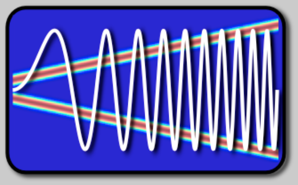

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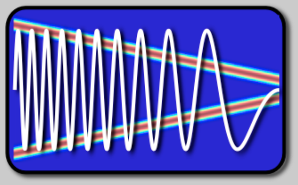


Digital Signal Processing

Lecture 2

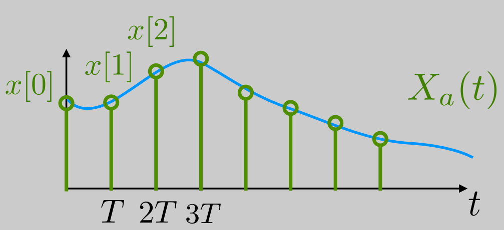
Today

- Last time:
  - Administration
  - Overview
- Today:
  - Another demo
  - Ch. 2 - Discrete-Time Signals and Systems

Discrete Time Signals

- Samples of a CT signal:

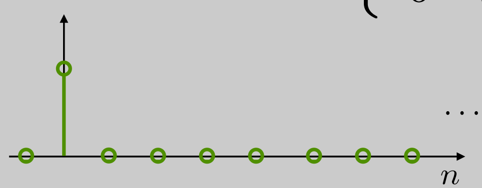
$$x[n] = X_a(nT) \qquad n = 1,\ 2, \cdots$$



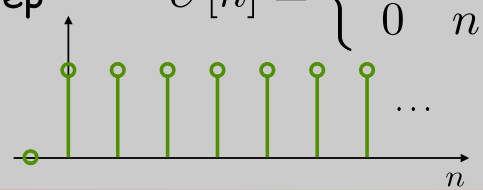
- Or, inherently discrete (Examples?)

Basic Sequences

- Unit Impulse $\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$

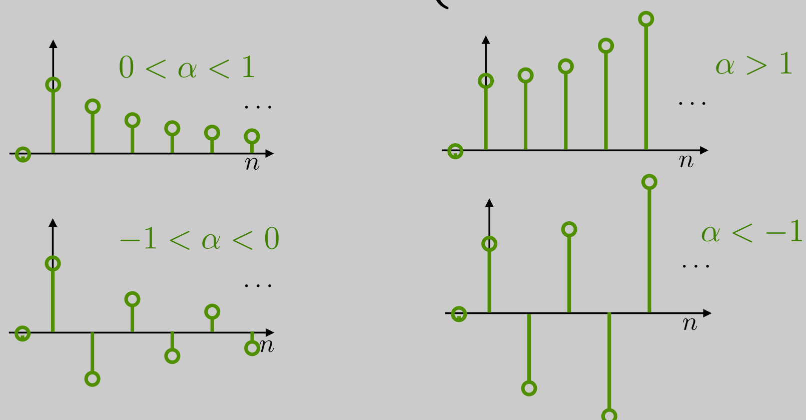


- Unit Step $U[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$



Basic Sequences

- Exponential $x[n] = \begin{cases} A\alpha^n & n \geq 0 \\ 0 & n < 0 \end{cases}$



Bounded unBounded

Discrete Sinusoids

$$x[n] \;=\; A\cos(\omega_0 n + \phi)$$

or, $x[n] \;=\; Ae^{j\omega_0 n + j\phi}$

Q: Periodic or not? $x[n+N] = x[n]$ for N integer

Discrete Sinusoids

$$x[n] \;=\; A\cos(\omega_0 n + \phi)$$

or, $x[n] \;=\; Ae^{j\omega_0 n + j\phi}$

Q: Periodic or not? $x[n+N] = x[n]$ for N integer

A: If ω_0/π is rational (Different than C.T.!)

