

Problem Set 1

EECS123: Digital Signal Processing

Prof. Ramchandran
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- (a) Prove that $\sum_{n=0}^N a^n = \frac{1-a^{N+1}}{1-a}$
(b) If $|a| < 1$, prove that $\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$
(c) Find a closed form expression for $\sum_{n=N_1}^{N_2} a^n$ valid for any $0 < N_1, N_2 < \infty$.

These expressions will be helpful in evaluating the \mathcal{Z} -transforms.

- Determine whether the following systems are linear, shift-invariant, and/or causal. In all cases, $x[n]$ is the input and $y[n]$ is the output of the system:
 - $y[n] = g[n]x[n]$ with $g[n]$ given.
 - $y[n] = ax[n] + b$.
 - $y[n] = x[-n]$.
 - $y[n] = e^{x[n]}$.

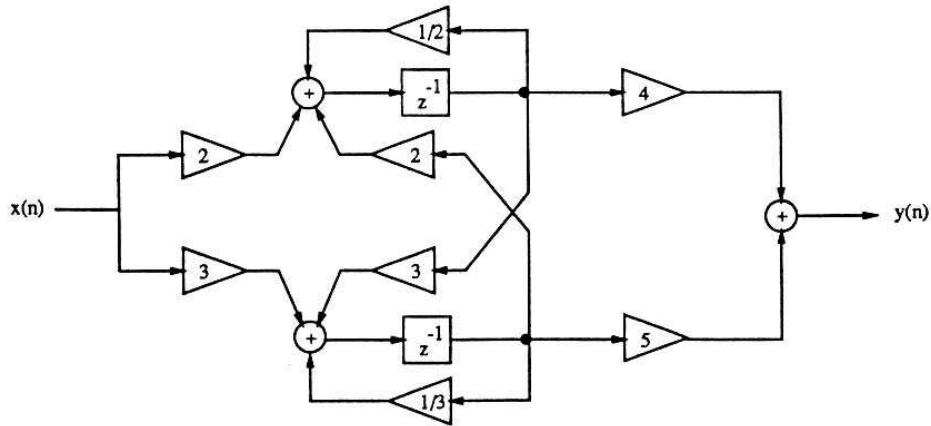
- Use the convolution formula to find $y_n = h_n * x_n$ when

$$h_n = \begin{cases} 1 & n \geq -3 \\ 3^n & n < -3 \end{cases} \quad x_n = \begin{cases} (1/3)^n & n \geq 3 \\ 3^n & n < 3 \end{cases} \quad (1)$$

- Consider the LSI system given by $y[n] = x[n] * h[n]$ where $h[n] = (j/4)^n u[n]$. If $x[n] = e^{j(\pi/6)n}$, draw the block diagram of a system which computes $y[n] = y_R[n] + jy_I[n]$, using only **real** signals, adders and multipliers, and delay registers to compute the output pair $(y_R[n], y_I[n])$.
- Let $h[n]$ be the impulse response of the system $y[n] = x[n] - y[n - 1]$.
 - Find the impulse response $h[n]$ of this system. Is it BIBO stable? Is it causal?
 - Let the series cascade of M systems with impulse response $h[n]$ as in Problem 5a be $h_M[n]$. What is the minimum value of M for which $|h_M[n]| \rightarrow \infty$, as $n \rightarrow \infty$?
 - The system is modified to

$$y[n] = x[n] - \alpha y[n - 1], \quad (2)$$

to improve stability. Let $g[n]$ be the impulse response for this filter. As a designer, what values of α will you choose to make the system BIBO stable?



6. Find the transfer function of the system shown in the figure above.

7. As discussed in class, an alternative interpretation of the sampling theorem is in terms of orthonormal bases for bandlimited functions. Consider the space S of functions bandlimited to $(-\pi, \pi)$ and the sinc function $\text{sinc}(t) = (\sin(t)/t)$. Define the inner product as $\langle f, g \rangle = \int_{-\infty}^{\infty} f(t)g^*(t)dt$.

(a) Show that the set $\{\text{sinc}(\pi(t - n)), n \in \mathbb{Z}\}$ is an orthonormal set, or

$$\langle \text{sinc}(\pi(t - n)), \text{sinc}(\pi(t - m)) \rangle = \delta(n - m)$$

(b) Show that any function $f \in S$ can be written as

$$f(t) = \sum_{n=-\infty}^{\infty} \alpha_n \text{sinc}(\pi(t - n))$$

where $\alpha_n = \langle \text{sinc}(\pi(t - n)), f(t) \rangle$. Hint: First show that $\alpha_n = f(n)$ because $f(t)$ is bandlimited.

8. Let P be an $n \times n$ matrix. Let $(\lambda_1, \dots, \lambda_n)$ be the eigenvalues and x_1, \dots, x_n be the corresponding linearly independent eigenvectors for the matrix P . Show that,

(a) If Q has the same eigenvectors and eigenvalues as P , then

$$P = Q. \tag{3}$$

(b) Write a formula for P in terms of the eigenvalues $(\lambda_1, \dots, \lambda_n)$ and eigenvectors x_1, \dots, x_n .

(c) In the lecture, we saw that a discrete LTI system can be represented by a matrix transform. Use Problem 8 (a) to argue (in words) why different LTI systems cannot have the same (discrete time) Fourier transform.