi}{M+1}$ | 0.0063 | 44 |
| Hamming | $\frac{8\pi}{M+1}$ | 0.0022 | 53 |
| Blackman | $\frac{12\pi}{M+1}$ | 0.0002 | 74 |

Most of these (Bartlett, Hann, Hamming) have a transition width that is twice that of the rect window.

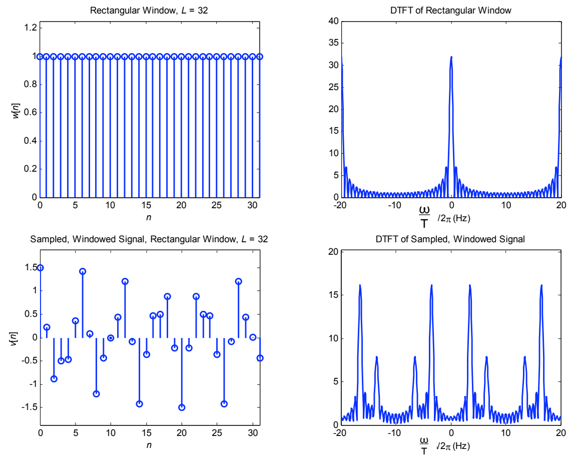
Warning: Always check what's the definition of M

Adapted from *A Course In Digital Signal Processing* by Boaz Porat, Wiley, 1997

# Windows Examples

Here we consider several examples. As before, the sampling rate is $\Omega_s/2\pi = 1/T = 20$ Hz.