- To find fundamental period N
  - Find smallest integers K,N :

$$\omega_0 N = 2\pi K$$

Discrete Sinusoids

- Example:

$\cos(5/7\pi n)$ $N = 14$ $(K = 5)$

$\cos(\pi/5n)$ $N = 10$ $(K = 1)$

$\cos(5/7\pi n) + \cos(\pi/5n) \;\Rightarrow\; N = \text{S.C.M}\{10, 14\} = 70$

Discrete Sinusoids

- Another Difference:

  Q: Which one is a higher frequency?

$$\omega_0 = \pi \quad \text{or} \quad \omega_0 = \frac{3\pi}{2}$$

Discrete Sinusoids

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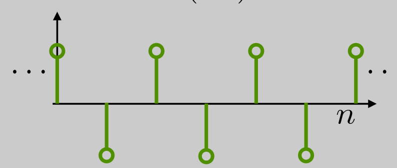
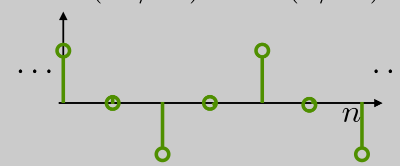
$\cos(\pi n)$



$\cos(3\pi/2n) = \cos(\pi/2n)$



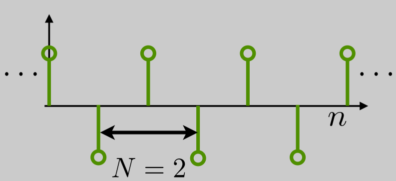
Discrete Sinusoids

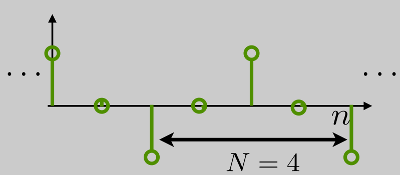
- Another Difference:

  Q: Which one is a higher frequency?

$$\omega_0 = \pi \quad \text{or} \quad \omega_0 = \frac{3\pi}{2}$$

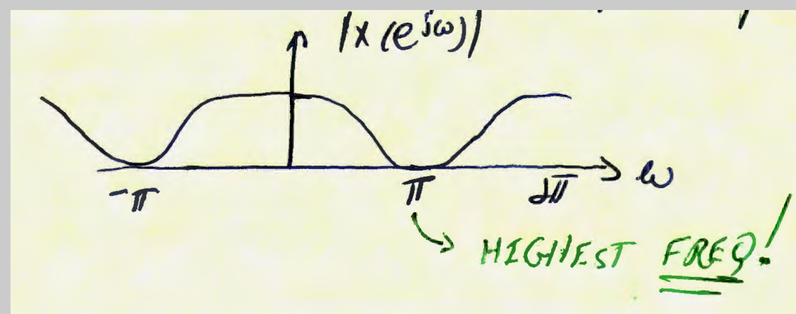
A: $\omega_0 = \pi$





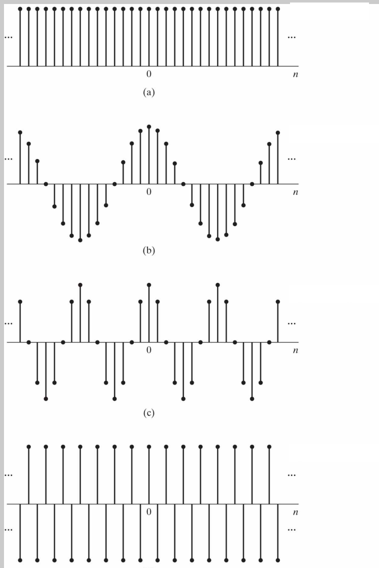
Discrete Sinusoids

- Recall the periodicity of DTFT



Discrete Sinusoids

$\cos(w_0 n)$



$\omega_0 = 0$

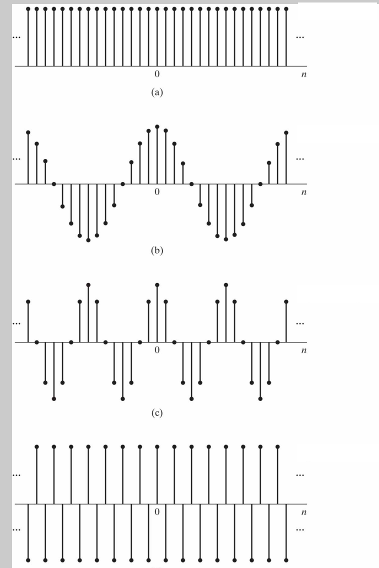
$\omega_0 = \pi/8$

$\omega_0 = \pi/4$

$\omega_0 = \pi$

Discrete Sinusoids

$\cos(w_0 n)$



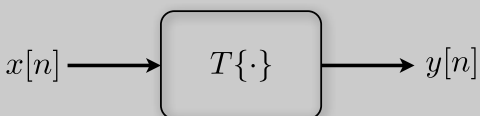
$\omega_0 = 2\pi$

$\omega_0 = \frac{15}{8}\pi$

$\omega_0 = \frac{7}{4}\pi$

$\omega_0 = \pi$

Discrete Time Systems



What properties?

