Professor Fearing EECS120/Problem Set 5 v 1.02 Fall 2016

Due at 4 pm, Fri. Sep. 30 in HW box under stairs (1st floor Cory)

Reading: O&W Ch4, 5. Note: $\Pi(t) = u(t+\frac{1}{2}) - u(t-\frac{1}{2})$, and $comb(t) = \sum_{n=-\infty}^{\infty} \delta(t-n)$.

1. (20 pts) Fourier Series using Fourier Transform (Lec 6,7, OW 4.2, Arcak Lec 4)

a. Let $H(j\omega) = \mathcal{F}\{h(t)\}$. Given periodic $x(t) = \sum_k a_k e^{jk\omega_o t}$ with period T_o and that $x(t) = h(t) * \sum_{n=-\infty}^{\infty} \delta(t - nT_o)$. Find a_k in terms of $H(j\omega)$.

Calculate the Fourier Series for the following signals using Fourier Transform properties. That is, find the complex scaling coefficients a_k and fundamental frequency $\omega_o = \frac{2\pi}{T_o}$. Sketch the line spectrum ($|a_k|$ vs. ω) for each signal.

b. $\Pi(2t) * comb(\frac{t}{5})$ c. $\Pi(2t) * \delta(t-1) * comb(\frac{t}{5})$

d. $\Pi(2t) * \Pi(2t) * comb(\frac{t}{5})$ e. $\Pi(2t) * \Pi(t) * comb(\frac{t}{5})$

2. (20 pts) Filtering with Fourier Transform (Lec 6,7, OW 4.4, 4.6)

Consider an auditorium with an echo which can be modelled as an impulse response:

$$h(t) = \sum_{k=0}^{\infty} e^{-kT} \delta(t - kT)$$

where e^{-kT} represents the attentuation of the kth echo. Find $G(j\omega)$ that cancels the effects of the echos; that is for input $X(j\omega)$, $H(j\omega)G(j\omega) = 1$. (That is, $G(j\omega)$ cancels the effects of the echos).Find g(t) from $G(j\omega)$ and show that $g(t) * h(t) = \delta(t)$.

3. (20 pts) Fourier Transform Duality (Lec7, OW 4.3, 4.6)

a. Given $X(j\omega) = \mathcal{F}\{x(t)\}$, show the duality property $\mathcal{F}\{X(t)\} = 2\pi x(-\omega)$.

Use the duality property to find the following Fourier Transform pairs:

b. If $X_1(j\omega) = \delta(\omega - 2\pi) + \delta(\omega + 2\pi)$, find $\mathcal{F}\{X_1(t)\}$.

c. If $X_2(j\omega) = \frac{\sin W\omega}{\pi\omega}$, find $\mathcal{F}\{X_2(t)\}$.

d. If $X_3(j\omega) = e^{-2\omega}u(\omega)$, find $\mathcal{F}\{X_3(t)\}$.

e. If $X_4(j\omega) = e^{-|\omega|}$, find $\mathcal{F}\{X_4(t)\}$.

4. (20 pts) Fourier Transforms (Lec 6, 7 OW 4, 4.5)

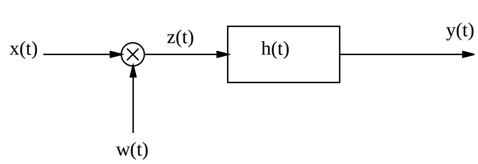
Given $x(t) = cos(100\pi t) + cos(400\pi t)$. Sketch $z(t), y(t)$ and the Fourier transforms $Z(j\omega), Y(j\omega)$ for the following, referring to the block diagram below. Sketch should label key heights and frequencies.

$$a. \quad w(t) = \Pi(10t) \quad h(t) = \delta(t) \tag{1}$$

$$b. \quad w(t) = \Pi(10t) \quad h(t) = \sum_{n=-\infty}^{\infty} \delta(t - n/5) \tag{2}$$

$$c. \quad w(t) = 1 \quad h(t) = \frac{\sin 200\pi t}{\pi t} \tag{3}$$

$$d. \quad w(t) = \cos(1100\pi t) \quad h(t) = \frac{\sin \cancel{1200}\underline{800}\pi t}{\pi t} \tag{4}$$



5. (20 pts) Discrete Time Fourier Transforms (Lec 7,8, OW 5, Arcak 6)

Compute the DTFT for the following signals:

a) $x[n] = \delta[n+2] + 2\delta[n+1] + 2\delta[n-1] + \delta[n-2]$

b) $x[n] = \sin(\frac{\pi}{4}n) + \cos(\frac{\pi}{8}n)$

Find the discrete time signal $h[n]$ for the LTI system which has frequency response for $0 \leq \omega \leq 2\pi$:

c) $H(e^{j\omega}) = \cos^2 \omega$

d) $H(e^{j\omega}) = 1$ for $0 \leq \omega \leq \pi/8$ and $= 1$ for $7\pi/8 \leq \omega \leq 2\pi$ and 0 else

e) $H(e^{j\omega}) = e^{-2j\omega} \cos(\omega)$