Due at 4 pm, Fri. Sep. 23 in HW box under stairs (1st floor Cory)

Reading: O&W Ch3, Ch4.

Note: $\Pi(t) = u(t+\frac{1}{2}) - u(t-\frac{1}{2})$, and $r(t) = tu(t)$ where $u(t)$ is the unit step, and $comb(t) = \sum_{n=-\infty}^{\infty} \delta(t-n)$.

1. (22 pts) LDE DT and Block Diag (Lec 3, OW 3.2, Arcak Lec 2)

An LTI system with scalar input $u[n]$ (here $u[]$ is not the unit step) and scalar output $y[n]$ is described by the state-space equations: $\mathbf{x}[n+1] = A\mathbf{x}[n] + Bu[n]$ and output equation $y[n] = C\mathbf{x}[n] + Du[n]$.

a. For each {A,B,C,D} below: draw the block diagram realization of this system using multiply by constant, summation blocks, and the minimum number of delay blocks.

b. Write the corresponding difference equation:

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{l=0}^{N} b_l u[n-l]$$

c. With initial conditions $y[n] = 0$ for $n < 0$, find the unit sample response <u>(that is, for $u[n] = \delta[n]$)</u> for $0 \le n \le 5$ (by hand is ok).

i.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1/2 & 1 & 1/2 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad C = [-1/2 \;\; 1 \;\; 1/2] \qquad D = [1]$$

ii.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad C = [1/2 \;\; -1 \;\; -1/2] \qquad D = [1]$$

2. (24 pts) Fourier Series and Fourier Transform (OW 3.3, 3.5, 4.4)

Calculate the Fourier Series for the following signals. That is, find the complex scaling coefficients a_k and fundamental frequency $\omega_o = \frac{2\pi}{T_o}$. Sketch the time function and line spectrum (a_k vs. $\omega = k\omega_o$) for each signal. Note that $\delta(at) = \frac{1}{a}\delta(t)$.

a. $x_1(t) = 2\Pi(t) * \Pi(t) * \frac{1}{2}comb(t/2)$

b. $x_2(t) = 4\Pi(t) * \Pi(t) * \frac{1}{4}comb(t/4)$

c. $x_3(t) = 8\Pi(t) * \Pi(t) * \frac{1}{8}comb(t/8)$

d. Compare the line spectra above to the Fourier transform $\mathcal{F}\{\Pi(t) * \Pi(t)\}$ (compare sketches).

3. (20 pts) Convolution and Fourier Transform (OW 3.8, 3.10, 4.2, 4.4)

This problem examines filtering of a periodic signal from time domain and frequency domain approaches. Given signal $x(t) = \Pi(t) * \frac{1}{8}comb(t/8)$, and LTI low pass filter with impulse response $h(t) = e^{-t}u(t)$.

a. Find $y_1(t) = x(t) * h(t)$ by convolution and include sketches.

b. Find the Fourier transforms $\mathcal{F}\{x(t)\} = X(j\omega), \mathcal{F}\{h(t)\} = H(j\omega)$, and $Y_2(j\omega) = H(j\omega)X(j\omega)$ using Fourier Transform properties and Tables 4.1 and 4.2 as appropriate.

c. Sketch the magnitude of the Fourier Transforms $X(j\omega), H(j\omega), Y_2(j\omega)$.

d. Explain why $\mathcal{F}\{y_1(t)\}$ and $Y_2(j\omega)$ are the same, considering the Fourier series for $y_1(t)$.

4. (18 pts) Fourier Transform (OW 4.4, 4.6, Lec 6)

For $x(t) = \frac{sin150\pi t}{\pi t}$ and $h(t) = \frac{sin200\pi t}{\pi t} - \frac{\sin 100\pi t}{\pi t}$,

find and sketch the Fourier Transform (real and imaginary parts) of:

a. $x(t)h(t)$ b. $x(t)cos(2\pi 10^2 t)$ c. $x(t) * h(t)$

d. $x^2(t)$ e. $x(t) * cos(2\pi 10^2 t)$ f. $[x(t)cos(2\pi 10^2 t)] * h(t)$

5. (16 pts) Fourier Transform (OW 4.3, 4.6, Lec 6)

a. Show that any signal $x(t)$ can be written as

$x(t) = x_e(t) + x_o(t)$

where $x_e(t)$ is even symmetric: $x_e(-t) = x_e(t)$, and $x_o(t)$ is odd symmetric: $x_o(-t) = -x_o(t)$.

b. Assuming $x(t)$ is real, show that

$\mathcal{F}\{x_e(t)\} = \mathrm{Re}\{X(j\omega)\}$ and $\mathcal{F}\{x_o(t)\} = j\mathrm{Im}\{X(j\omega)\}$.

c. Find $x_e(t)$ and $x_o(t)$ and calculate their Fourier tranforms for $x(t) = t\Pi(t - 1/2)$.