UNIVERSITY OF CALIFORNIA

College of Engineering
Department of Electrical Engineering
and Computer Sciences

Professor Varaiya Spring 2000

EECS 120

MODULATION

Fourier Transform Defined

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt, \qquad \omega - \text{rad/sec}$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega, \quad t - \text{sec}$$

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt, \qquad f - \text{Hz}$$

$$x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}df, \qquad t - \text{sec}$$

Important Fourier Transforms

$x(t) \equiv 1$	$X(\omega) = 2\pi\delta(\omega)$	$X(f) = \delta(f)$
$x(t) = \delta(t)$	$X(\omega) \equiv 1$	$X(f) \equiv 1$
$x(t) = \text{sgn}(t)$	$X(\omega) = \frac{2}{j\omega}$	$X(f) = \frac{1}{j\pi f}$
$x(t) = u(t)$	$X(\omega) = \frac{1}{j\omega} + \pi\delta(\omega)$	$X(f) = \frac{1}{j2\pi f} + \frac{1}{2}\delta(f)$
$x(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_0)$	$X(\omega) = \frac{2\pi}{T_0}\sum \delta(\omega - k\omega_0)$	$X(f) = \frac{1}{T_0}\sum \delta(f - kf_0)$
$x(t) = \sum X_n e^{jn\omega_0 t}$	$X(\omega) = 2\pi\sum X_n\delta(\omega - n\omega_0)$	$X(f) = \sum X_n\delta(f - nf_0)$

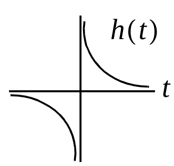
Fourier Transform Properties

	Signal x, y	**FT(f)** $X(f), Y(f)$	**FT(ω)** $X(\omega), Y(\omega)$
Linearity	$\alpha x + \beta y$	$\alpha X + \beta Y$	$\alpha X + \beta Y$
Delay	$D_\tau x$	$X(f)e^{-j2\pi f\tau}$	$X(\omega)e^{-j\omega\tau}$
Flip	$x(-t)$	$X(-f)$	$X(-\omega)$
Conjugate	x^*	$X^*(-f)$	$X^*(-\omega)$
Modulation	$e^{j2\pi f_0 t}x(t)$	$X(f-f_0)$	$X(\omega-\omega_0)$, $\omega_0 = 2\pi$
Time scale	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{f}{a}\right)$	$\frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$
Convolution	$x * y$	$X(f)Y(f)$	$X(\omega)Y(\omega)$
Multiply	$x(t)y(t)$	$(X * Y)(f)$	$\frac{1}{2\pi}(X * Y)(\omega)$
Differentiate	$\dot{x}$	$(j2\pi f)X(f)$	$(j\omega)X(\omega)$
Integrate	$\int_{-\infty}^{t} x(s)ds = u * x(t)$	$\frac{1}{j2\pi f}X(f) + \frac{1}{2}X(0)\delta(f)$	$\frac{1}{j\omega}X(\omega + \pi X/0)\delta$
Duality	$x(t) \leftrightarrow$ $X(t) \leftrightarrow$	$X(f)$ $x(-f)$	$x(t) \leftrightarrow X(\omega)$ $X(t) \leftrightarrow 2\pi x(-\omega)$
Real	$x(t)$ real	$X(f) = X(-f)^*$	$X(\omega) = X(-\omega)^*$
Real, Even	$x(t) = x(-t)$, real	$X(f) = X(f)$, real	$X(\omega) = X(-\omega)$, real
Real, Odd	$x(t) = -x(-t)$, real	$X(f) = -X(-f)$, imag	$X(\omega) = -X(-\omega)$, imag

Hilbert Transform: $x \in$ Cont Signals $\rightarrow \hat{x} \in$ Cont Signals

$$\hat{x} = x * \left\{\frac{1}{\pi t}\right\}$$

$$h(t) = \frac{1}{\pi t}$$



$$\hat{X}(\omega) = X(\omega) \cdot \mathrm{FT}\left\{\frac{1}{\pi t}\right\} = -j\ \mathrm{sgn}(\omega) \cdot X(\omega)$$

$$\hat{X}(f) = X(f) \cdot \mathrm{FT}\left\{\frac{1}{\pi t}\right\} = -j\ \mathrm{sgn}\ (f) \cdot X(f)$$

Let $z = x + j\hat{x}$

$$Z(\omega) = X(\omega) + j\hat{X}(\omega) = 2X(\omega)u(\omega)$$

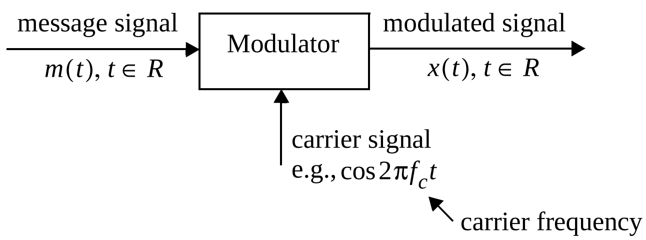
$$Z(f) = X(f) + j\hat{X}(f) = 2X(f)u(f)$$

