Problem Set 4

EECS123: Digital Signal Processing

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1. Problem 8.31 from Oppenheim, Schafer, and Buck.

2. Problem 8.35 from Oppenheim, Schafer, and Buck.

3. Problem 8.36 from Oppenheim, Schafer, and Buck.

4. When John Smith plugged in his new digital TV, he was dismayed to see ghosts on the screen. He figured they are created by a secondary delayed RF propagation path. Some measurements revealed that this problem could be modeled by his digital video signal being filtered by an LTI system with unit pulse response $h[n] = \delta[n] - 0.2\delta[n - k_0]$.

   To correct the problem, he wishes to process his signal by an inverse filter with unit pulse response $g[n]$, so that the effect of $h[n]$ is canceled. To determine $g[n]$ he computes the N-point DFT $\{H[k]\}_{k=0}^{N-1}$, with $N = 4k_0$, of the sequence $h[n]$ and then defines $g[n]$ as the inverse DFT of $G[k] = 1/H[k]$, for $k = 0, 1, \ldots, N - 1$. Determine $g[n]$ and the convolution $g[n] * h[n]$. Is the system with $g[n]$ the inverse of the one with unit pulse response $h[n]$? Find the transfer function of an exact inverse, and comment on the relation to $g[n]$.

5. (MATLAB) Let $\omega_0 = \frac{\pi}{16}$. Consider three values $N = 16$, $N = 64$, and $N = 256$. Let $x_N[n] = \cos(\omega_0 n)$ for $n = 0, \ldots, N - 1$. Let $X_N[k]$ be the N-point DFT coefficients.

   (a) Plot $|X_N[k]|$ for the above three values of N. You will find the `fft()` function useful. Use `subplot()` to compare the results for different values of N.

   (b) Explain the behavior of $|X_N[k]|$ as a function of N.

   Note: Remember to attach the MATLAB code and the generated plots to your homework submission.

6. Let V be a vector space over real number field. Let S be a subspace of V. Let energy of $v \in V$ be defined as $||v||^2 := \langle v, v \rangle$. Let $P(v) : V \to S$ be an orthogonal projection of v on S, i.e., $(v - P(v)) \perp S$.

   (a) Show that $||P(v)||^2 + ||v - P(v)||^2 = ||v||^2$. (This is analogous to the well known Pythagoras Theorem of geometry).

   (b) Let $u, v \in V$. Prove or disprove, if $P(u) = P(v)$, then $(v - P(v)) = c(u - P(u))$, for some $c \in \mathbb{R}$.