Professor Fearing EECS120/Problem Set 2 v 1.02 Fall 2016

Due at 4 pm, Fri. Sep. 9 in HW box under stairs (1st floor Cory)

Reading: O&W Ch 1, Ch2.

Note: $\Pi(t) = u(t+\frac{1}{2}) - u(t-\frac{1}{2})$, and $r(t) = tu(t)$ where $u(t)$ is the unit step.

1. (10 pts) What are signals and systems? (Lec 1)

For each example below, identify the input x, system $H\{\}$, and output y. Which of (x, H, y) are known *a priori*, which would need to be calculated or designed? *Example: NASDAQ market manipulation. Answer: Input: buy and sell orders. System: security dealer network plus investor behavior plus stock order book, e.g. limit orders. Output: share price. System known, output directly measured, design input to control price.*

a) atomic force microscope b) a heart pacemaker

c) a prosthetic arm reaching for a cup d) a voice synthesizer

2. (12 pts) algebra review

For $f(t) = [u(t+1) - r(t+1) + r(t-1)]u(3-t)$, sketch:

a) $f(t)$ b) $f(\frac{t}{2})$

c) $f(\frac{3}{2}t+1)$ d) $f(-\frac{3}{2}t+1)$

e) $f(t) - f(-t)$ f) $f(t) + f(-t)$

3. (20 pts) DT and CT Convolution (Lec 2, OW 2.1, 2.2, Arcak Lec 1)

Consider a DT or CT linear time invariant operator $H\{\}$.

Assume that in discrete time $H\{\delta[n]\} = h[n]$. In continuous time, $H\{\delta(t)\} = h(t)$.
Given $x[n] = \delta[n] + \frac{1}{2}\delta[n-1] + \frac{1}{4}\delta[n-2]$, let $y[n] = H\{x[n]\}$.
Also, $x(t) = \delta(t) + \frac{1}{2}\delta(t-1) + \frac{1}{4}\delta(t-2)$, and let $y(t) = H\{x(t)\}$.

a. Using LTI properties directly of $H\{x[n]\}$, find $y[n]$ in terms of $h[n]$.

b. Explicitly using $x_1[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$, and LTI properties, find $H\{x_1[n]\}$.

c. Using LTI properties directly of $H\{x(t)\}$, find $y(t)$ in terms of $h(t)$.

d. Explicitly using $x_1(t) = \int_{\tau=-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$ and LTI properties, find $y_1(t) = H\{x_1(t)\}$.

e. Repeat part d. above, but now use $h(t) = e^{-t}u(t)$, and sketch $y_1(t)$.

4. (18 pts) BIBO Stability (Lec 2, OW 2.3, Arcak Lec 1)

For each of the following impulse responses, determine whether the system is BIBO stable. If the system is not BIBO stable, find a bounded input $x(t)$ or $x[n]$ which gives an unbounded output, and show that the output is unbounded for this input. (Here $u(t)$ is unit step.)

a. $h(t) = \sum_{n=-\infty}^{\infty} \Pi(t-6n)$ b. $h(t) = u(t-1)$

c. $h(t) = t^{-1}u(t-1)$ d. $h[n] = e^{-n}u[n+4]$

e. $h(t) = \sin(2\pi t)u(t)$ f. $h[n] = \sum_{k=-\infty}^{\infty} \delta[n-2k]$

5. (20 pts) LDE CT and Block Diag (Lec 3, OW 3.2, Arcak Lec 2)

An LTI system with input $x(t)$ and output $y(t)$ is described by the LDE:

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + 101y(t) = 100\frac{dx(t)}{dt} \tag{1}$$

a. Draw a block diagram realization of this system using multiply by constant, summation blocks, and the minimum number of integration blocks.
b. What is the steady state response of the system (i.e. the response after all transients have died away) described by the LDE in eq.(1) to a complex exponential input $x(t) = e^{j\omega t}$? (Hint: $e^{j\omega t}$ is an eigenfunction, and thus you can assume $y(t) = He^{j\omega t}$ where H is a complex constant for a given ω.)
c. What are the eigenvalues (determined from H)? Is the system B.I.B.O. stable?

6. (20 pts) LDE DT and Block Diag (Lec 3, OW 3.2, Arcak Lec 2)

An LTI system with input $u[n]$ (here $u[]$ is not the unit step) and output $y[n]$ is described by the LDE:

$$y[n] - \frac{9}{16}y[n-2] = u[n] + 10u[n-1] + 25u[n-2] \tag{2}$$

a. Draw a block diagram realization of this system using multiply by constant, summation blocks, and the minimum number of delay blocks.
b. What is the steady state response of the system (i.e. the response after all transients have died away) described by the LDE in eq.(2) to a complex exponential input $u[n] = e^{j\omega n}$? (Hint: $e^{j\omega n}$ is an eigenfunction and thus you can assume $y[n] = He^{j\omega n}$ where H is a complex constant for a given ω.)
c. Re-write the linear difference equation in state space form:
$\mathbf{x}[n+1] = A\mathbf{x}[n] + \cancel{B\mathbf{u}[n]}\ \underline{Bu[n]}$ and output equation $y[n] = C\mathbf{x}[n] + Du[n]$, specifying the 4 matrices A, B, C, D. <u>Note that A is 3x3, $\mathbf{x}$ is 3x1, u is scalar, C is 1x3, and D is 1x1.</u>
~~Note $\mathbf{u}[n] = [u[n-2]\ \ u[n-1]\ \ u[n]]^T$.~~

Also, use A of the form:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ ? & ? & ? \end{bmatrix}$$

where the botton row needs to be determined. (This is controller canonical form).