- Causality:
  - $y[n_0]$ depends only on x[n] for $\infty \le n \le n_0$

Properties of D.T. Systems Cont.

- Memoryless:
  - y[n] depends only on x[n]

    Example: $y[n] = x[n]^2$
- Linearity:
  - Superposition:

    $$T\{x_1[n] + x_2[n]\} = T\{x_1[n]\} + T\{x_2[n]\} = y_1[n] + y_2[n]$$
  - Homogeneity:

    $$T\{ax_1[n]\} = aT\{x[n]\} = ay[n]$$

Properties of D.T. Systems Cont.

- Time Invariance:
  - If: $y[n] = T\{x[n]\}$
    Then: $y[n-n_0] = T\{x[n-n_0]\}$

- BIBO Stability
  - If: $|x[n]| \le B_x < \infty \quad \forall\ n$
    Then: $|y[n]| \le B_y < \infty \quad \forall\ n$

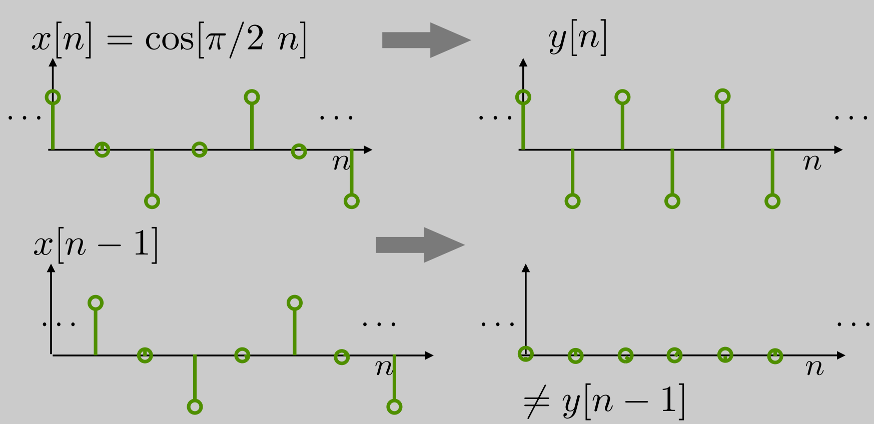
Example:

	Causal	L	TI	memoryless	BIBO stable
Time Shift $y[n] = x[n-n_d]$	if nd>=0	Y	Y	if nd=0	Y
Accumulator $y[n] = \sum_{k=-\infty}^{n} x[k]$	Y	Y	Y	N	N
Compressor $y[n] = x[Mn]$ $M > 1$	N	Y	N	N	Y

Examples

Why the compressor is NOT Time Invariant?

