EECS 123 Digital Signal Processing — University of California, Berkeley: Fall 2007
Gastpar — November 7, 2007

Exam 2

Last name	First name	SID

- You have 1 hour and 45 minutes to complete this exam.
- The exam is closed-book and closed-notes; calculators, computing and communication devices are *not* permitted.
- No form of collaboration between the students is allowed. If you are caught cheating, you may fail the course and face disciplinary consequences.
- However, four single-sided US letter pages of handwritten and *not photocopied* notes are allowed.
- Additionally, you receive Tables 2.1, 2.2, 2.3, 3.1, 3.2, 8.1, 8.2 from the class textbook.
- If we can't read it, we can't grade it.
- We can only give partial credit if you write out your derivations and reasoning in detail.
- You may use the back of the pages of the exam if you need more space.

*** GOOD LUCK! ***

Problem	Points earned	out of
Problem 1		20
Problem 2		20
Problem 3		20
Problem 4		25
Problem 5		15
Total		100

Partial Credit.

Partial credit will be given only if there is sufficient information in your work. In general, a good way to show that you understand what's going on is for example to provide plots, sketches, and formulas for *intermediate* signals. That way, if you make an error somewhere along the way, we can trace it and evaluate whether or not you understood the basics of the problem.

Useful Formulae.

- For the continuous-time box function,

$$b(t) = \begin{cases} 1, & -T \leq t \leq T \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

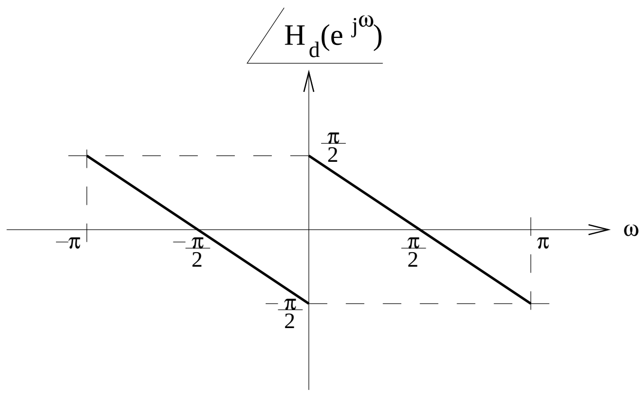
  the (continuous-time) Fourier transform is given by

$$B(j\Omega) = \frac{2\sin(\Omega T)}{\Omega}. \tag{2}$$

- $\tan(\pi/4) = 1$

Problem 1 *(Phase Properties.)* 20 Points

Given the phase characteristics of a generalized linear phase FIR filter $H_d(e^{j\omega})$ shown below, answer the following questions. Include brief explanations to get credit.



(a) (6 pts) Is this a symmetric (i.e., Type-I or Type-II) or an anti-symmetric (i.e., Type-III or Type-IV) filter? Why?

(b) (8 pts) Can the filter length be determined from the given information? If yes, what is the length? If not, why not?

(c) (6 pts) The filter magnitude response has a DC gain of 1. True or False? Why?

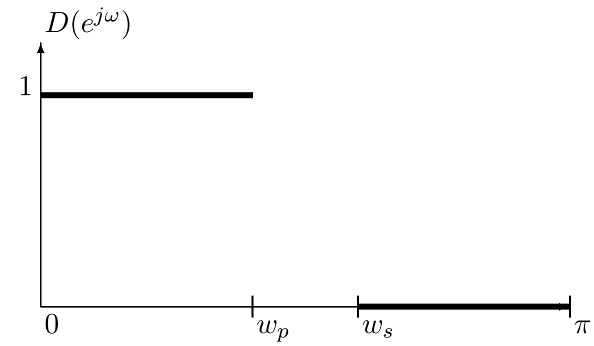
Problem 2 *(Filter Design.)* 20 Points

(a) (12 pts) A continuous-time filter is given by $H_a(s) = \frac{2}{2+s}$. We want to use this as a prototype filter to design a discrete-time filter *via the bilinear transform* with a suppression of $1/\sqrt{2}$ at $\omega_c = \pi/2$. (Note that the filter has a gain of 1 at frequency zero.) Give $H(z)$ explicitly.

(b) (8 pts) Sketch $|H(e^{j\omega})|$ for the filter designed in part (a) over the interval $0 \le \omega \le \pi$.

Problem 3 *(Filter Design.)* 20 Points

(a) (5 pts) We want to approximate the lowpass filter in Figure **??** with the optimal minimax (Parks-McClellan) Type-I filter $h(n)$. Just like in class, the band $0 \leq \omega \leq \omega_p$ is the desired passband, and the band $\omega_s \leq \omega \leq \pi$ is the desired stopband. In Figure **??**, provide a sketch the form of $H(e^{j\omega})$ when $h(n)$ has length 3. Recall that the amplitude of a Type-I filter has the form $A(e^{j\omega}) = \sum_{k=0}^{M/2} a_k \cos(k\omega)$.