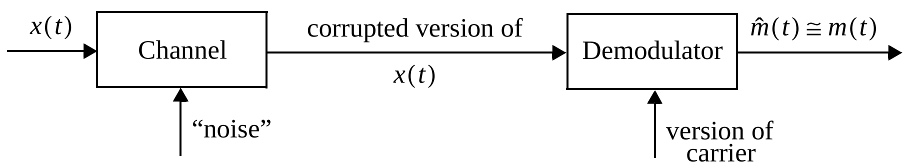
Let $w = x - j\hat{x}$

$$W(\omega) = X(\omega) - j\hat{X}(\omega) = 2X(\omega)u(-\omega)$$

$$W(f) = X(f) - j\hat{X}(f) = 2X(f)u(-f)$$

1. What is modulation?



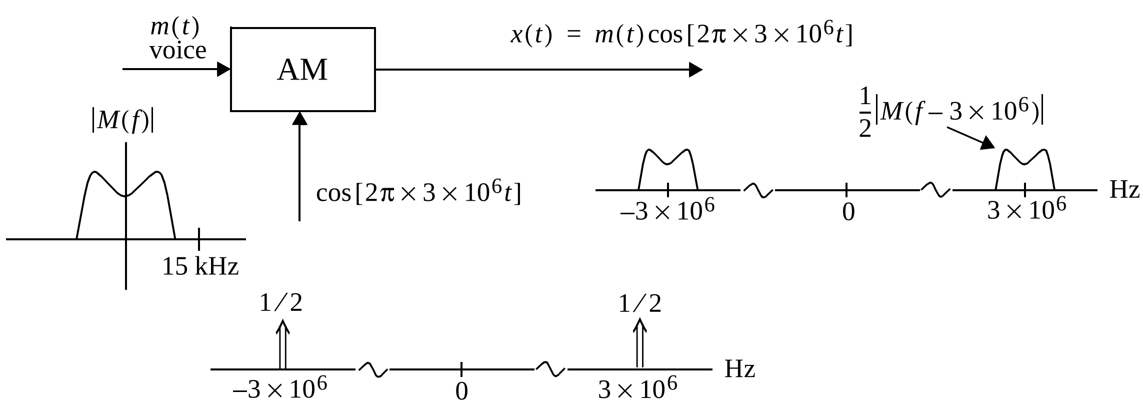


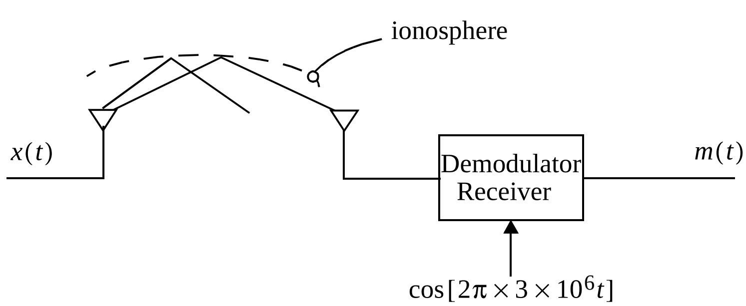
A modulator is a system:

$$\underset{\text{message signal}}{[H(m(t))]}(t) = \underset{\text{modulated signal}}{x(t)}$$

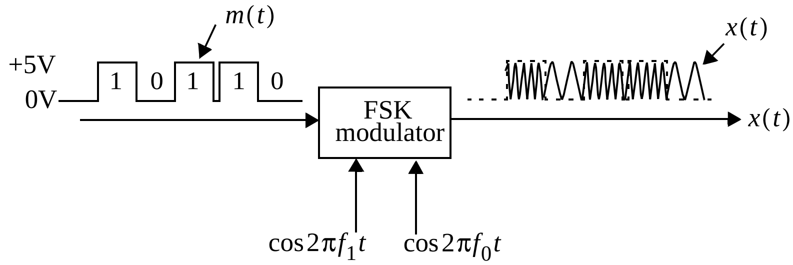
H is usually time-varying; may be linear (AM) or non-linear (FM)

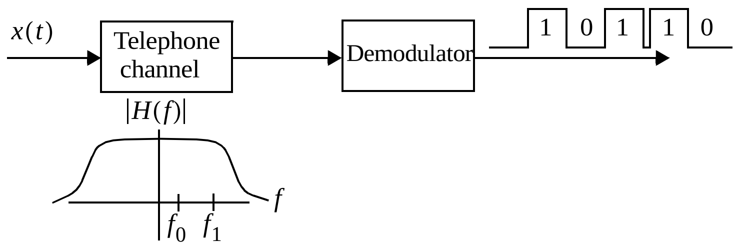
Example 1 – (Shortwave broadcast) AM





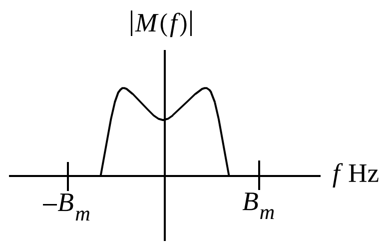
Example 2 – "Cheap" modem



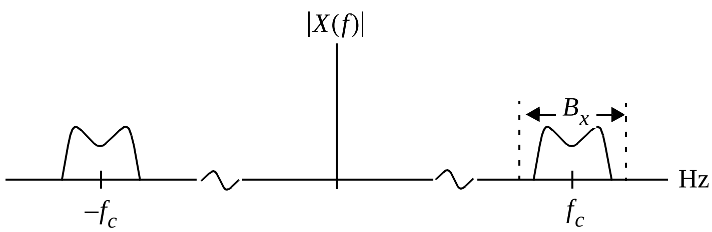


2. Assumptions

A. $m(t) \leftrightarrow M(f)$. Assume m is **bandlimited** to $|f| < B_m$ Hz. Also, $|m(t)| \leq 1$, all t.



B. $x(t) \leftrightarrow X(f)$. Assume x is **bandpass** signal centered around f_c and with bandwidth B_x.

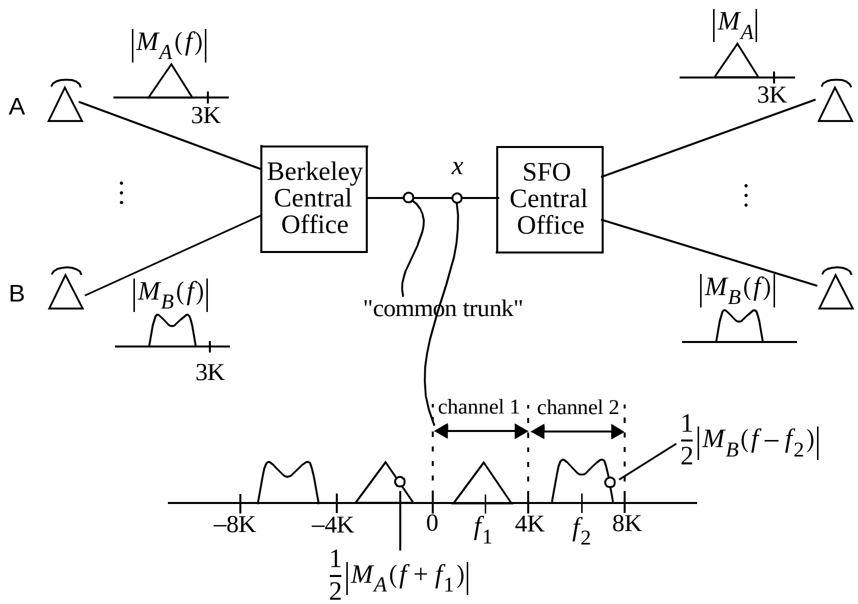


C. $B_x \ll f_c$. We say that x is a **narrowband** signal.

3. Why modulation?

A. *Propagation.* It may not be possible to transmit a **baseband** signal.

B. *Channel sharing*

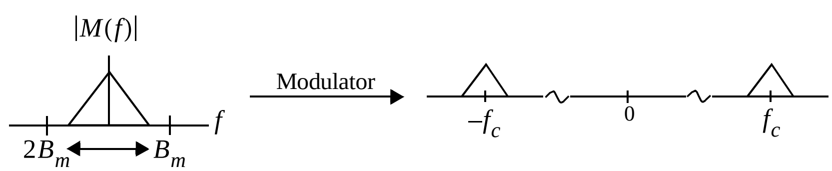


C. *Noise immunity:* FM better than AM. Go to EECS 121.

<u>4. Linear Modulation</u> (Modulation is a linear system) (but not time invariant!)