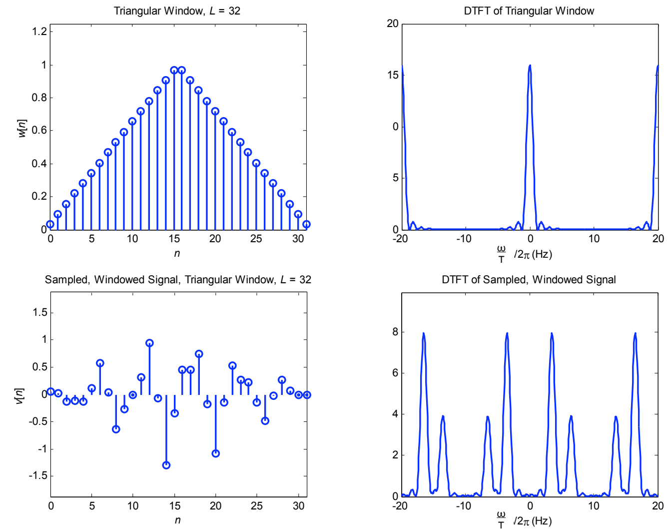
**Rectangular Window,** $L = 32$






# Windows Examples

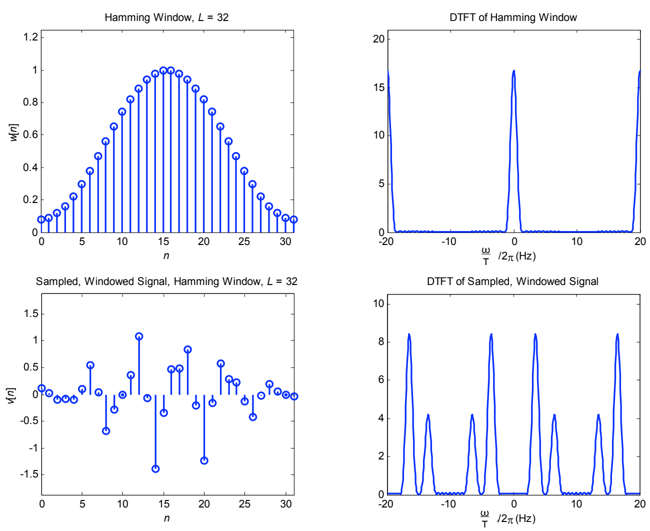
**Triangular Window,** $L = 32$






# Windows Examples

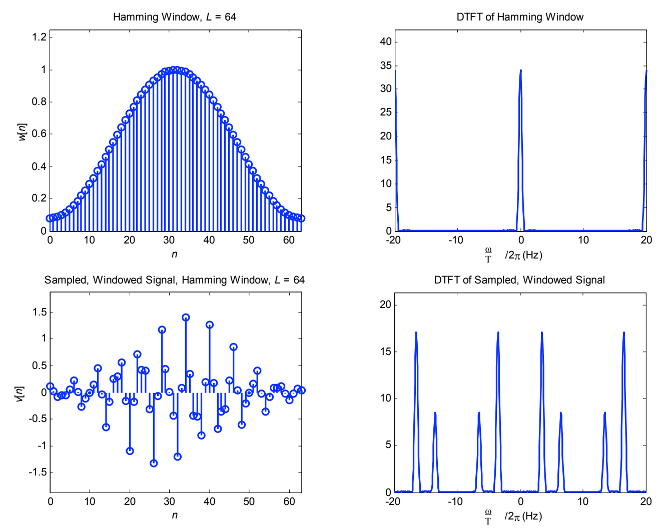
**Hamming Window,** $L = 32$






# Windows Examples

**Hamming Window,** $L = 64$



## Optimal Window: Kaiser

- Minimum main-lobe width for a given side-lobe energy %

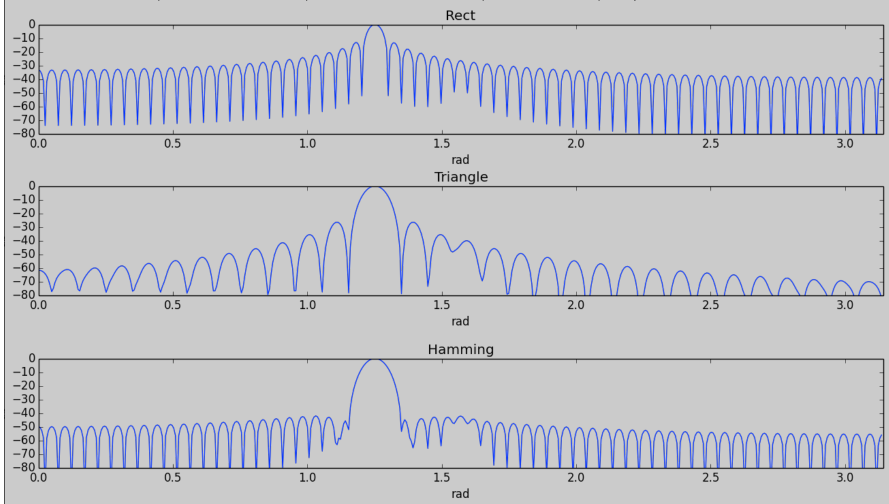
$$\frac{\int_{\text{sidelobes}} |H(e^{j\omega})|^2 d\omega}{\int_{-\pi}^{\pi} |H(e^{j\omega})|^2 d\omega}$$

- Window is parametrized with L and β   OS Eq 10.12
  - β determines side-lobe level
  - L determines main-lobe width



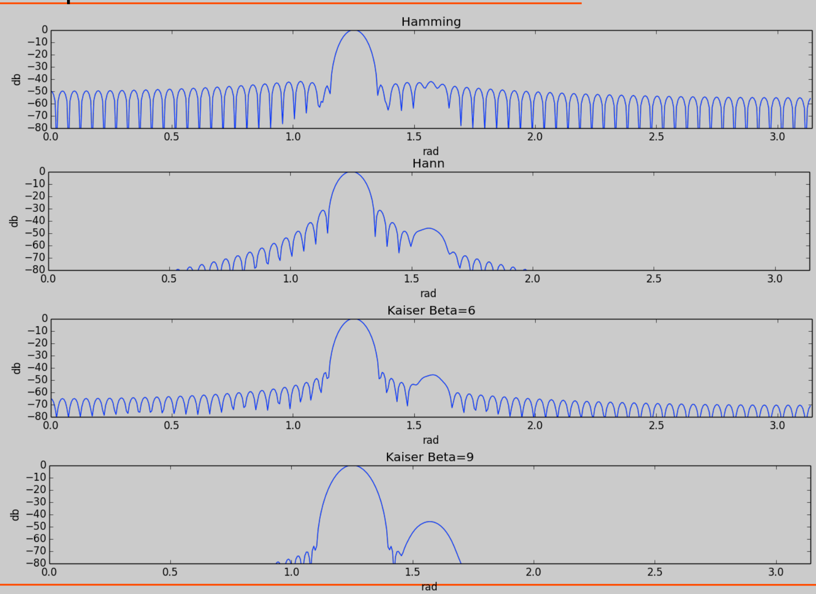
## Example

$$y = \sin(2\pi 0.1992n) + 0.005\sin(2\pi 0.25n) \quad | \quad 0 \le n < 128$$






## Example






## Zero-Padding

- In preparation for taking an $N$-point DFT, we may zero-pad the windowed block of signal samples to a block length $N \ge L$:

$$\begin{cases} v[n] & 0 \le n \le L-1 \\ 0 & L \le n \le N-1 \end{cases}$$

- This zero-padding has no effect on the DTFT of $v[n]$, since the DTFT is computed by summing over $-\infty < n < \infty$.

*Effect of Zero Padding*

- We take the $N$-point DFT of the zero-padded $v[n]$, to obtain the block of $N$ spectral samples:

$$V[k], \quad 0 \le k \le N-1$$

## Zero-Padding

- Consider the DTFT of the zero-padded $v[n]$. Since the zero-padded $v[n]$ is of length $N$, its DTFT can be written:

$$V(e^{j\omega}) = \sum_{n=0}^{N-1} v[n]e^{-jn\omega}, \qquad -\infty < \omega < \infty$$

The $N$-point DFT of $v[n]$ is given by:

$$V[k] = \sum_{n=0}^{N-1} v[n]W_N^{kn} = \sum_{n=0}^{N-1} v[n]e^{-j(2\pi/N)nk}, \qquad 0 \le k \le N-1$$

We see that $V[k]$ corresponds to the samples of $V(e^{j\omega})$:

$$V[k] = V(e^{j\omega})\big|_{\omega = k\frac{2\pi}{N}}, \qquad 0 \le k \le N-1$$

To obtain samples at more closely spaced frequencies, we zero-pad $v[n]$ to longer block length $N$. The spectrum is the same, we just have more samples.



## Frequency Analysis with DFT

- Note that the ordering of the DFT samples is unusual.

$$V[k] = \sum_{n=0}^{N-1} v[n]W_N^{nk}$$

The DC sample of the DFT is $k = 0$

$$V[0] = \sum_{n=0}^{N-1} v[n]W_N^{0n} = \sum_{n=0}^{N-1} v[n]$$

- The positive frequencies are the first $N/2$ samples
- The first $N/2$ negative frequencies are circularly shifted

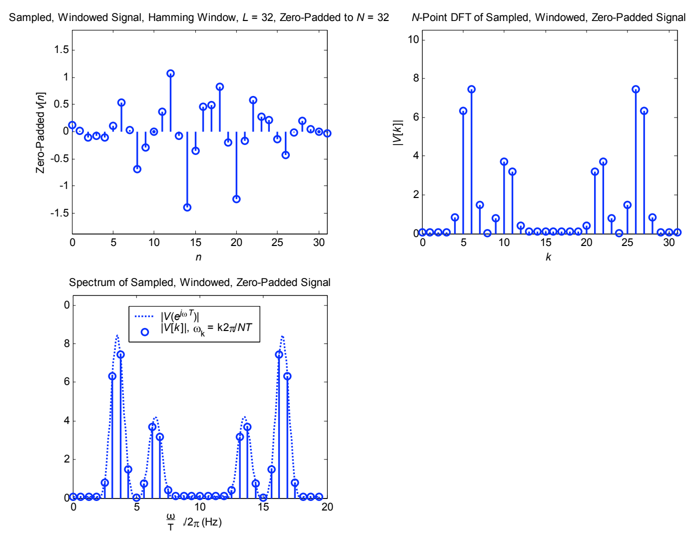
$$((-k))_N = N - k$$

so they are the last $N/2$ samples. (Use `fftshift` to reorder)



