EE123

Digital Signal Processing

Lecture 3

A couple of things

- Read Ch 2 2.0-2.9
- It's OK to use 2nd edition
- Class webcasted in bcourses.berkeley.edu
- My office hours: posted on-line
  - W 11a-12 (EE225E priority), Th 3p-5p Cory 506
- Frank Ong
  - Th 5p-6 504 Cory (this week W 2p-3 Cory 400)
  - Lab Bash - Tu 2p-3p Cory 521
- Reward: 1$ for every typo/errors in my slides/slide

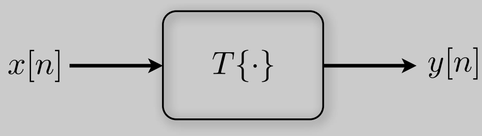
Ham Stuff

- Exam:
  - Wednesday, Feb 26th at 6:00 PM, Location TBD
  - Bring your own pencil, pen
  - Bring $15 or a check for $15 made out to ARRL-VEC
  - Bring a legal photo ID (passport, driver's license); school ID is not sufficient alone, but must be combined with social security card, birth certificate, or other documents (see http://www.arrl.org/what-to-bring-to-an-exam-session)
  - Apply for a Federal Registration Number (FRN) before the exam, and have that number with you at the exam
  - If you already have a license, bring both original and photocopy of the license (or CSCE) to the exam
  - You could also get licensed on your own: http://www.arrl.org/find-an-amateur-radio-license-exam-session

More Ham Stuff

- Get the book.
- Ham exam preparation lectures
  - Next 3 weeks
- Wednesday noon, demonstration of satellite communication in Memorial Glade.

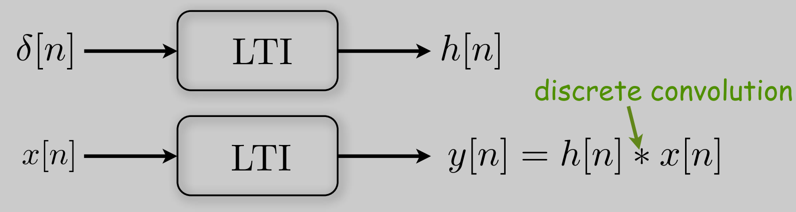
Discrete Time Systems



- Causality
- Memoryless
- Linearity
- Time Invariance
- BIBO stability

Discrete-Time LTI Systems

- The impulse response h[n] completely characterizes an LTI system "DNA of LTI"



$$y[n] = \sum_{\substack{N=-\infty \\ \text{m}}}^{\infty} h[m]x[n-m]$$

Sum of weighted, delayed impulse responses!

BIBO Stability of LTI Systems

- An LTI system is BIBO stable iff h[n] is absolutely summable

$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty$$

BIBO Stability of LTI Systems

- Proof: "if"

$$|y[n]| = \left| \sum_{k=-\infty}^{\infty} h[k]x[n-k] \right|$$

$$\leq \sum_{k=-\infty}^{\infty} |h[k]| \cdot \underbrace{|x[n-k]|}_{\leq B_x}$$

$$\leq B_x \sum_{k=-\infty}^{\infty} |h[k]| < \infty$$

BIBO Stability of LTI Systems

- Proof: "only if"
  - suppose $\sum_{k=-\infty}^{\infty} |h[k]| \ = \ \infty$
    show that there exists bounded x[n] that gives unbounded y[n]
  - Let: $\quad x[n] \ = \ \dfrac{h[-n]}{|h[-n]|} = \text{Sign}\{h[-n]\}$

$$y[n] = \sum h[k]x[n-k]$$

$$y[0] = \sum h[k]x[-k] = \sum h[k]h[k]/|h[k]| = \sum |h[k]| = \infty$$

Discrete-Time Fourier Transform (DTFT)

$$X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} x[k]e^{-j\omega k}$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

Why one is sum and the other integral?

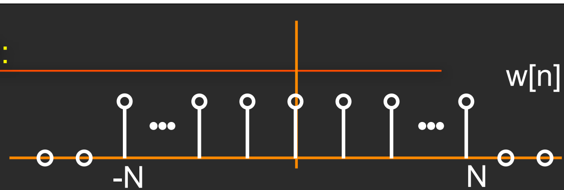
Why use one over the other?

Alternative

$$X(f) = \sum_{k=-\infty}^{\infty} x[k]e^{-j2\pi fk}$$

$$x[n] = \int_{-0.5}^{0.5} X(f)e^{j2\pi fn}df$$

Example 1:



w[n] "window"

DTFT:

$$W(e^{j\omega}) \ = \ \sum_{k=-N}^{N} e^{-j\omega k}$$

$$= \ e^{-j\omega N}\left(1 + e^{j\omega} + \cdots + e^{j\omega 2N}\right)$$

Recall: $\quad 1 + p + p^2 + \cdots + p^M = \dfrac{1-p^{M+1}}{1-p} \qquad p = e^{j\omega}, \quad M = 2N$

Example 1 cont.