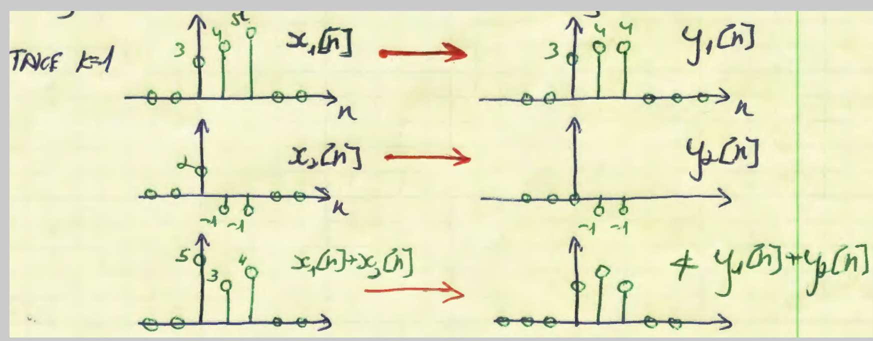
Suppose M=2,



Examples

Non-Linear system: Median Filter

$$y[n] = \mathrm{MED}\{x[n-k],\ \cdots,\ x[n+k]\}$$



Example: Removing Shot Noise

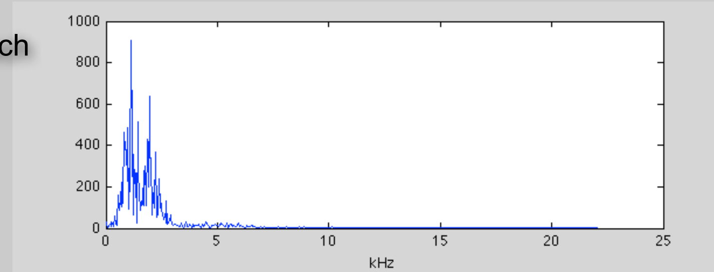
- From NPR This American Life, ep.203
  "The Greatest Phone Message in the World"

em.. (giggle) There comes a time in life, when .. eh... when we hear the greatest phone mail message of all times and .... well here it is... eh.. you have to hear it to believe it..

