1. (20 points) The following values from the 8-point DFT of a length-8 real sequence $x[n]$ are known:

$X[0] = 3,\ X[2] = 0.5 - 4.5j,\ X[4] = 5,\ X[5] = 3.5 + 3.5j,\ X[7] = -2.5 - 7j.$

a) (5 points) Find the missing values $X[1]$, $X[3]$, $X[6]$.

b) (5 points) Evaluate $x[0]$.

c) (10 points) Find the 4-point DFT of the length-4 sequence $w[n]$ given by:

$$w[n] = x[n] + x[n+4] \quad n = 0, 1, 2, 3.$$

Hint: Derive a general formula that relates $W[k]$ to $X[k]$ so you don't have to calculate $x[n]$ and $w[n]$.

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2. (20 points) The continuous-time signals $x(t)$ below are sampled to generate the corresponding discrete-time signals $x[n]$. Specify a choice for the sampling period T consistent with each pair. In addition, indicate whether the choice of T is unique. If not, specify a second choice of T.

a) (10 points) $x(t) = \sin(10\pi t) \quad \rightarrow \quad x[n] = \sin(\pi n/4)$

b) (10 points) $x(t) = \frac{\sin(10\pi t)}{10\pi t} \quad \rightarrow \quad x[n] = \frac{\sin(\pi n/2)}{\pi n/2}$

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3. (20 points) Consider the system below, where

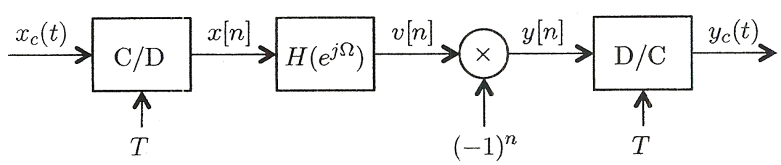
$$H(e^{j\Omega}) = \begin{cases} 1, & |\Omega| < \pi/2 \\ 0, & \pi/2 < |\Omega| \leq \pi. \end{cases}$$

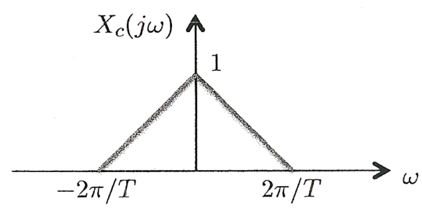
and assume the CTFT of the input, $X_c(j\omega)$, is as shown below.

a) (10 points) Sketch the DTFT for $x[n]$, $v[n]$ and $y[n]$.

b) (5 points) Sketch the CTFT for the output, $Y_c(j\omega)$, assuming an ideal D/C converter.

c) (5 points) Sketch the magnitude $|Y_c(j\omega)|$ assuming, this time, a zero-order hold D/C converter.





Additional workspace for Problem 3

magnitude T

4. a) (15 points) Specify the transfer function of a stable and causal LTI system whose frequency response has the magnitude:

$$|H(j\omega)| = \frac{1}{\sqrt{1+\omega^4}}.$$

b) (5 points) Is the answer to part (a) unique? If not, specify another stable and causal LTI system whose frequency response has the same magnitude but a different phase.

Additional workspace for Problem 4.

5. (20 points) When the input to an LTI is:

$$x[n] = \left(\frac{1}{2}\right)^n u[n] + 2^n u[-n-1]$$

the output is:

$$y[n] = 6\left(\frac{1}{2}\right)^n u[n] - 6\left(\frac{3}{4}\right)^n u[n].$$

a) (10 points) Find the transfer function of $H(z)$ and indicate the region of convergence.

b) (5 points) Is the system causal? Is it stable?

b) (5 points) Write the difference equation that characterizes the system.