Problem Set 5

EECS123: Digital Signal Processing

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1. Problem 8.43 from Oppenheim, Schafer, and Buck.

2. Problem 9.32 from Oppenheim, Schafer, and Buck.

3. Calculate the DFT of the length–8 sequence by hand { 1, 2, 3, 4, 5, 6, 7, 8 } using the radix–2 decimation-in-time algorithm. Check your answer by calculating the DFT by direct computation or by using MATLAB.

4. Problem 9.49 (a) & 9.49 (b) from Oppenheim, Schafer, and Buck.

5. In problems (a)-(b), $x_a(t)$ is an analog signal consisting of a sum of sinusoids, and it is desired to determine the number and frequencies of the sinusoids in the sum. N samples of $x_a(t)$ are taken at intervals T, generating the sequence $\{x_a(nT)\}_{n=0}^{N-1} = \{x[n]\}_{n=0}^{N-1}$.

   NOTE: You may want to use the `subplot()` command of MATLAB to produce multiple plots on a single page, to facilitate comparison and save paper and time.

   (a) Let $x_a(t) = \cos(8\pi t) + 0.75\cos((30/7)\pi t)$. In this exercise, we investigate how our choice of N and T affects the analysis of the spectrum. We will use a 256-point DFT to get a fine sampling of $X_d(e^{j\omega})$ for each choice of N and T. (Recall that $X[m] = X_d(e^{\frac{j2\pi m}{N}})$.)

   In order to conveniently vary N and T, create the file "test.m" to generate N samples of $x_a(t)$ at intervals of T. You may want to use the following code:

```
function x=test(N,T)
x=zeros(1,256);
for i=1:N
x(i)=cos(8*pi*(i-1)*T) + 0.75*cos((30/7)*pi*(i-1)*T);
end
return
```

   Note: the returned vector is zero-padded to length 256, so that a 256-point DFT can always be used.

   Given this new function, you can plot the magnitude of the 256-point DFT for $N = 64$ and $T = 1/30$ sec. by typing:

```
x=test(64,1/30);
plot(abs(fft(x)))
```

Compute and plot the magnitude of the 256-point DFT for each of the following cases:

- N=64, T=1/240
- N=64, T=1/30
- N=64, T=1/5
- N=32, T=1/120
- N=32, T=1/30
- N=32, T=1/15

i. Use the expression for the DFT of a single sinusoid to explain the effect of the number N of samples and of the sampling interval T on the resulting plots.

ii. Given a 2 second long segment of $x_a(t)$, how would you choose the sampling interval T to best resolve the sinusoidal components?

iii. Given $x_a(t)$ for $-\infty < t < \infty$, and that only 128 samples are to be acquired, how would you choose T to best resolve the sinusoidal components?

iv. Given that $T = 1/30$, use your previous reasoning to give an estimate of the minimum number N_{min} of samples required to resolve the sinusoids. Also determine N_{min} experimentally (i.e. using `test(N,1/30)`, how small can `N` be before you cannot tell that there are two sinusoids?)

(b) In this exercise, we will investigate how the size of the DFT affects our analysis of the signal spectrum. For this exercise, we will fix $N = 64$ and $T = 1/30$, and vary the DFT size and the input sinusoidal frequencies. It may help to create a file "test2.m" for this, as shown below:

```
function x=test2(M,F1,F2)
x=zeros(1,M);
for i=1:64
x(i)=cos(F1*pi*(i-1)/30) + 0.75*cos(F2*pi*(i-1)/30);
end
return
```

Now, `M` will be the DFT size (or size of the returned vector), and `F1*pi` and `F2*pi` will be the frequencies of the two sinusoids.

i. Plot the 64-point DFT of the signal in problem (1), using:

```
x=test2(64,8,30/7);
plot(abs(fft(x)))
```

Using this plot, determine the analog frequencies and magnitudes of the analog sinusoidal components in $x_a(t)$. Note: instead of reading values from the plot, you can display the numerical spectrum values with `abs(fft(x))`.

Compare your computed values for the frequencies and magnitudes with the actual values.

ii. Now let $x_a(t) = cos(7.5\pi t) + 0.75cos(3.75\pi t)$, and $T = 1/30, N = 64$. Plot the magnitude of the 64-point DFT of the signal using

```
x2=test2(64,7.5,3.75);
plot(abs(fft(x2)))
```

Repeat (i) for this data.

iii. Why are the two plots in (i) and (ii) different? To support your explanation, refer to your N=64, T=1/30 plot above and to a plot of the magnitude of a 256-point DFT of $\{x(n)\}_{n=0}^{63}$ Note that you can generate this last plot by using

```
x3=test2(256,7.5,3.75);
plot(abs(fft(x3)))
```

iv. Under what conditions on the analog sinusoidal frequencies f_1 and f_2 and on T and N will the plot of samples of $x_a(t) = cos(2\pi f_1 t) + 0.75 cos(2\pi f_2 t)$ look similar to that of (ii)? Prove your answer analytically.

6. **Supplementary Problem (optional):** Problem 9.48 from Oppenheim, Schafer, and Buck. (This problem will not be graded and its solution will not be provided).

7. **Supplementary Problem (optional):** Derive a radix–3 decimation-in-time FFT algorithm for a length–9 DFT. Sketch a pictorial representation of your algorithm, showing the connections between the length–3 DFTs in the two stages, the ordering of the input and output data, and the twiddle factors. (This problem will not be graded and its solution will not be provided).