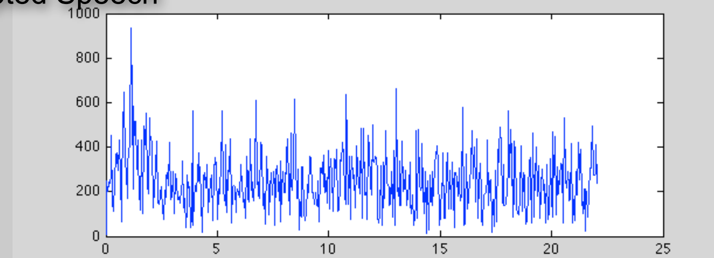
corrupted message

Spectrum of Speech

Speech

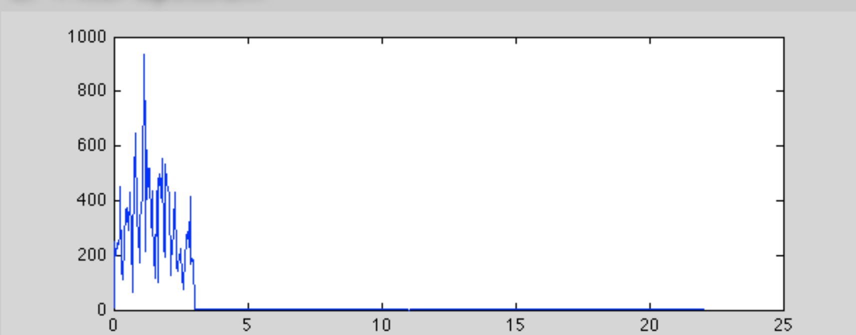


Corrupted Speech



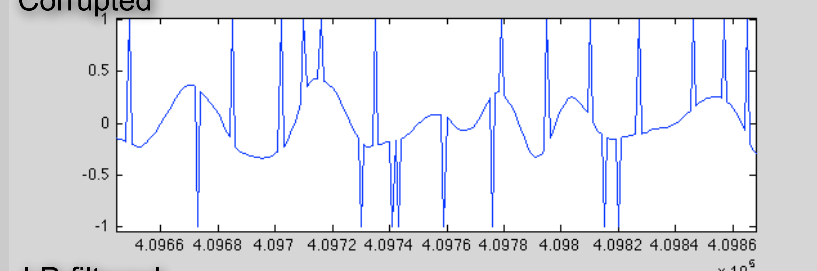
Low Pass Filtering

LP-Filter Spectrum

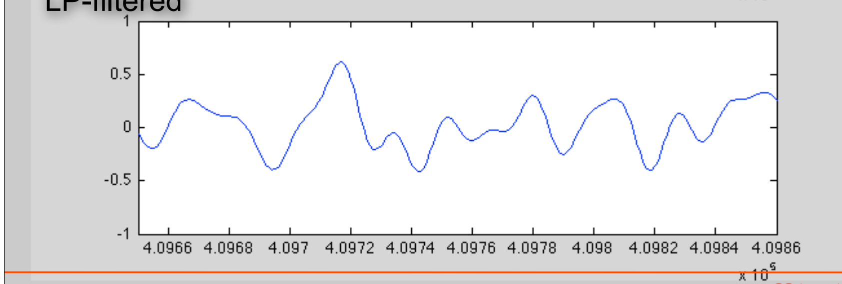


Low-Pass Filtering of Shot Noise

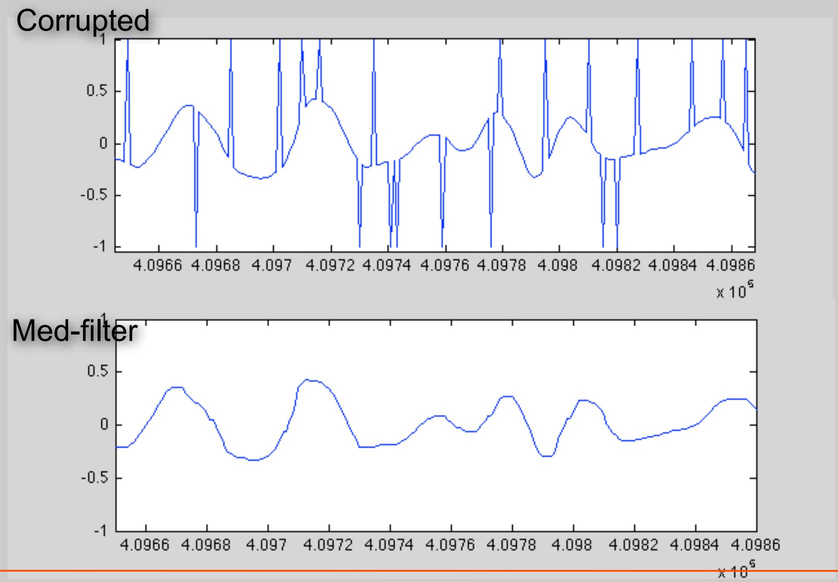
Corrupted



LP-filtered



Low-Pass Filtering of Shot Noise

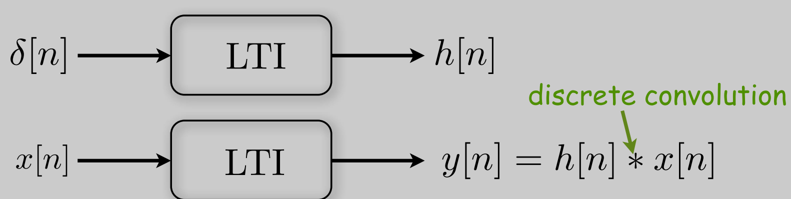


The Greatest Message of All Times...

- I thought you'll get a kick out of a message from my mother......
  Hi Fred, you and the little mermaid can go blip yourself. I told you to stay near the phone... I can't find those books.. you have other books here.. it must be in Le'hoya.. call me back... I'm not going to stay up all night for you... .... Bye Bye...

Discrete-Time LTI Systems

- The impulse response h[n] completely characterizes an LTI system "DNA of LTI"



$$y[n] = \sum_{N=-\infty}^{\infty} h[m]x[n-m]$$

Sum of weighted, delayed impulse responses!

BIBO Stability of LTI Systems

- An LTI system is BIBO stable iff h[n] is absolutely summable

$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty$$

BIBO Stability of LTI Systems

- Proof: "if"

$$|y[n]| = \left| \sum_{k=-\infty}^{\infty} h[k]x[n-k] \right|$$

$$\leq \sum_{k=-\infty}^{\infty} |h[k]| \cdot |x[n-k]|$$

$$\leq B_x$$

$$\leq B_x \sum_{k=-\infty}^{\infty} |h[k]| < \infty$$

BIBO Stability of LTI Systems

- Proof: "only if"
  - suspect $\sum_{k=-\infty}^{\infty} |h[k]| = \infty$ show that there exists bounded x[n] that gives unbounded y[n]
  - Let: $x[n] = \dfrac{h[-n]}{|h[-n]|} = \text{Sign}\{h[-n]\}$

$$y[n] = \sum h[k]x[n-k]$$

$$y[0] = \sum h[k]x[-k] = \sum h[k]h[k]/|h[k]| = \sum |h[k]| = \infty$$