DTFT:

Example 1 cont.

DTFT:

$$W(e^{j\omega}) = e^{-j\omega N}\left(1 + e^{j\omega} + \cdots + e^{j\omega 2N}\right)$$

$$= e^{-j\omega N}\frac{1 - e^{j\omega(2N+1)}}{1 - e^{j\omega}}$$

$$= \frac{e^{-j\omega N} - e^{j\omega N}e^{j\omega}}{1 - e^{j\omega}} \qquad \frac{\times\ e^{-j\frac{\omega}{2}}}{\times\ e^{-j\frac{\omega}{2}}}$$

$$= \frac{e^{-j\omega(N+\frac{1}{2})} - e^{j\omega(N+\frac{1}{2})}}{e^{-j\frac{\omega}{2}} - e^{j\frac{\omega}{2}}}$$

$$= \frac{\sin[(N+\frac{1}{2})\omega]}{\sin(\frac{\omega}{2})}$$

periodic sinc

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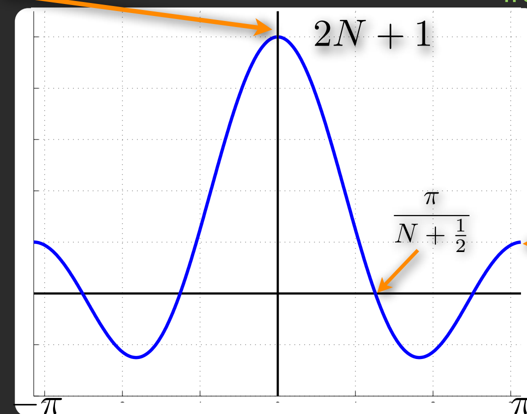
Example 1 cont.

$$W(e^{j\omega}) = \frac{\sin[(N+\frac{1}{2})\omega]}{\sin(\frac{\omega}{2})} \rightarrow (2N+1) \quad \text{as} \quad \omega \rightarrow 0$$

from l'Hôpital

also, Σx[n]



=1, why?

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Properties of the DTFT

Periodicity: $X(e^{j(\omega+2\pi)}) = X(e^{j\omega})$

Conjugate Symmetry:

$X^*(e^{j\omega}) = X(e^{-j\omega})$ if x[n] is real

$$\mathcal{R}e\left\{X(e^{-j\omega})\right\} = \mathcal{R}e\left\{X(e^{j\omega})\right\}$$

$$\mathcal{I}m\left\{X(e^{-j\omega})\right\} = -\mathcal{I}m\left\{X(e^{j\omega})\right\}$$

Big deal for: MRI, Communications, more....

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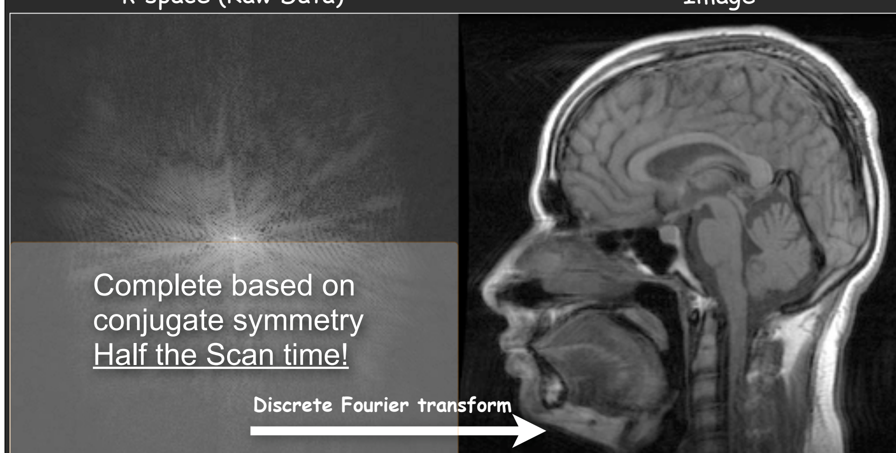
Half Fourier Imaging in MR

k-space (Raw Data) Image



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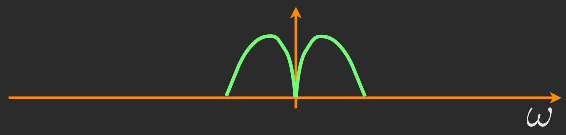
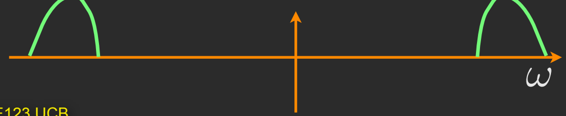
SSB Modulation

Real Baseband signal has conjugate symmetric spectrum



AM modulation

SSB-SC reduced power, half bandwidth



SSB

http://www.youtube.com/watch?v=y0qi9Fr2j6Y&list=PLA5FE5E811C57CF77