Problem Set 2

EECS123: Digital Signal Processing

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1. Find the inverse $\mathcal{Z}$-transform of the following:
   (a) $\frac{3z^{-3}}{(1-\frac{1}{4}z^{-1})^2}$, $|z| < 1/4$.
   (b) $\sin(z)$.
   (c) $\frac{z^7-2}{1-z^{-7}}$, $|z| > 1$.

2. Find $y[n] = h[n] * x[n]$, using $\mathcal{Z}$-transforms, where:

$$x[n] = \begin{cases} n & n \geq 0 \\ 0 & n < 0 \end{cases} \qquad h[n] = \begin{cases} (-1)^n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

3. Consider the signal $x[n] = \{-3, 1, -2, \overset{\downarrow}{0}, 2, -1, 3\}$ with DTFT $X(e^{j\omega})$. The arrow denotes the $n = 0$ location. The signal is zero outside. Compute the following quantities, without explicitly computing $X(e^{j\omega})$:
   (a) $X(e^{j0})$
   (b) Give the possible values for $\angle X(e^{j\omega})$
   (c) $\int_{-\pi}^{\pi} X(e^{j\omega})\, d\omega$
   (d) $X(e^{j\pi})$
   (e) $\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$

4. Two systems with impulse responses

$$h_1[n] = \frac{4}{7}2^{n-1}u[-n] + \frac{11}{7}\left(\frac{1}{4}\right)^{n-1}u[n-1] \qquad h_2[n] = \delta[n] - 3^{n-1}u[-n]$$

   are connected in cascade. For each of the individual systems, as well as for the cascade, determine whether it is causal and/or BIBO stable.

5. The input $x[n] = 2^n(u[n] - 3u[n-1])$ to an unknown LTI system produces the output $y[n] = (3^n - 2^n)u[n]$.
   (a) Determine the unit-pulse (impulse) response $\{h[n]\}_{n=-\infty}^{\infty}$ of the system.

(b) Is the solution unique? What if the system is known to be unstable? What if it is causal?

6. The correlation between $x[n]$ and $y[n]$ is defined as

$$R_{XY}[n] = \sum_{k=-\infty}^{\infty} x^*[k]y[n+k].$$

Let the Fourier transform of $R_{XY}[n]$ be $S_{XY}(e^{j\omega})$. Show that,

$$S_{XY}(e^{j\omega}) = X^*(e^{j\omega})Y(e^{j\omega}).$$

Further, show that $S_X(e^{j\omega}) := S_{XX}(e^{j\omega}) = |X(e^{j\omega})|^2$.