Professor Fearing EECS120/Problem Set 6 v 1.01 Fall 2016

Due at 4 pm, Fri. Oct. 7 in HW box under stairs (1st floor Cory)

Reading: O&W Ch 5, 6.

1. (12 pts) DTFT properties (Lec 8,9 OW Ch 5)

Given $\mathcal{F}\{x[n]\} = X(e^{j\omega})$, find the DTFT of the following signals in terms of $X(e^{j\omega})$ for $x[n]$ complex.

a. $Im\{x[n]\}$

b. $x[-n]$

c. $Odd\{x[n]\}$

2. (28 pts) DTFT exercises (Lec 8,9,10 OW 5.5, 5.8)

Given an LTI system with input $x[n]$ and output $y[n]$ described by the LDE $y[n] + \frac{1}{2}y[n-1] = x[n]$.

a. Determine the frequyency response $H(e^{j\omega})$ of this system.

b. Find output $y[n]$ for $x_1[n] = (\frac{1}{2})^n u[n]$.

c. Find output $y[n]$ for $x_2[n] = (-\frac{1}{2})^n u[n]$.

d. Find output $y[n]$ for $x_3[n] = \delta[n] + \frac{1}{2}\delta[n-1]$.

e. Find output $y[n]$ for $x_4[n] = \delta[n] - \frac{1}{2}\delta[n-1]$.

f. Find output $y[n]$ to input with DTFT $X_5(e^{j\omega}) = \frac{1-\frac{1}{4}e^{-j\omega}}{1+\frac{1}{2}e^{-j\omega}}$.

g. Find output $y[n]$ to input with DTFT $X_6(e^{j\omega}) = 1 + 2e^{-3j\omega}$.

3. (25 pts) Digital receiver, quadrature modulation Lec 9. OW 5, 5.5

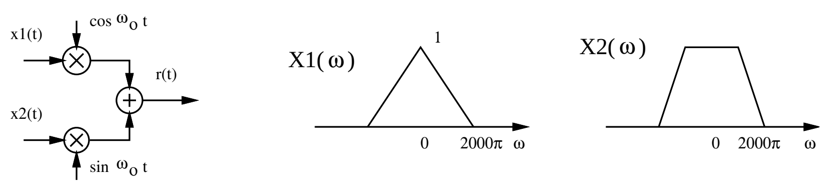
Two bandlimited real signals can be simultaneously transmitted in the same frequency band using quadrature modulation as shown in the figure below. Let $\omega_o = 2\pi 1MHz$.

a. Sketch real and imaginary parts of $R(\omega)$, the Fourier transform of $r(t)$.

b. In a digital receiver $r(t)$ is sampled at rate $3\omega_o$ giving $r[n]$. Sketch real and imaginary parts of $R(e^{j\omega})$, the Discrete Time Fourier transform of $r[n]$.

c. Draw a block diagram of a system which could be used to recover $x_1[n]$ and $x_2[n]$ from $r[n]$. Blocks can include ideal digital filters, sum, multiply by discrete sine or discrete cosine.

d. Verify the operation of your system by sketching the real and imaginary DTFT of the relevant signals in your block diagram.



4. (20 pts) LDE and DTFT (Lec 9, OW 5.8, 6)
Given a causal LTI system with input $x[n]$ and output $y[n]$ with frequency response

$$H(e^{j\omega}) = e^{-j\omega} \frac{1 - \frac{1}{2}e^{j\omega}}{1 - \frac{1}{2}e^{-j\omega}}$$

a. Show that $|H(e^{j\omega})|$ is 1 for all ω.
b. Find the phase $\angle H(e^{j\omega})$.
c. What is the output of this filter when input is $\cos(\frac{\pi n}{3})$?
d. Find the corresponding causal linear difference equation for this system, and determine minimum number of additions, multiplies, and delays required for implementation.

5. (15 pts) Bode Plots (Lec 10, OW 6)
a. A system has impulse response $h(t) = e^{-2t}u(t)$. Find $H_1(j\omega)$ and sketch magnitude and phase on linear axes.
b. Consider a DT system with unit sample response $h_2[n] = (1 - e^{-0.1})e^{-0.1n}u[n]$. Find the DTFT $H_2(e^{j\omega})$ and sketch magnitude and phase on linear axes.
c. Compare $H_1(j\omega)$ and $H_2(e^{j\omega})$. In particular, for what frequency range would the discrete filter $H_2(e^{j\omega})$ be a reasonable approximation to $H_1(j\omega)$?