## Frequency Analysis with DFT Examples:

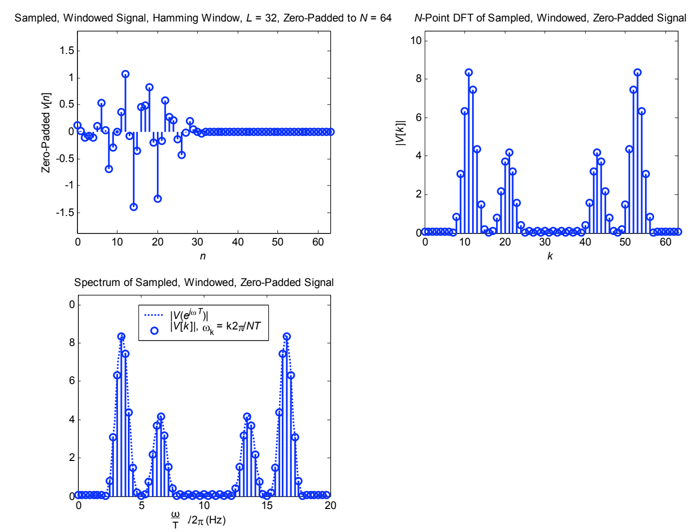
**Hamming Window, $L = 32$, $N = 32$**

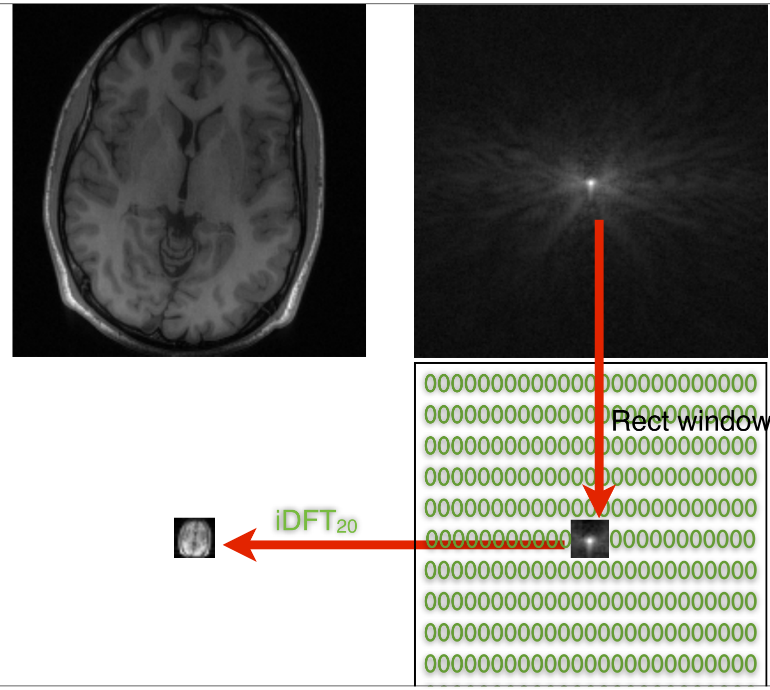




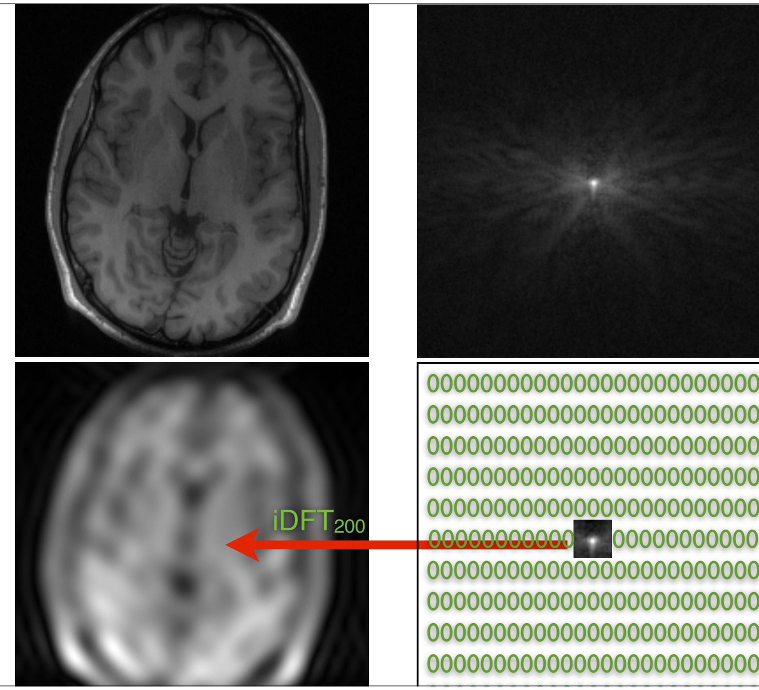

## Frequency Analysis with DFT Examples:

**Hamming Window, $L = 32$, Zero-Padded to $N = 64$**

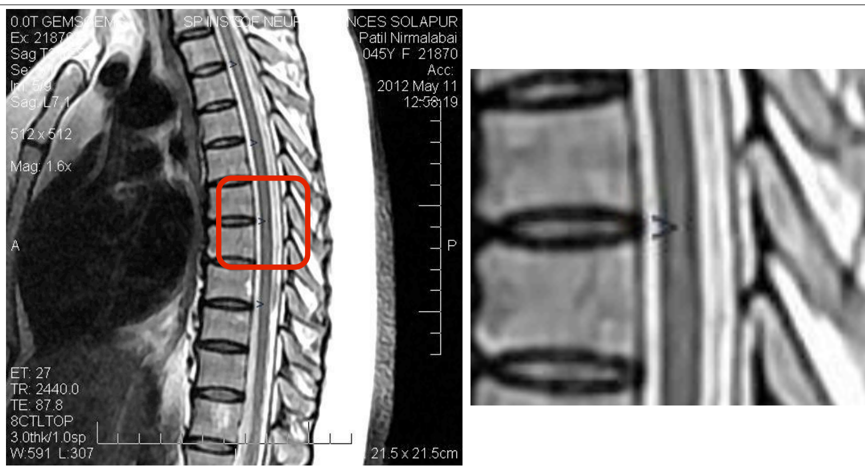









A 40 yo pt with a history of lower limb weakness referred for mri screening of brain and whole spine for cord. MRI sagittal T2 screening of dorsal region shows a faint uniform linear high signal at the center of the cord. The signal abnormality likely to represent:

(1) Cord demyelination.
(2) Syrinx (spinal cord disease).
(3) Artifact.

**Answer** : Its an artifact, known as truncation or Gibbs artifact

http://www.neuroradiologycases.com



## Frequency Analysis with DFT

- Length of window determines spectral resolution
- Type of window determines side-lobe amplitude. (Some windows have better tradeoff between resolution-sidelobe)
- Zero-padding approximates the DTFT better. Does not introduce new information!

