Problem Set 11

EECS123: Digital Signal Processing

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1. This problem sketches a method used for prevention of data-loss due to scratches on CD (compact discs). The same method is used in satellite communication links.

   Consider a real signal $x[n]$ of length 12. Let $\bar{x}[n]$ be a 16 length signal obtained from $x[n]$ by padding four zeros. Let $\{X[k]\}_{k=0}^{15}$ be the 16-pt DFT of $\bar{x}[n]$. In the following questions, $X[k]$ is corrupted by additive noise $W[k]$. Answer the following:

   (a) Let $Y_1[k] = X[k] + W_1[k], k = 0, \ldots, 15$. Assuming that $W_1[k]$ is non-zero for four *known* values of k, describe a procedure to obtain $x[n]$ from $Y_1[k]$. Assume that $0 \leq i_1 < i_2 < i_3 < i_4 \leq 15$ are the four locations where $W_1[k]$ is non-zero.

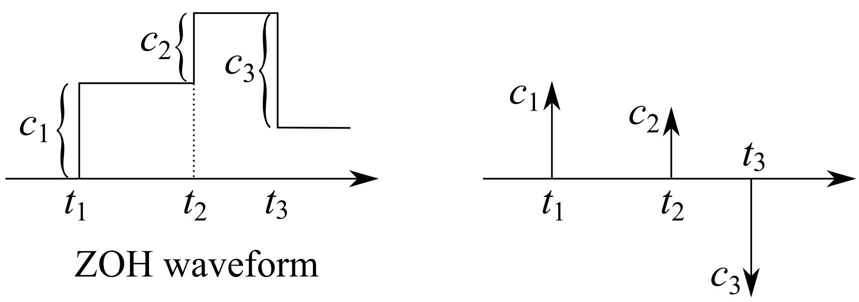
   (b) Let $Y_2[k] = X[k] + W_2[k], k = 0, \ldots, 15$. You are told that $W_2[k]$ is non-zero only at two places $k = i_1$ and $k = i_2$, where i_1, i_2 are unknown. Describe (in detail) a procedure to obtain $x[n]$ from $Y_2[k]$.

   (c) In this part, you will design the zero-padding to tolerate more errors. You are told that $W[k]$ will have t non-zero values at unknown locations. How much zero-padding should be added in $x[n]$ to successfully reconstruct $x[n]$ from $X[k] + W[k]$? Describe in detail.

   (d) (MATLAB) Apply the technique in (a) to `Y1.mat` and the technique in (b) to `Y2.mat` to obtain $\bar{x}[n]$. If you did it right, you should see the same $\bar{x}[n]$ as the solution.
   The locations where $W_1[k]$ is non-zero are $k = 1, k = 5, k = 7$, and $k = 15$. (This noise affects `Y1.mat`).

   The CD connection: $x[n]$ is the data to be stored. $X[k]$ is the data that is actually stored. The scratches in CD can be thought of as successive errors in $W[k]$. And finally, the algebra (computations) happen in a Galois field (instead of the real number field as in this problem).

2. Consider a ZOH waveform shown in the following figure:

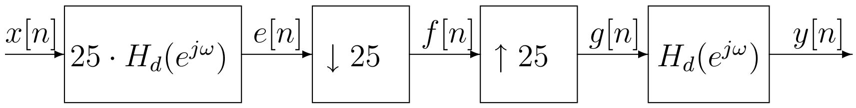


By differentiation, the ZOH output is converted into a sequence of delta functions. In this problem, the constants $\{c_1, c_2, c_3\}$ and $\{t_1, t_2, t_3\}$ are unknown. The finite stream of delta functions is observed after passing through a filter with impulse response $g(t) = e^{-\frac{t^2}{9}}$. Let $y(t)$ be the output of the filter.

Using the samples in `y.mat`, which contains $y(0), y(1), \ldots, y(5)$, find $\{c_1, c_2, c_3\}$ and $\{t_1, t_2, t_3\}$ using the annihilation filter method. (Use `zplane()` for factorization of the annihilation filter).

3. Problem 4.46 from Oppenheim, Schafer, and Buck.

4. Consider the system



where $H_d(e^{j\omega})$ is an (approximate) FIR LPF $\{h[n]\}_{n=0}^{N-1}$ with cutoff frequency at $\frac{\pi}{25}$ and $X_d(e^{j\omega}) = \pi - |\omega|,\ |\omega| \leq \pi$. For sketching spectrum, assume that $H_d(e^{j\omega})$ is ideal.

(a) Sketch $E_d(e^{j\omega}),\ F_d(e^{j\omega}),\ G_d(e^{j\omega}),$ and $Y_d(e^{j\omega})$.

(b) What does this overall system implement?

(c) For accurate low-pass filtering, N must be very large, say $N = 200$, to accurately realize the narrowband filter $H_d(e^{j\omega})$. Why might the above implementation be preferred over a more direct implementation?