Problem Set 8

EECS123: Digital Signal Processing

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Spring 2008

Revision: Typos in Problem 4(e) and Problem 5(d) have been corrected (03/31/08).

Note: Problem 1 has been extended. You should submit only Problem 1(a) and 1(b) for grades. Any innovative solutions to Problem 1(c) are welcome for discussions.

1. This problem deals with the identification of complex exponentials with by using minimum number of samples. You should use the annihilation-filter method.

   (a) It is known that $y_1[n] = a_0 e^{j\omega_0 n} + a_1 e^{j\omega_1 n}$. Using this prior information, find the unknown $a_0, a_1, \omega_0, \omega_1$ using the samples $\{y_1[n]\}_0^3$ stored in `y1.mat`.

   (b) Consider another signal $y_2[n]$. It is known that $y_2[n] = \sum_{k=0}^{N-1} a_k e^{j\omega_k n}$, where the only information available about N is that $N \leq 5$. Using this information, find the unknown $\{a_k, \omega_k\}_0^4$ using the samples $\{y_2[n]\}_0^9$ stored in `y2.mat`.

   (c) **Optional:** Consider a third signal $z[n] = a_0 e^{j\omega_0 n} + a_1 e^{j\omega_1 n}$. This time, $z[n]$ is only available through two i.i.d. Gaussian noise affected copies of $z[n]$, viz., $z_3[n]$ and $z_4[n]$. To combat noise, you are given 16 and 100 samples, respectively, of the noise-affected signal. The signals $\{z_3[n]\}_0^{15}$ and $\{z_4[n]\}_0^{99}$ are stored in `z3.mat` and `z4.mat`, respectively. Estimate ω_0 and ω_1. (Solution to this problem will not be provided).

   Note: All the `.mat` files can be found on the course webpage. You can load the variables using the `load` command in MATLAB.

   Note: For factorizing at $N = 2$, you can use quadratic formula. For larger values of N, plot $|H_d(e^{j\omega})|$ to obtain the roots of $H_d(e^{j\omega}) = 0$. Some people had success with the `roots()` function in MATLAB.

2. Problem 5.42 from Oppenheim, Schafer, and Buck.

3. Problem 5.44 from Oppenheim, Schafer, and Buck.

4. Recall that discrete-time filters with frequency response of the form $H_d(e^{j\omega}) = |H_d(e^{j\omega})|e^{-jM\omega}$ are termed *linear phase*, whereas those with $H_d(e^{j\omega}) = R(\omega)e^{-jM\omega + j\beta}$ with $R(\omega)$ real are termed *generalized linear phase*. For each of the following filters, determine whether it is a generalized linear phase filter. If it is, then find $R(\omega)$, M, and β, and indicate whether it is also a linear phase filter.

(a) $\{h[n]\}_{n=0}^{2} = \{2, 1, 2\}$

(b) $\{h[n]\}_{n=0}^{2} = \{1, 2, 3\}$

(c) $\{h[n]\}_{n=0}^{2} = \{-1, 3, 1\}$

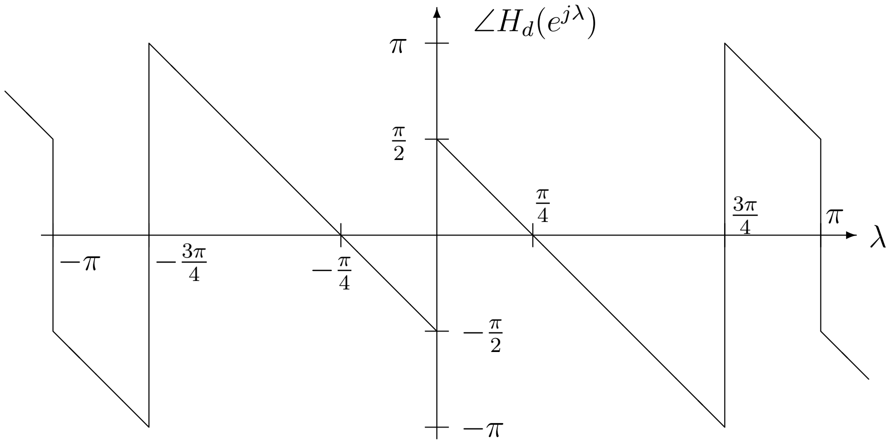
(d) $\{h[n]\}_{n=0}^{4} = \{1, 1, 1, -1, -1\}$

(e) $\{h[n]\}_{n=0}^{2} = \{1, 0, -1\}$

(f) $\{h[n]\}_{n=0}^{3} = \{2, 1, 1, 2\}$

In each case, the remaining terms of the unit pulse response of the filter are zero.

5. Given the following phase response $\angle H_d(e^{j\lambda})$ of a generalized linear-phase FIR filter, answer the following questions. Explain your answers.



(a) Is the filter (i) Type-I GLP, (ii) Type-II GLP, (iii) neither Type I nor Type II GLP, or (iv) is the given information insufficient to make any of the preceding statements?

(b) Can you characterize the filter as being (i) possibly low-pass, (ii) possibly high-pass, (iii) neither high-pass nor low-pass, or (iv) is the given information insufficient to make any of the preceding statements?

(c) Can the filter length be determined from the given information? If so, what is it?

(d) You are now asked to find the filter with the given phase characteristic given two *additional* pieces of information: (i) The filter has a magnitude gain of 4 at $\lambda = \pi/2$, i.e. $|H_d(e^{j\pi/2})| = 4$; and (ii) the value of the integral:

$$\int_{-\pi}^{\pi} |H_d(e^{j\lambda})|^2 d\lambda = 20\pi$$

[Hint: use symmetry properties of GLP filter and Parseval's theorem.]

(e) Is your filter of (d) unique?