Example showing going from desired DTFT $H(e^{j\omega})$ to linear difference equation to block diagram (using delay, scale, sum), and then to state space form.

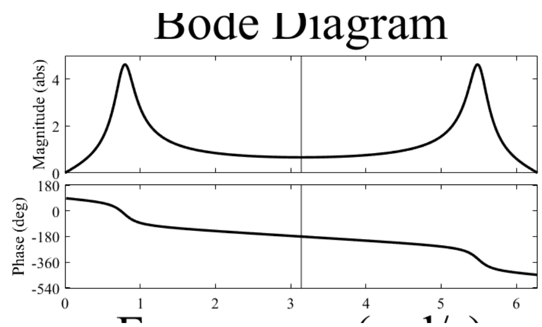


Fig. 1. Magnitude and phase response for filter $H(e^{j\omega})$.

Consider a DT LTI filter with input $u[n]$ (not unit step) and output $y(n)$. Given the DTFT of the desired filter function:

$$\frac{Y(e^{j\omega})}{U(e^{j\omega})} = H(e^{j\omega}) = \frac{e^{j\omega} - 1}{e^{2j\omega} - (7\sqrt{2}/8)e^{j\omega} + (7/8)^2}. \tag{1}$$

We know there is a corresponding linear difference equation with the same transfer function.

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{l=0}^{N} b_l u[n-l] \tag{2}$$

Using $u[n] = e^{j\omega n}$ as the eigenfunction of eq.(2), and thus $y[n] = H(e^{j\omega})e^{j\omega n}$, and hence $y[n-k] = H(e^{j\omega})e^{-j\omega k}e^{j\omega n}$.

Eq.(2) becomes

$$\sum_{k=0}^{N} a_k H(e^{j\omega})e^{-j\omega k}e^{j\omega n} = \sum_{l=0}^{N} b_l e^{-j\omega l}e^{j\omega n} \tag{3}$$

Thus the general Discrete Time Fourier Transform from eq.( ) is:

$$H(e^{j\omega}) = \frac{\sum_{l=0}^{N} b_l e^{-j\omega l}}{\sum_{k=0}^{N} a_k e^{-j\omega k}} \tag{4}$$

Multiplying out eq.(1:

$$Y(e^{j\omega})(e^{2j\omega} - (7\sqrt{2}/8)e^{j\omega} + (7/8)^2) = U(e^{j\omega})(e^{j\omega} - 1) \tag{5}$$

We can write the difference equation from eq.(1) noting that multiply by $e^{j\omega}$ corresponds to a “delay” of -1. Thus:

$$y[n+2] - (7\sqrt{2}/8)y[n+1] + (7/8)^2 y[n] = u[n+1] - u[n] \tag{6}$$

Use the properties of time invariance to express the output $y[n]$ in terms of previous values:

$$y[n] = (\frac{7}{8}\sqrt{2})y[n-1] - (\frac{7}{8})^2 y[n-2] + u[n-1] - u[n-2] \tag{7}$$

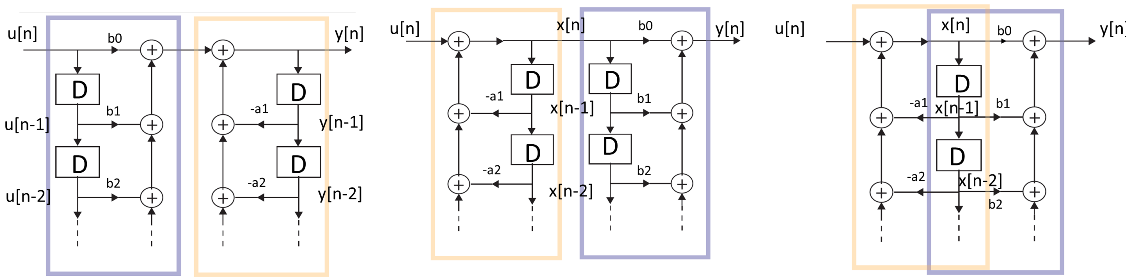


Figure 1: Equivalent block diagrams for LDE, reducing to minimal states.

From the block diagram, the state equations can be read off.

$$\mathbf{x}[n+1] = \begin{bmatrix} x[n-1] \\ x[n] \end{bmatrix} = A\mathbf{x}[n] + Bu[n] = \begin{bmatrix} 0 & 1 \\ -a_2 & -a_1 \end{bmatrix} \begin{bmatrix} x[n-2] \\ x[n-1] \end{bmatrix} \quad + \quad \begin{bmatrix} 0 \\ 1 \end{bmatrix} u[n] \quad (8)$$

Similarly, the output equations can be read off:

$$y[n] = C\mathbf{x}[n] + Du[n] = [b_2 - b_0 a_2 \quad b_1 - b_0 a_1] \begin{bmatrix} x[n-2] \\ x[n-1] \end{bmatrix} \quad + \quad b_0 u[n] \quad (9)$$

Substituting in parameters $a_1 = -\frac{7}{8}\sqrt{2},\ a_2 = (\frac{7}{8})^2,\ b_0 = 0,\ b_1 = 1,\ b_2 = -1$ we get:

$$\mathbf{x}[n+1] = \begin{bmatrix} 0 & 1 \\ -(\frac{7}{8})^2 & \frac{7}{8}\sqrt{2} \end{bmatrix} \begin{bmatrix} x[n-2] \\ x[n-1] \end{bmatrix} \quad + \quad \begin{bmatrix} 0 \\ 1 \end{bmatrix} u[n] \quad (10)$$

and

$$y[n] = [-1 \quad 1] \begin{bmatrix} x[n-2] \\ x[n-1] \end{bmatrix} \quad (11)$$