A. Double side band AM (DSB-AM)

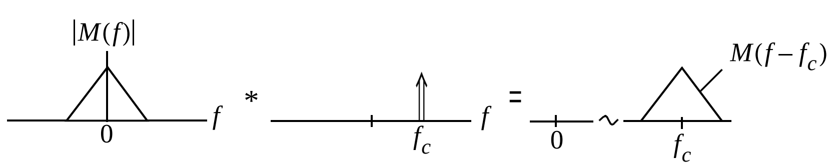
Objective



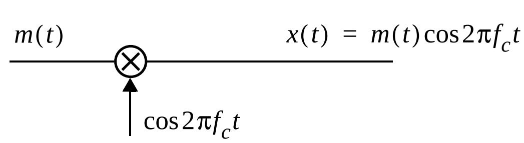
Idea

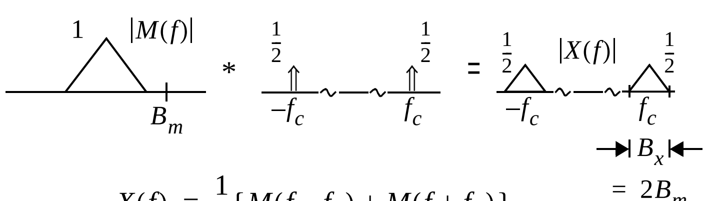
$$m(t) \cdot e^{j2\pi f_c t} \leftrightarrow \mathcal{F}[m] * \mathcal{F}[e^{j2\pi f_c t}]$$

$$= M(f) * \delta(f-f_c) = M(f-f_c)$$



Better idea

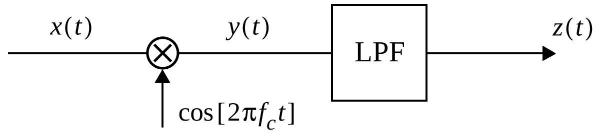


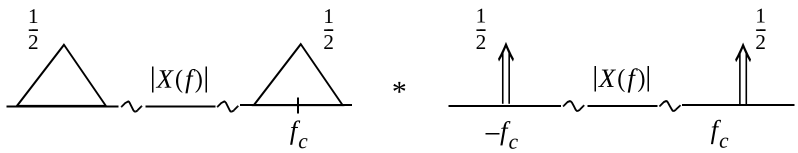


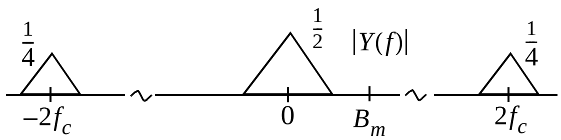
$$X(f) = \frac{1}{2}\{M(f-f_c) + M(f+f_c)\}$$

Note, $B_x \ll f_c$.

Demodulation





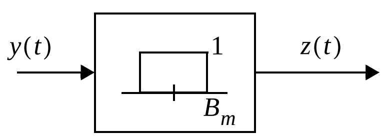


$$Y(f) = X(f) * \frac{1}{2}[\delta(f-f_c) + \delta(f+f_c)]$$

$$= \frac{1}{2}M(f) + \frac{1}{4}\{M(f-2f_c) + M(f+2f_c)\}$$

$$y(t) = x(t)\cos 2\pi f_c t = m(t)(\cos 2\pi f_c t)^2$$

$$= \frac{1}{2}m(t) + \frac{1}{2}m(t)\cos 4\pi f_c t$$



$$Z(f) = \frac{1}{2}M(f)$$

$$z(t) = \frac{1}{2}m(t)$$

Difficulty A

Demodulation must have local oscillator (at f_c) where phase is “locked” to carrier. This requires a phase-locked loop: Suppose there is a phase difference:

$$y(t) = x(t) \cdot \cos[2\pi f_c t + \theta],$$

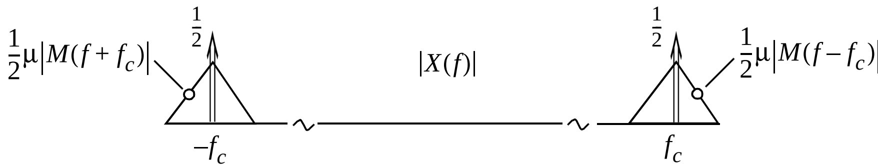
then

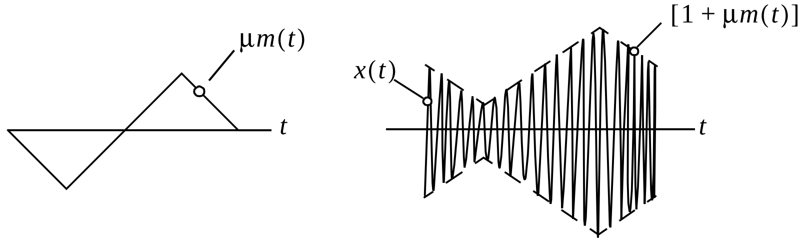
$$z(t) = \left[\underbrace{\frac{1}{2}\cos\theta}_{\text{can drift!!!}}\right] \cdot m(t)$$

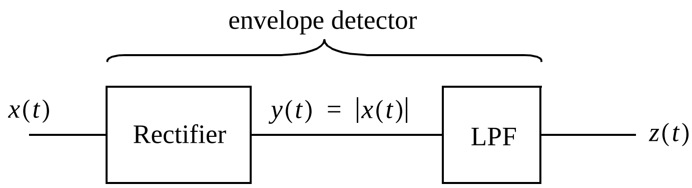
Cheap Solution: DSB with large carrier (DSB-LC)

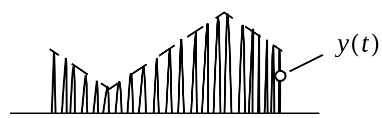
Suppose $|m(t)| \le 1$. Let $\mu < 1 \quad (\simeq 0.8)$ **modulation index**. Let

$$x(t) = [1 + \mu m(t)]\cos(2\pi f_c t)$$
$$= \cos 2\pi f_c t + \mu m(t)\cos 2\pi f_c t$$









$x(t)$ is high frequency (f_c) sinusoid whose "envelope" is $[1 + \mu m(t)]$.

$z(t) \cong [1 + \mu m(t)]$ is the "envelope."

<u>Exercise</u>: Determine $Y(f)$, $Z(f)$ and show the envelope detector works.

