Problem Set 9

EECS123: Digital Signal Processing

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Reading Assignment: Section 7.1 and Section 7.4 of the textbook Oppenheim, Schafer, and Buck.

1. We wish to approximate the following *causal* filter by using a rectangular window.

   $$H(e^{j\omega}) = \frac{1}{1 - \alpha e^{-j\omega}}, \quad \alpha = 0.977.$$

   Use rectangular windows of lengths L=32,64,128, and 256. Plot each in a separate plot and plot the "desired" response in each plot using a dashed line. Comment on the effect of the length of the window. Comment on the presence or absence of Gibbs' phenomenon.

   Note: For each plot, please use a log scale for the magnitude.

2. Problem 7.37 from Oppenheim, Schafer, and Buck.

3. For this problem, we will examine an FIR bandpass filter.

   (a) An FIR bandpass filter with lower and upper cutoff frequencies at $\frac{\pi}{4}$ and $\frac{3\pi}{4}$ respectively, can be obtained with the MATLAB code below:

   ```
   n=35;
   c=[0.25,0.75];
   a=fir1(n,c);
   ```

   Here, the filter length is $(n + 1)$, and its normalized cutoff frequencies in $[0, \pi]$ are at $\lambda_c = \pi c$. The filter coefficients are returned in a.

   (b) Plot the magnitude and phase of your filter's frequency response using

   ```
   [h,w]=freqz(a,1,512);
   mag = abs(h);
   phase = angle(h);
   plot(w,mag);
   plot(w(175:340),mag(175:340));
   semilogy(w,mag);
   plot(w,phase)
   ```

Here, `semilogy();` produces a plot with a logarithmic vertical scale. What is the role of `plot(w(175:340),mag(175:340));`?

Interpret the appearance of the magnitude and phase plots. In particular:

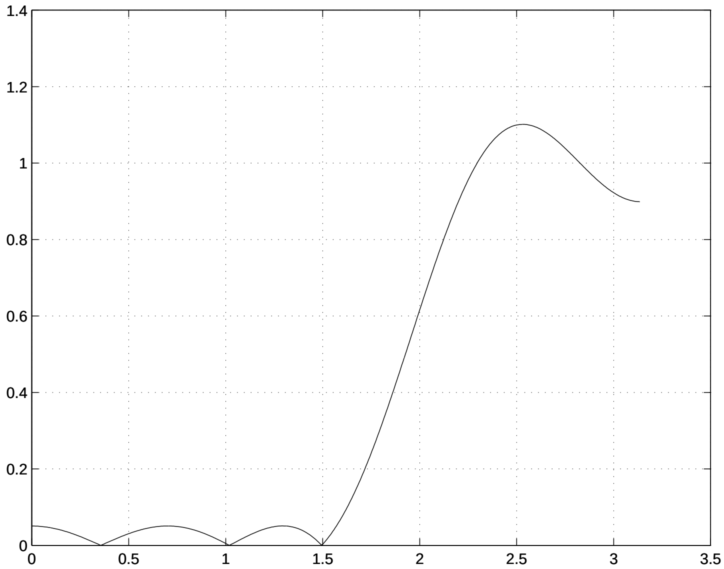
i. What are the passband ripple and stopband attenuations of your filter in dB, what is the transition bandwidth, and how accurate are the positions of the band edges relative to the design values?

ii. How many zeros does the filter have? How many of them are on the unit-circle?

iii. How many poles does the filter have, and where?

iv. Explain the significance of the slope of the phase and the location and height of all jumps.

4. Smart Alec designed a length 11 FIR filter using the Parks-McClellan algorithm. Its magnitude frequency response is shown below:



(a) Determine (as accurately as you can from the plot) the specifications that Smart Alec used in the design. What were the weights used for the pass and stop bands?

(b) Smart Alec forgot to plot the phase response. He only remembers that $H_d(e^{j0}) > 0$. Sketch the complete phase response below the given magnitude response. Clearly label in your plot the positions of the transitions and their magnitude.

5. We wish to design a bandpass filter using Parks-McClellan algorithm. The specifications are $\omega_{s1} = 0.16\pi$, $\omega_{p1} = 0.32\pi$, $\omega_{p2} = 0.67\pi$, $\omega_{s2} = 0.83\pi$, $\delta_p = \delta_{s1} = \delta_{s2} = 0.01$. Find the smallest filter order N that meets this specification. Plot the impulse response of this filter.

For any value of `N`, the following MATLAB code will be helpful:

```
f = [ 0 0.16 0.32 0.67 0.83 1];
m = [0 0 1 1 0 0];
q = [1 1 1];
a = remez(N,f,m,w);
% newer version of MATLAB use firpm() instead of remez()
```

The vector `f` corresponds to $[0,\ \omega_{s1}/\pi,\ \omega_{p1}/\pi,\ \omega_{p2}/\pi,\ \omega_{s2}/\pi,\ \pi/\pi]$, the vector `m` contains the desired values of the magnitude at the frequencies in `f`, and the vector `q` is the error weighting in the passband and stopbands.

6. The goal of this assignment is to compare a low-pass filter produced using the Window method with a similar low-pass filter produced using Parks-McClellan algorithm. Use MATLAB.

   (a) Low-pass filter using Windowing: Design a length-21 low-pass filter with passband $\{0 \leq |\omega| \leq 0.4\pi\}$ and stopband $\{0.5\pi \leq |\omega| \leq \pi\}$. You should set the cutoff frequency to the center of the transition band when using the fir1 function. Apply the following three windows: Rectangular, Hamming, and Blackman.

      i. Plot the impulse response, magnitude response and zero locations of the three filters.
      ii. Compare the transition bandwidths of the three filters.
      iii. Calculate the minimax error in the passband and stopband for each filter. Use at least 100 samples of the DTFT for this calculation.

   (b) Low-pass filter using Parks-McClellan: Design a length-21 low-pass filter with the same specs as in part (a). Again, set the passband to $\{0 \leq |\omega| \leq 0.4\pi\}$ and the stopband $\{0.5\pi \leq |\omega| \leq \pi\}$. We want a desired response of 1 in the passband and 0 in the stopband.

      i. Plot the impulse response, frequency response and zero locations for the filter.
      ii. How many extremal frequencies are there (places where the ripples are the same maximum size)?
      iii. How many "small ripples" are there that do not give extremal frequencies, and if any, are they in the passband or the stopband?
      iv. Compute the minimax error in the passband and stopband. Again, use at least 100 points of the DTFT.

   Finally, compare the passband and stopband deviation with each of the windowed filters.

   Useful functions for completing this problem are listed below:

`firpm, remez`	Parks-Mcclellan Filter
`fir1, fir2`	Filter design using a specific window
`window`	Outputs a specific window
`pzmap`	Gives the Poles and Zeros for any transfer function
`max`	Computes the maximum value of elements in a matrix
`freqz`	Calculates DTFT at a finite number of points