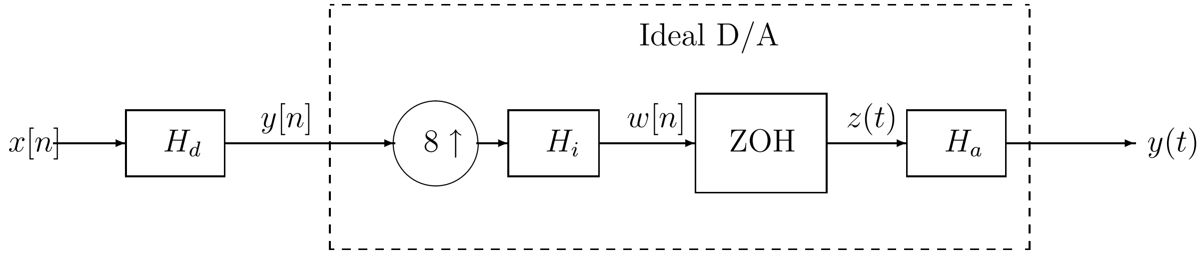
Assume that $\omega_s = \pi - \omega_p$

Figure 1:

(b) (10 pts) Determine the filter $h(n)$. *Hint:* If you find it easier, you may start by assuming that $\omega_p = \pi/3$ and thus, $\omega_s = 2\pi/3$.

(c) (5 pts) What is the largest value of ω_p such that the maximum error is $\delta \leq 1/6$?

Problem 4 *(Multirate System.)* 25 Points



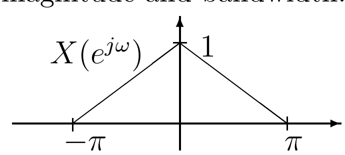
$$H_d(e^{j\omega}) = \begin{cases} 1, & |\omega| \le \frac{\pi}{2} \\ 0, & \text{else} \end{cases} \qquad H_i(e^{j\omega}) = \begin{cases} 8, & |\omega| \le \frac{\pi}{8} \\ 0, & \text{else} \end{cases}$$

The ZOH operates at interval T but produces pulses of width $\frac{T}{4}$, i.e.

$$g(t) = \begin{cases} 1, & 0 \le t \le \frac{T}{4} \\ 0, & \text{else,} \end{cases}$$

and the output is $z(t) = \sum_{n=-\infty}^{\infty} w[n]g(t - nT)$.

(a) (9 pts) For $X(e^{j\omega})$ pictured, sketch $W(e^{j\omega})$. Label the magnitude and bandwidth.



(b) (7 pts) For the same $X(e^{j\omega})$, sketch $|Z(j\Omega)|$ for $\Omega = [\frac{-8\pi}{T}, \frac{8\pi}{T}]$.

(c) (7 pts) For this part, we are interested in making the dashed block (with input $y[n]$ and output $y(t)$) an ideal D/A converter for arbitrary $y[n]$, i.e. ignore the effects of H_d.

What is the "cheapest"[1] filter $H_a(j\Omega)$ such that between $y[n]$ and $y(t)$ we have an ideal D/A. Sketch $|H_a(j\Omega)|$ and specify its value where necessary.

(d) (2 pts) Explain (in words) the advantages and disadvantages of this D/A converter design over a direct implementation (as we have discussed it in class).

[1]the filter having the largest transition band (as we have seen in class, the smaller (i.e., steeper) the transition band, the more filter coefficients are necessary)

Problem 5 *(Filter Bank.)* 15 Points

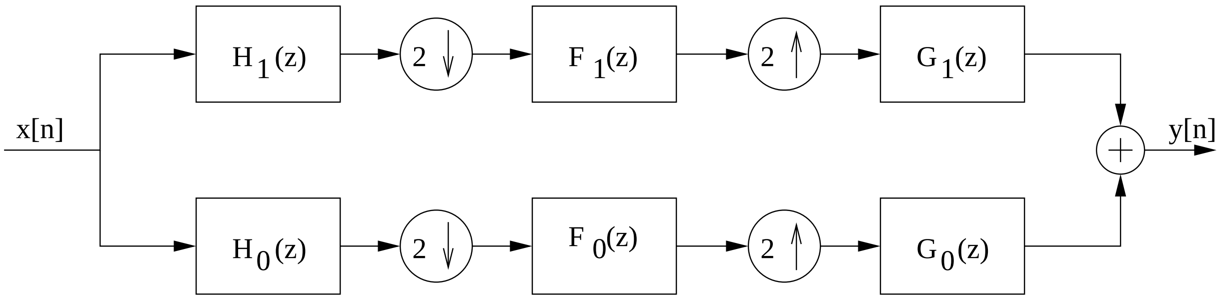


Figure 2: A two-channel filter bank.

For the filter bank in Figure **??**, find the conditions for perfect reconstruction, i.e., the conditions on the filters $F_i(z), G_i(z), H_i(z)$ (for $i = 0, 1$) such that $y[n] = x[n]$.