**Potential Problems and Solutions**

| Problem | Possible Solutions |
|---|---|
| 1. Spectral error from aliasing Ch.4 | a. Filter signal to reduce frequency content above $\Omega_s/2 = \pi/T$.<br>b. Increase sampling frequency $\Omega_s = 2\pi/T$. |
| 2. Insufficient frequency resolution. | a. Increase $L$<br>b. Use window having narrow main lobe. |
| 3. Spectral error from leakage | a. Use window having low side lobes.<br>b. Increase $L$ |
| 4. Missing features due to spectral sampling. | a. Increase $L$,<br>b. Increase $N$ by zero-padding $v[n]$ to length $N > L$. |



iSpectrum Example



## Discrete Transforms (Finite)

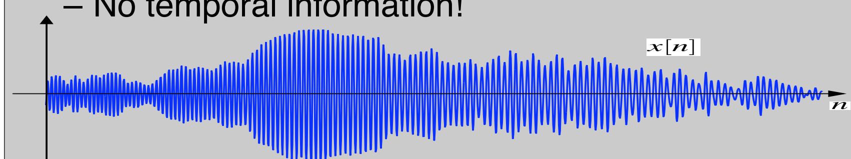
- DFT is only one out of a LARGE class of transforms
- Used for:
  - –Analysis
  - –Compression
  - –Denoising
  - –Detection
  - –Recognition
  - –Approximation (Sparse)

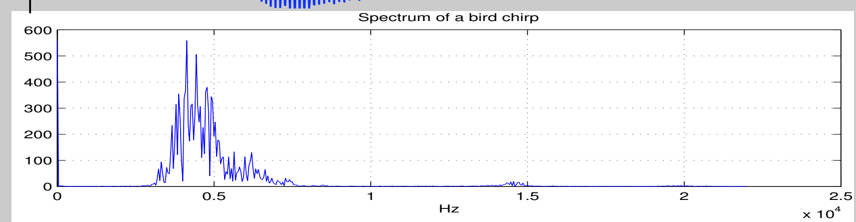
Sparse representation has been one of the hottest research topics in the last 15 years in sp



## Example of spectral analysis

- Spectrum of a bird chirping
  - – Interesting,.... but...
  - – Does not tell the whole story
  - – No temporal information!



