Professor Fearing EECS120/OWProblem Set 3 v 1.02 Fall 2016

Due at 4 pm, Fri. Sep. 16 in HW box under stairs (1st floor Cory)

Reading: O&W Ch2, Ch3.

Note: $\Pi(t) = u(t+\frac{1}{2}) - u(t-\frac{1}{2})$, and $r(t) = tu(t)$ where $u(t)$ is the unit step, and $comb(t) = \sum_{n=-\infty}^{\infty} \delta(t-n)$.

1. (24 pts) Graphical Convolution (OW 2.2)

Use graphical convolution method (flip-drag-integrate) with appropriate sketches, determine the output $y(t) = x(t) * h(t)$ for the following input and impulse response pairs:

a. $x(t) = \delta(t-\frac{1}{4})$ $h(t) = \cos(2\pi t)u(t)$

b. $x(t) = \delta(t-\frac{1}{4})$ $h(t) = \cos(3\pi t)u(t)$

c. $x(t) = \Pi(t-2)$ $h(t) = u(t-1)$

d. $x(t) = e^{-t}u(t)$ $h(t) = e^{-2t}u(t-2)$

2. (24 pts) CT Fourier Series (Lec 4, OW 3.3, Arcak Lec 3)

Calculate the Fourier Series for the following signals. That is, find the complex scaling coefficients a_k and fundamental frequency $\omega_o = \frac{2\pi}{T_o}$. Sketch the time function and line spectrum (a_k vs. $\omega = k\omega_o$) for each signal. Note that $\delta(at) = \frac{1}{a}\delta(t)$.

Updated to comb(t/4)

a. $x_1(t) = \Pi(t/2) * comb(t/4)$

b. $x_2(t) = \Pi(t/6) * comb(t/4)$

c. $x_3(t) = \Pi(t/2) * \Pi(t-1) * comb(t/4)$

3. (32 pts) FS, Parseval (Lec 4, OW 3.5)

In frequency doubling, a signal with fundamental frequency ω_o is passed through a nonlinear function to generate higher harmonics. (In optics, an IR light frequency could be doubled to generate green light.)

a. Find the Fourier Series coefficients $a_k = \frac{1}{T_o}\int_{T_o} x(t)e^{-jk\omega_o t}$ and sketch the line spectra for $x_1(t) = \cos(2\omega_o t)$ and $x_2(t) = |\cos(\omega_o t)|$.

b. Find the the time average power in $x_1(t)$ and $x_2(t)$.

c. For $x_2(t)$, neglecting the a_0 term, what fraction of the time average power is at the desired frequency $2\omega_o$?

d. Consider an LTI filter with transfer function $H(j\omega) = \frac{1}{1+j\frac{\omega}{2\omega_o}}$ with input $x_2(t)$ and output $y(t)$. Sketch the line spectra for $y(t)$. Neglecting the DC (y_0 term, what fraction of the time average power for $y(t)$ is at the desired frequency $2\omega_o$?

4. (20 pts) DT Fourier Series (Lec 4, OW 3.3, Arcak Lec 3)

Calculate the Discrete Time Fourier Series for the following periodic signals with period N. Find complex scaling coefficients a_k for each signal, and sketch .

a. $x[n] = \frac{1}{2}\delta[n+1] + \delta[n] + \frac{1}{2}\delta[n-1]$ with $N = 16$.

b. $x[n] = 1$ for $n = \{0, 1, 2, 3\}$ with $N = 8$.