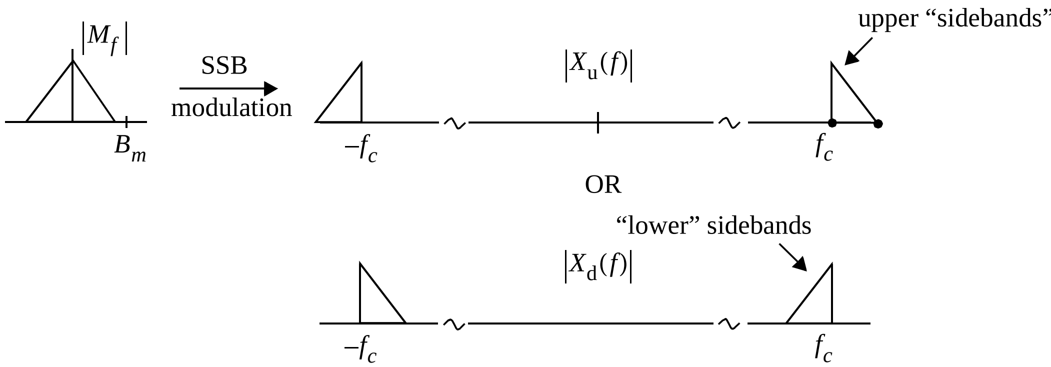
Drawback: $P_x = \underbrace{\frac{1}{2}}_{P_{\text{carrier}}} + \underbrace{\frac{1}{2}\mu^2 P_m}_{P_{\text{signal}}}$

$$\text{Efficiency} = \frac{P_{\text{signal}}}{P_x} = \frac{\mu^2 P_m}{1 + \mu^2 P_m} < 50\%$$

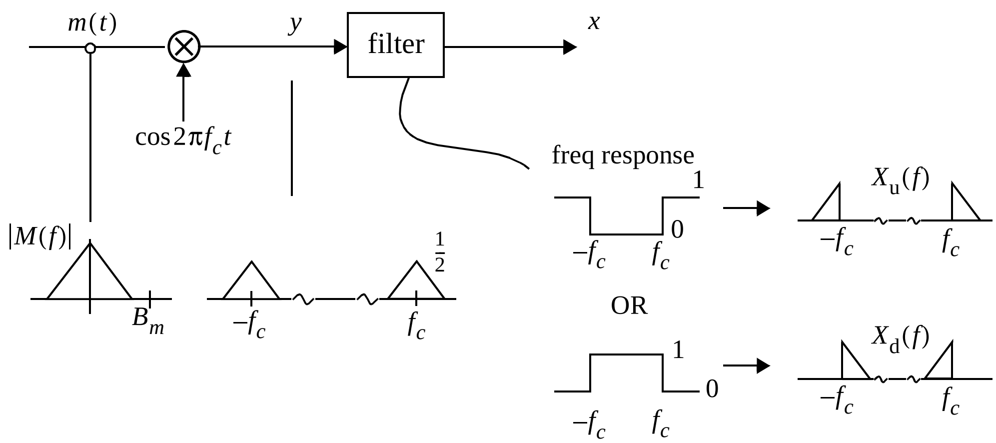
Difficulty B

DSB and DSB-LC consume bandwidth $B_x \simeq 2B_m$, i.e., twice message bandwidth.

Idea: Make $B_x \simeq B_m$ by:



Scheme 1

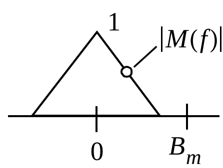


Scheme 1 requires filters with sharp cut-off.

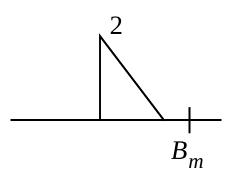
Scheme 2 – Phase Shift Method

Idea – Review Hilbert Transform

$$m(t) \leftrightarrow M(f)$$

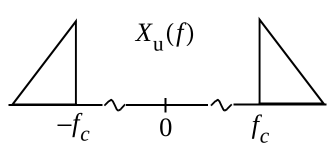


$$m(t) + j\hat{m}(t) \leftrightarrow M(f) + j\hat{M}(f)$$
$$= 2M(f)u(f)$$



$$\{m(t) + j\hat{m}(t)\}e^{j2\pi f_c t} \leftrightarrow 2M(f-f_c)u(f-f_c)$$

$$x_u(t) := Re[m((t) + j\hat{m}(t))e^{j2\pi f_c t}]$$
$$= M(f-f_c)u(f-f_c) + M(-f-f_c)u(-f-f_c)$$



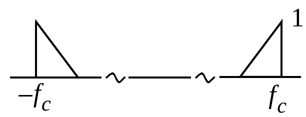
$$x_u(t) = Re[m(t) + j\hat{m}(t)e^{j2\pi f_c t}]$$

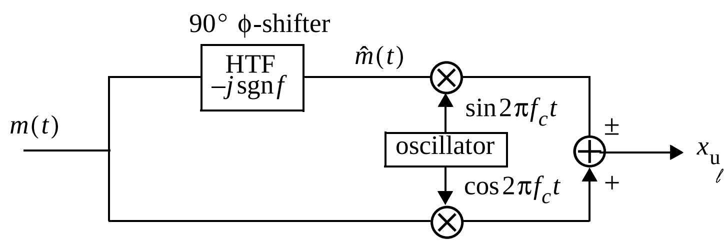
$$= m(t)\cos 2\pi f_c t - \hat{m}(t)\sin 2\pi f_c t$$

Similarly:

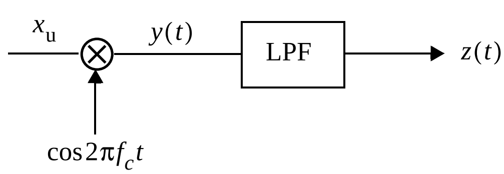
$$x_d(t) = m(t)\cos 2\pi f_c t + \hat{m}(t)\sin 2\pi f_c t$$

$\leftrightarrow$





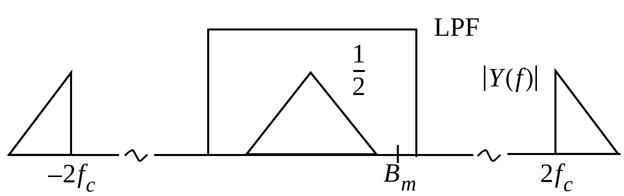
SSB-U Demodulator



$$x_u(t) = m(t)\cos 2\pi f_c t - \hat{m}(t)\sin 2\pi f_c t$$

$$y(t) = \frac{1}{2}m(t) + \frac{1}{2}m(t)\cos 4\pi f_c t - \frac{1}{2}\hat{m}(t)\sin 4\pi f_c t$$

$$z(t) = \frac{1}{2}m(t)$$

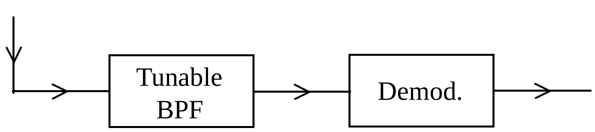


AM Receiver: Superheterodyne

AM band: $\begin{cases} \text{550 - 1600 kHz} \\ \text{10 kHz = Maximum frequency} \end{cases}$

AM Receiver:

