Problem Set 3

EECS123: Digital Signal Processing

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Spring 2008

1. Let $x[n]$ be a finite real sequence, which is non-zero only for $|n| \leq M$. Let

$$\bar{x} = \frac{1}{2M+1}\sum_{n=-M}^{M} x[n], \quad P_x = \frac{1}{2M+1}\sum_{n=-M}^{M} x^2[n], \text{and} \quad \sigma_x^2 = \frac{1}{2M+1}\sum_{n=-M}^{M} (x[n]-\bar{x})^2.$$

   (a) Show that $\sigma_x^2 = P_x - (\bar{x})^2$.

   (b) Let,

$$E(x[n], c) = \frac{1}{2M+1}\sum_{n=-M}^{M} (x[n]-c)^2, \quad c \in \mathbb{R}.$$

   Show that $E(x[n], c)$ is minimized at $c = \bar{x}$.

   (c) Write a formula for σ_x^2 in terms of $X(e^{j\omega})$.

   Note: σ_x^2 can be thought of as the mean-square error when $x[n]$ is approximated by a constant sequence $\bar{x}$.

2. Consider the frequency response (DTFT) $H(e^{j\omega})$ of a discrete-time LTI system. Let $h[n]$ be the impulse response. Assume that $h[n]$ satisfies the following five properties:

   (i) The system is causal.

   (ii) $H(e^{j\omega}) = H^*(e^{-j\omega})$.

   (iii) The DTFT of the sequence $h[n+1]$ is real.

   (iv) $\frac{1}{2\pi}\int_{-\pi}^{\pi} |H(e^{j\omega})|^2 d\omega = 2$.

   (v) $H(e^{j\pi}) = 0$.

   Answer the following questions:

   (a) Show that properties (i)–(iii) imply that $h[n]$ is non-zero for only a finite duration.

   (b) Find all possible discrete signals $h[n]$ that satisfy properties (i)–(v).

3. Let $0 < |a| < |b|$. Find the inverse $\mathcal{Z}$-transform of

$$X(z) = \log\left(\frac{z+a}{z+b}\right), |z| > |b|,$$

where log is the natural log or log to the base e.

4. Determine the region of convergence of $Y(z)$ where,

(a) $Y(z) = X_1(z) + X_2(z)$,

$$\begin{aligned} X_1(z) &= \frac{z}{z+1}, \quad |z| > 1 \\ X_2(z) &= \frac{z^{-2}}{z+1}, \quad |z| > 1. \end{aligned}$$

(b) $Y(z) = H(z)X(z)$,

$$\begin{aligned} x[n] &= \delta[n] + 2\delta[n-1] \\ H(z) &= \frac{1}{z^2+7z+10}, \quad |z| > 5. \end{aligned}$$

(c) $Y(z) = z^{-2}X(z)$,

$$x[n] = 2^n u[-n-1].$$

5. Suppose that the 8-point DFT of a sequence $\{x_n\}_{n=0}^{7} = \{a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7\}$ is given by $\{X_m\}_{m=0}^{7} = \{A_0, A_1, A_2, A_3, A_4, A_5, A_6, A_7\}$. If the 4-point DFT of another sequence $\{y_n\}_{n=0}^{3} = \{b_0, b_1, b_2, b_3\}$ is given by $\{Y_m\}_{m=0}^{3} = \{A_0, A_2, A_4, A_6\}$, find the b_k's in terms of the a_k's.

6. Determine z_n, the cyclic convolution of x_n and y_n for the following cases:

(a) $\{x_n\}_{n=0}^{5} = \{1, 2, 3, 4, 5, 6\}$ and $\{y_n\}_{n=0}^{5} = \{1, 0, 0, 1, 0, 0\}$.

(b) $\{x_n\}_{n=0}^{8} = \{1, 2, 3, 4, 5, 6, 0, 0, 0\}$ and $\{y_n\}_{n=0}^{8} = \{1, 0, 0, 1, 0, 0, 0, 0, 0\}$.