## Time Dependent Fourier Transform

- To get temporal information, use part of the signal around every time point

$$X[n,\omega) = \sum_{m=-\infty}^{\infty} x[n+m]w[m]e^{-j\omega m}$$

*Also called Short-time Fourier Transform (STFT)

- Mapping from 1D ⇒ 2D, n discrete, *w* cont.
- Simply slide a window and compute DTFT



## Time Dependent Fourier Transform

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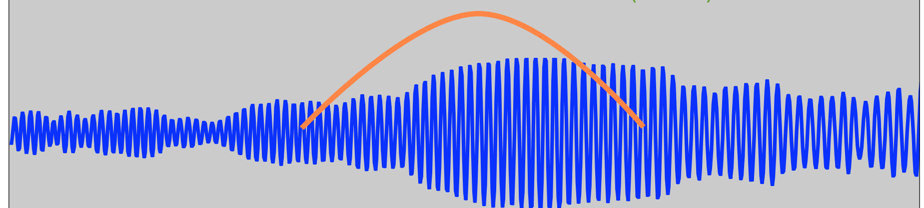
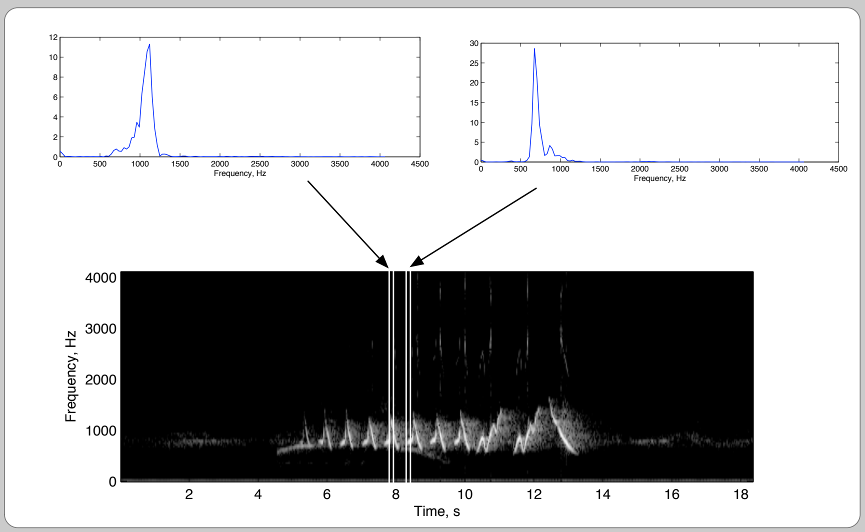




## Spectrogram





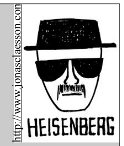

## Discrete Time Dependent FT

$$X_r[k] = \sum_{m=0}^{L-1} x[rR+m]w[m]e^{-j2\pi km/N}$$

- L - Window length
- R - Jump of samples
- N - DFT length

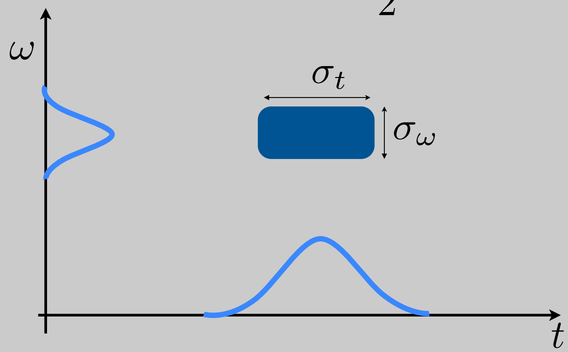
- Tradeoff between time and frequency resolution

## Heisenberg Boxes




- Time-Frequency uncertainty principle

$$\sigma_t \cdot \sigma_\omega \geq \frac{1}{2}$$






## DFT

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N}$$

$$\Delta\omega = \frac{2\pi}{N}$$

$$\Delta t = N$$

$$\Delta\omega \cdot \Delta t = 2\pi$$