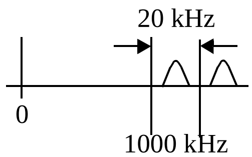
In principle,



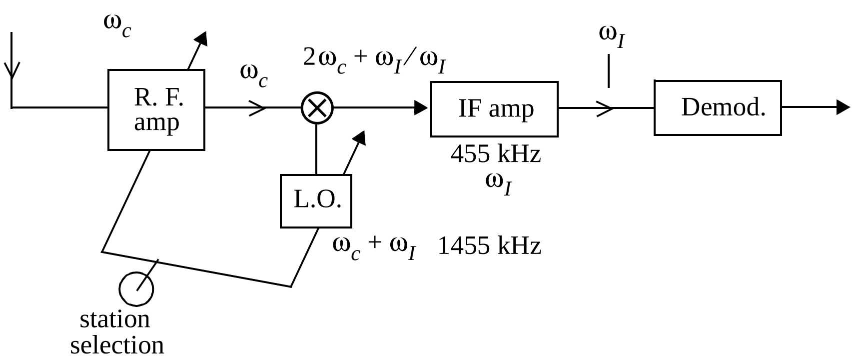
Difficulties:

1) large gain

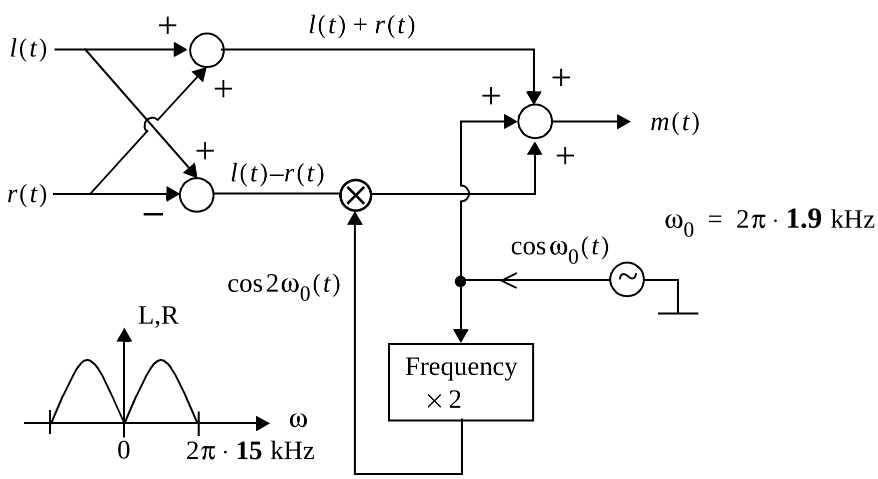
2) sharp filters

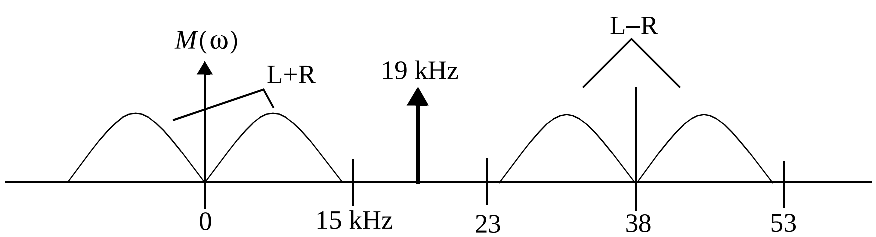


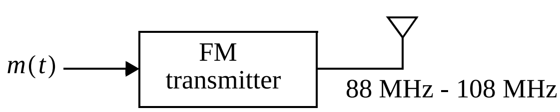
Solution: ($\omega_c + 2\omega_I$: image) Let $\omega_c = 2\pi$ (1000 kHz)

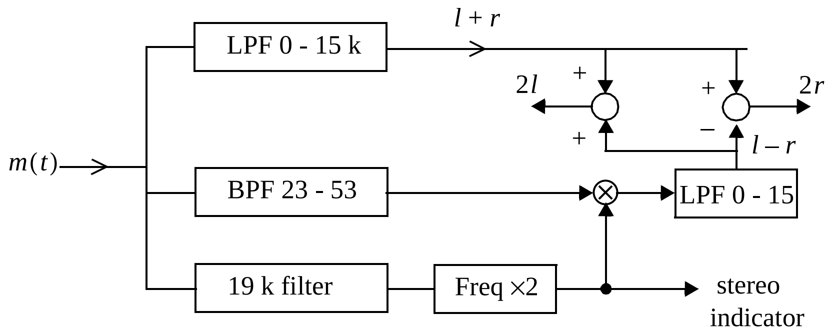


Stereo









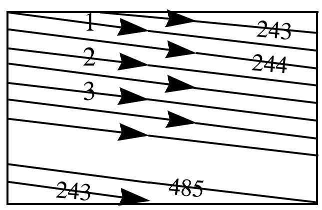
Note:

- compatible
  (stereo on mono receiver ⇒ mono reception)
- reverse compatible
  (mono on stereo receiver ⇒ mono reception)

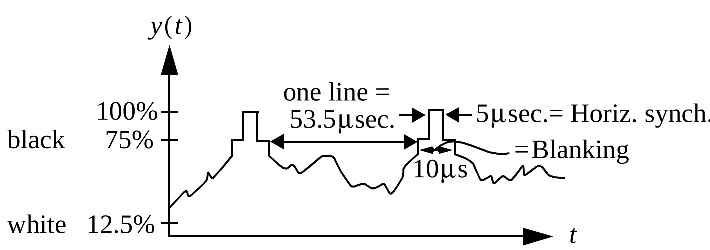
TV

BW: 485 lines in 2 fields + 2×20 line durations (vertical retrace)

$\times$ 30/sec. ⇒ line frequency: $f_2 = 15750$ lines/sec



3×4



Bandwidth:

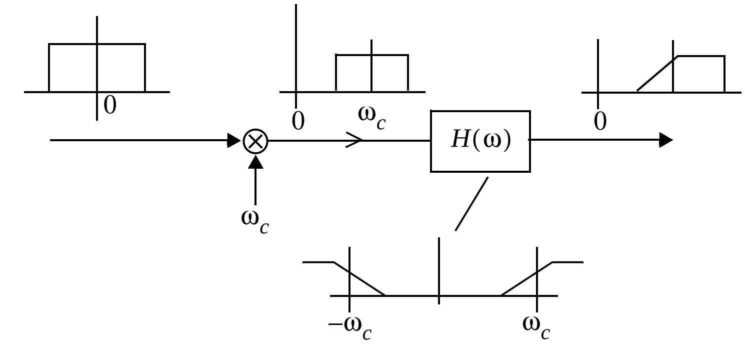
$$525 \times 525 \times \frac{4}{3} \times 30 \text{ points/sec.}$$

worst case: BWBW ...

