Professor Fearing EECS120/Problem Set 1 v 1.03 Fall 2016

Due at 4 pm, Fri. Sep. 2 in HW box under stairs (1st floor Cory) (Up to 2 people may turn in a single homework writeup with both names listed.)

Reading: EE16AB notes. This problem set should be review of material from EE16AB. (Please note, vector notation here is $\mathbf{x} = \vec{x}$, u is a scalar, and A is a matrix.)

1. (18 pts) Complex review.

Given $z = x + jy = re^{j\theta}$. Derive the following relations:

a. $zz^* = r^2$ b. $\frac{z}{z^*} = e^{j2\theta}$

c. $(z_1 z_2)^* = z_1^* z_2^*$ d. $(\frac{z_1}{z_2})^* = \frac{z_1^*}{z_2^*}$

e. Show that $\frac{1}{2}(e^{j\omega t} + e^{-j\omega t})^2 - 1 = \cos 2\omega t$

f. Find all integer $k \in \{0, 1, ...15\}$ for which $\sum_{n=0}^{15} e^{j2\pi nk/16} = 0$.

2. (24 pts) Phasors and Operational Amplifiers

Consider the circuit in Fig. 2. Use the "golden rules" (ideal) ideal op amp assumptions.

a. Using phasors, determine the transfer function $H = \frac{V_o}{V_i}$.

b. Sketch the magnitude Bode plot (log-log scale) for $H(\omega)$.

c. What filter function is performed (low pass, high pass, etc.)?

3. (18 pts) State space

For the circuit shown in Fig. 1, let the state variables x_1 be $v_c(t)$, and x_2 be the inductor current. The input $u = v_i(t)$ and output $y(t)$ is the voltage across the 1 $M\Omega$ resistor. Let $C = 10^{-6}F$ and $R_2 = 10M\Omega$.

a. Write the differential equation for the circuit in state space form (find A,B,C,D):

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu \qquad y = C\mathbf{x} + Du$$

b. Determine the eigenvalues ~~and eigenvectors~~ for A. (MatLAB or python ok). Is the system stable?

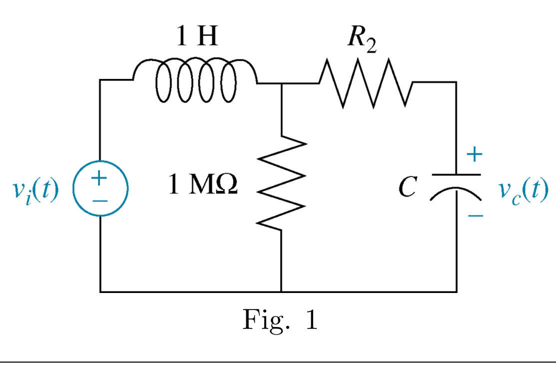

Fig. 1

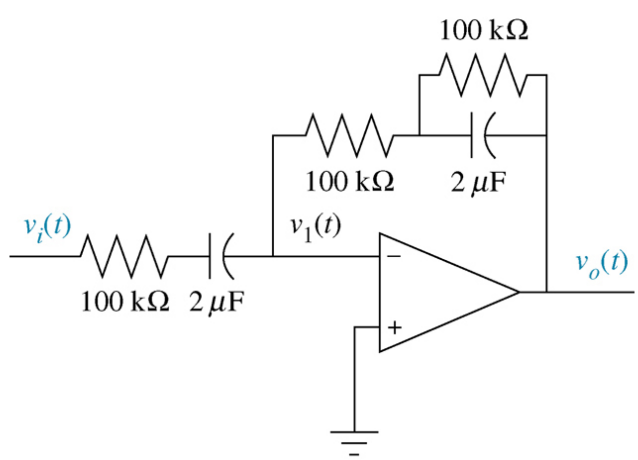

Fig. 2

4. (20 pts) LDE solutions

Consider $\dot{\mathbf{x}} = A\mathbf{x}$ where initial condition $\mathbf{x}_o = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$ and $A = \begin{bmatrix} 0 & 1 \\ -10 & -11 \end{bmatrix}$

a. Show that the general solution is $\mathbf{x}(t) = c_1 e^{\lambda_1 t}\mathbf{v}_1 + c_2 e^{\lambda_2 t}\mathbf{v}_2$ where λ_1, λ_2 are eigenvalues of A with corresponding eigenvectors $\mathbf{v}_1, \mathbf{v}_2$, and find eigenvalues and eigenvectors.
b. Plot each component of the solution and the phase portrait <u>for the given initial condition</u>. The phase portrait is a 2 dimensional plot of $x_1(t)$ vs $x_2(t)$. (Hand sketch is fine.)

5. (20 pts) DFT basics

The Discrete Time Fourier Series (DTFS) is defined as

$$a[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{\frac{-j2\pi nk}{N}} = \frac{1}{N}\sum_{n=0}^{N-1} x[n]W_N^{nk}$$

where $W_N \equiv e^{-j2\pi/N}$, and $k \in \{0, 1, ..., N-1\}$.
a. Show that the DTFS can be written as $\mathbf{a} = U\mathbf{x}$ for $N = 4$, and write U in terms of W_4.
b. What special property or properties do the columns of U have?
c. For $N = 4$, and $x[n] = \cos\frac{\pi n}{2}$, find $\mathbf{a}$ in terms of W_4 (simplify).
d. For $N = 8$, and $\mathbf{x} = [0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ ]^T$, find $\mathbf{a}$ in terms of W_8 (simplify).