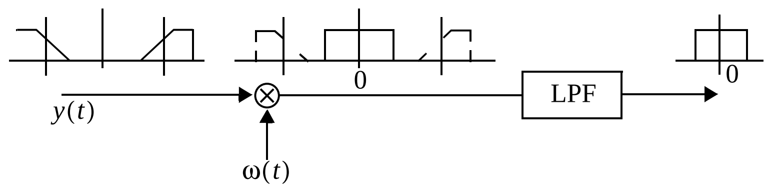
$$\Rightarrow \quad \frac{1}{2}\, 525 \times 525 \times \frac{4}{3} \times 30 = 5.5 \text{ MHz}$$

In practice: 4.2 MHz

VSB (Vestigial Side Band)



$H(\omega + \omega_c) + H(\omega - \omega_c) = 1$

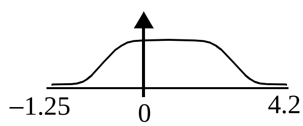


Envelope Detection: (with residual carrier)

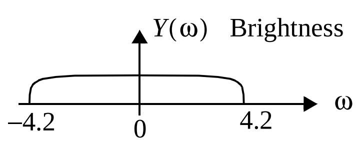
$$y(t) = k\cos\omega_c t + \underbrace{\text{SSB} - g(t)\sin\omega_c t}_{\text{VSB}}$$

$$= (k + m(t))\cos\omega_c t - (\hat{m}(t) + g(t))\sin 2\pi f_0$$

Envelope for k large $\simeq k + m(t)$, e.g., TV:



Thus:

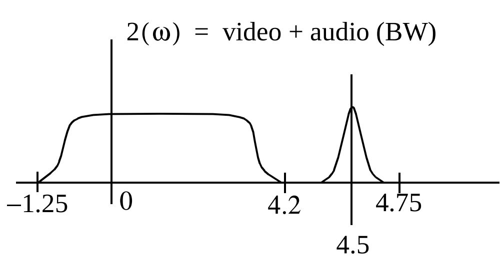


Also: Sound = 10 kHz max.

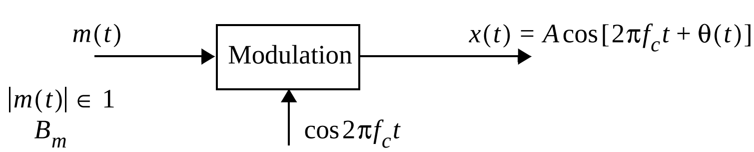
$B \simeq 80$ kHz

FM

Modulation: $(z(t))$



Exponential Modulation



A. Phase Modulation

phase deviation constant

$$\theta(t) = \phi_\Delta \cdot m(t)$$

$$x(t) = A\cos[2\pi f_c t + \phi_\Delta \cdot m(t)]$$

$$[|\phi_\Delta| \le \pi]$$

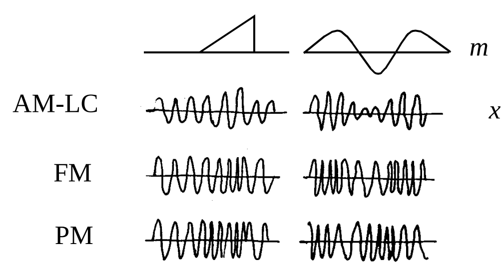
B. Frequency Modulation

frequency deviation constant

$$\theta(t) = 2\pi f_\Delta \int_0^t m(s)ds$$

or $$\dot{\theta}(t) = 2\pi f_\Delta \cdot m(t)$$

$$x(t) = A\cos\left[2\pi f_c t + 2\pi f_\Delta \int_0^t m(s)ds\right]$$



Narrowband PM/FM

$$x(t) = \cos[2\pi f_c t + \theta(t)] = Re\{e^{j[2\pi f_c t + \theta(t)]}\}$$

Assume $|\theta(t)| \ll 1$, so

$$e^{j\theta(t)} = 1 + j\theta(t) + \underbrace{\frac{j^2}{2!}\theta^2(t) + \frac{j^3}{3!}\theta^3(t) + \ldots}_{\text{neglect}}$$

$$\cong 1 + j\theta(t)$$

So

$$x(t) \cong Re\{[1 + j\theta(t)]e^{j2\pi f_c t}\}$$

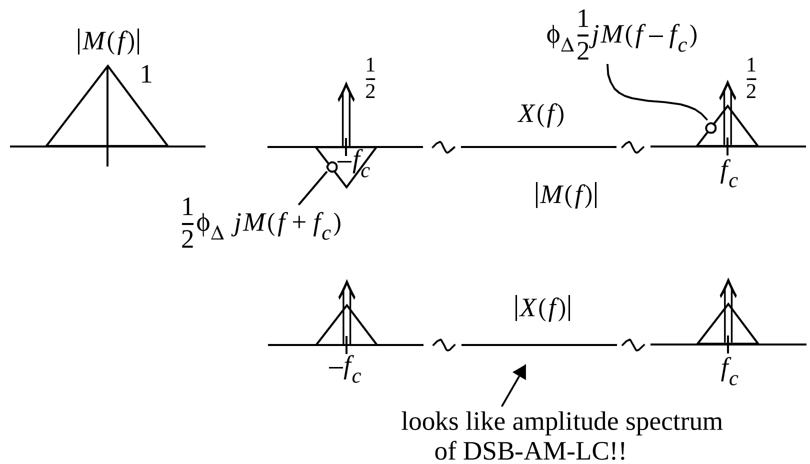
$$= \cos 2\pi f_c t - \theta(t)\sin 2\pi f_c t$$

NB-PM: $\theta(t) = \phi_\Delta m(t)$. So,

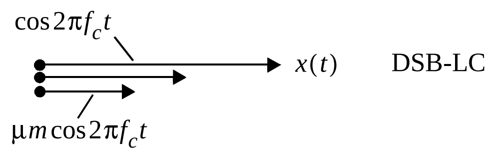
$$x(t) = \cos 2\pi f_c t - \phi_\Delta m(t) \sin 2\pi f_c t$$

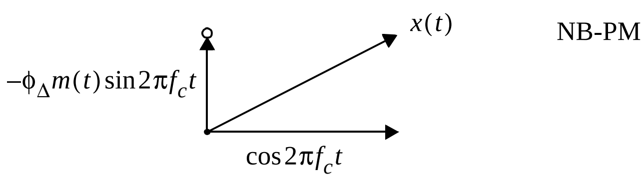
$$X(f) = \frac{1}{2}\{\delta(f-f_c) + \delta(f+f_c)\} - \phi_\Delta \cdot \frac{1}{2j}\{M(f-f_c) - M(f+f_c)\}$$

$$= \frac{1}{2}\{\delta(f-f_c) + \delta(f+f_c)\} + \frac{1}{2}\phi_\Delta \cdot j\{M(f-f_c) - M(f+f_c)\}$$



Time domain comparison between DSB-LC and narrowband PM:





[Amplitude of phasor in NB-PM case is $(1 + \phi_\Delta^2 m(t)^2)^{1/2} \simeq (1 + \theta^2(t))^{1/2} \simeq 1$!]

NB-FM

$$x(t) = \cos[2\pi f_c t + \theta(t)] \underset{\downarrow}{\overset{|\theta(t)| \ll 1}{\cong}} \cos 2\pi f_c t - \theta(t) \sin 2\pi f_c t \quad (1)$$

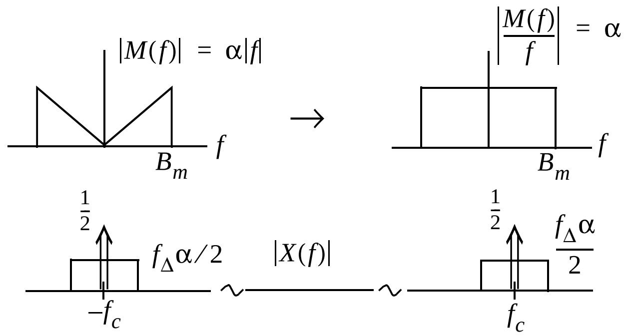
$$\theta(t) = 2\pi f_\Delta \cdot \int_0^t m(s) ds \quad (2)$$

$[|\theta(t)| \ll 1 \Rightarrow m$ has no dc component, $M(f)|_{f=0} = 0]$

$$(1) \Rightarrow X(f) = \frac{1}{2}\{\delta(f-f_c) + \delta(f+f_c)\} - \frac{1}{2j}\{\Theta(f-f_c) - \Theta(f+f_c)\}$$

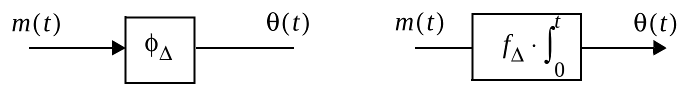
$$(2) \Rightarrow \Theta(f) = 2\pi f_\Delta \cdot \frac{M(f)}{j2\pi f} = -j\, f_\Delta \cdot \frac{M(f)}{f}$$

So, $$X(f) = \frac{1}{2}\{\delta(f-f_c) + \delta(f+f_c)\} + \frac{f_\Delta}{2}\left\{\frac{M(f-f_c)}{f-f_c} - \frac{M(f+f_c)}{f+f_c}\right\}$$

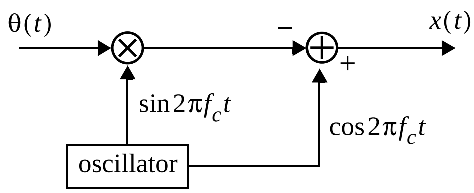


NB-Modulation

$$x(t) = \cos 2\pi f_c t - \theta(t)\sin 2\pi f_c t$$



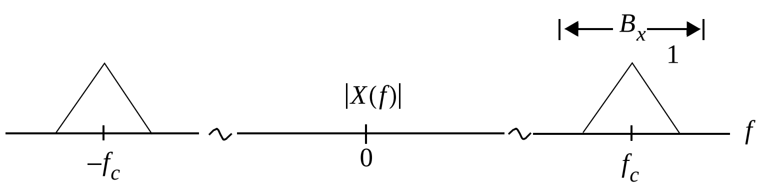
PM FM



NB-FM-Demodulator

First, do **representation of NB signals**.

Definition: $x(t)$, $t \in R$, is a real NB signal with carrier f_c if $|X(f)| = 0$ for $|f - f_c| > \frac{1}{2}B_x$ and $B_x \ll f_c$:



Theorem: Let x be a real NB signal with carrier f_c. Let $\hat{x} = \text{HT}$ of x. Then $x, \hat{x}$ have the representation

$$x(t) = A(t)\cos[2\pi f_c t + \theta(t)]$$

$$\hat{x}(t) = A(t)\sin[2\pi f_c t + \theta(t)]$$

where A, θ vary slowly compared with f_c. Let

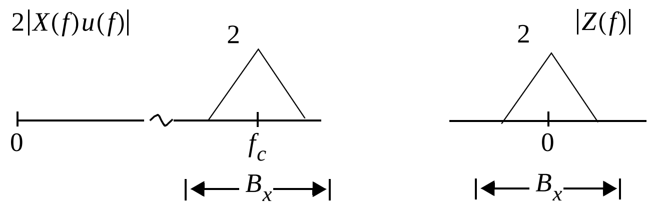
$$z(t) := [x(t) + j\hat{x}(t)]e^{-j2\pi f_c t}$$

then $$z(t) = A(t)e^{j\theta(t)} \quad \text{and} \quad |Z(f)| = 0, \quad |f| > \frac{1}{2}B_x.$$

Proof

$$x(t) + j\hat{x}(t) \leftrightarrow X(f) + j\hat{X}(f) = 2X(f)u(f)$$

$$z(t) \leftrightarrow Z(f) = 2X(f+f_c)u(f+f_c)$$

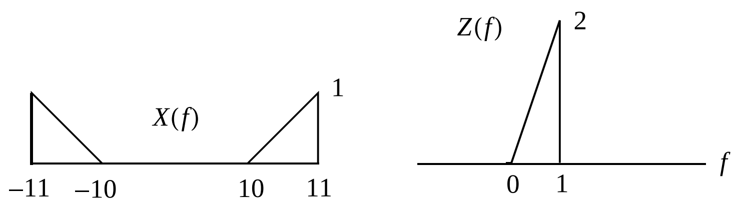


Suppose $z(t) = A(t)e^{j\theta(t)}$, then A, θ vary slowly. Also

$$x(t) = Re[z(t) \cdot e^{j2\pi f_c t}] = A(t)\cos[2\pi f_c t + \theta(t)]$$

$$\hat{x}(t) = Im[z(t)e^{j2\pi f_c t}] = A(t)\sin[2\pi f_c t + \theta(t)]$$

<u>Example</u>



$$f_c = 10, B_x = 2$$

$$z(t) = f^{-1}[Z] = \int_{-\infty}^{\infty} Z(f)e^{j2\pi ft}df = 2\int_0^1 fe^{j2\pi ft}df$$

$$= \ldots = \frac{1}{\pi t^2}\{e^{j2\pi t}(1 - 2\pi jt) - 1\}$$

$$= \frac{1}{\pi t^2}\{(\cos 2\pi t + 2\pi t\sin 2\pi t - 1) + j(\sin 2\pi t - 2\pi t\cos 2\pi t)\}$$

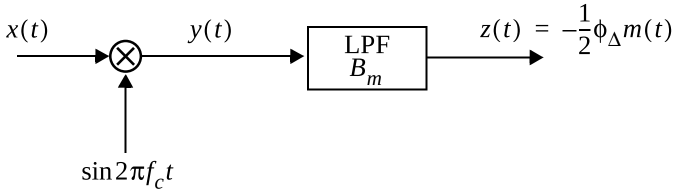
$$A(t) = \frac{1}{\pi t^2}[(\cos 2\pi t + 2\pi t\sin 2\pi t - 1)^2 + (\sin 2\pi t - 2\pi t\cos 2\pi t)^2\}^{1/2}$$

$$\theta(t) = \tan^{-1}\frac{\sin 2\pi t - 2\pi t\cos 2\pi t}{\cos 2\pi t + 2\pi t\sin 2\pi t - 1}$$

$$x(t) = A(t)\cos[20\pi t + \theta(t)]$$

$$\hat{x}(t) = A(t)\sin[20\pi t + \theta(t)]$$

Demodulating NB-PM



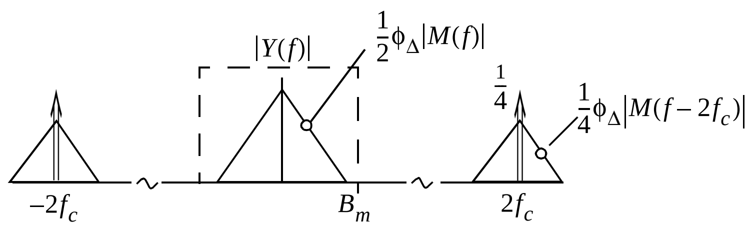
$$x(t) = \cos 2\pi f_c t - \phi_\Delta m(t) \sin 2\pi f_c t$$

$$y(t) = \cos 2\pi f_c t \cdot \sin 2\pi f_c t - \phi_\Delta m(t) [\sin 2\pi f_c t]^2$$

$$= -\frac{1}{2}\phi_\Delta m(t) + \frac{1}{2}\sin 4\pi f_c t + \frac{1}{2}\phi_\Delta m(t) \cos 4\pi f_c t$$

$$\text{where } -\frac{1}{2}\phi_\Delta m(t) = z(t)$$

$$Y(f) = -\frac{1}{2}\phi_\Delta M(f) + \frac{1}{4j}\{\delta(f - 2f_c) - \delta(f + 2f_c)\}$$
$$+ \frac{1}{4}\phi_\Delta \{M(f - 2f_c) + M(f + 2f_c)\}$$



"Wideband" PM/FM

$$\text{Suppose } m(t) = \begin{cases} A_m \sin 2\pi f_m t & \text{PM} \\ A_m \cos 2\pi f_m t & \text{FM} \end{cases}$$

$$\text{Then} \quad \theta(t) = \begin{cases} A_m \phi_\Delta \sin 2\pi f_m t & \text{PM} \\ A_m \dfrac{f_\Delta}{f_m} \sin 2\pi f_m t & \text{FM} \end{cases}$$

"index of modulation" =: $\beta \sin 2\pi f_m t$

$$x(t) = \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$$

$$= \underbrace{\cos(2\pi f_c t) \cos(\beta \sin 2\pi f_m t)}_{\text{periodic with period } f_m^{-1}} - \sin(2\pi f_c t) \underbrace{\sin(\beta \sin 2\pi f_m t)}$$

$$\underset{\text{even}}{\cos(\beta \sin 2\pi f_m t)} = \underbrace{J_0(\beta)}_{a_0} + \sum_{\substack{n>0 \\ \text{even}}} \underbrace{2J_n(\beta)}_{a_n} \cos n2\pi f_m t$$

$$= a_0 + \sum_n a_n \cos n2\pi f_m t$$

$$\underset{\text{odd}}{\sin(\beta \sin 2\pi f_m t)} = \sum_{\substack{n>0 \\ n\ \text{odd}}} \underbrace{2J_n(\beta)}_{b_n} \cos n2\pi f_m t$$

$$= \sum_n b_n \sin n2\pi f_m t$$

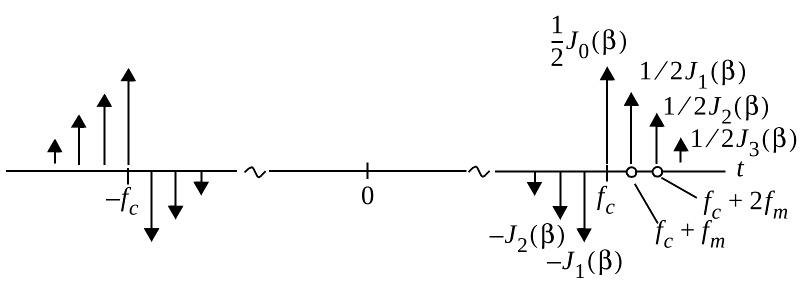
<u>Exercise</u>

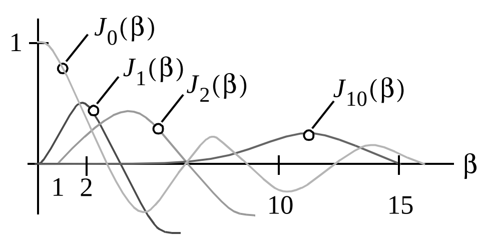
$$J_n(\beta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j[\beta \sin\lambda - n\lambda]}dx$$

So for pure tone message signal:

$$x(t) = J_0(\beta)\cos 2\pi f_c t + \sum_{\substack{n>0 \\ n\ \text{even}}} J_n(\beta)\{\cos[2\pi(f_c + nf_m)t] + \cos[2\pi(f_c - nf_m)t]\}$$

$$+ \sum_{\substack{n>0 \\ n\ \text{odd}}} J_n(\beta)\{\cos([2\pi(f_c + nf_m)t] - \cos[2\pi(f_c - nf_m)t])\}$$





Approximate B_x

For FM, define $D = \dfrac{f_\Delta}{B_m}$ derivative ratio

For PM, define $D = \phi_\Delta$

(Note: For pure tone $B_m = f_m, D = \beta$)

Carson's Rule: If $D \ll 1$ or $D \gg 1$, then

$$B_x \cong 2(D+1)B_m$$

Example – For commercial FM broadcast

$$B_m = 15 \text{ kHz}, \quad f_\Delta = 75 \text{ kHz}, \quad D = f_\Delta / B_m = 5$$

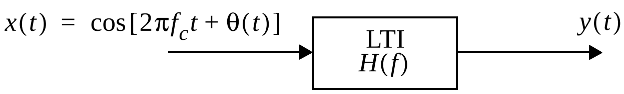
So Carson's rule gives

$$B_x = 2 \times 6 \times 15 = 180 \text{ kHz}$$

Actually $B_x \simeq 240$ kHz

Compare B_x for FM vs. AM!!!

FM-Demodulator



Suppose $H(f) = a + b2\pi|f| = a + b(j2\pi f)(-j\,\mathrm{sgn} f)$

$$Y(f) = H(f)X(f) = aX(f) + b(j2\pi f)(-j\,\mathrm{sgn} f)(X(f))$$

$$\begin{aligned} y(t) &= ax(t) + b\frac{d}{dt}(\hat{x}(t)) \\ &= a\cos[2\pi f_c t + \theta(t)] + b\frac{d}{dt}\{\sin(2\pi f_c t + \theta(t))\} \\ &= [a + b2\pi f_c + b\dot{\theta}(t)]\cos[2\pi f_c t + \theta(t)] \end{aligned}$$

where $\dot{\theta}(t) = f_\Delta m(t)$

$a + b2\pi f_c + bf_\Delta m(t)$ obtained by envelope detection, then suppress dc to get $bf_\Delta m(t)$: